

# Internal Force-based Coordinated Motion Control of Dual Redundant Manipulator

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**Abstracts** Internal Force based control of dual redundant manipulator is proposed. One is resolved acceleration type control in the decoupled joint space which includes null motion space and the other is in the impedance control fashion in which the desired impedances are decoupled in three subspace, internal motion controlled space, orthogonal to that space, and the null motion controlled space. The internal force is formulated with its basis set meaningful. The object dynamics is also briefly evolved beforehand.

**Keywords** Dual Manipulator, Internal Force, Redundant Manipulator, Impedance Control

## 1. INTRODUCTION

In this article the dynamic equation describing two cooperating redundant robot arms which simultaneously working on the same object is established by considering the whole system as a closed kinematic chain. The basic objective of two cooperating robot arms is to design a control system which is able to command both arms in such a way that the two arms operate in kinematically and dynamically coordinated fashion generating required forces for the manipulation of the object.

The coordination, load distribution between the robot arms through the dynamic equation, the resultant force generated by external forces or the environmental constraints and the adequate control schemes based on internal forces are considered in the following sections. The internal force control mechanism keeps the internal forces on the object being manipulated at a desired value.

Various controllers for cooperating robots have been proposed during recent years. They may be generally classified as position/force control [Hayati, Uchiyama, and Goldenberg, etc.] or impedance control [Bonitz, Hsu, Kosuge, and Cannon, etc.].

In this paper two control schemes is implemented with the redundant manipulators in the decoupled joint space. One is resolved acceleration type control in the decoupled joint space and the other is in the impedance control fashion in which the desired impedances are decoupled in three subspace, internal motion controlled space, orthogonal to that space, and the null motion controlled space. In both case, the null motion is controlled for redundant manipulators for the secondary task, though which is manipulability maximization here, it could be a collision free motion or overcoming hardware limitations.

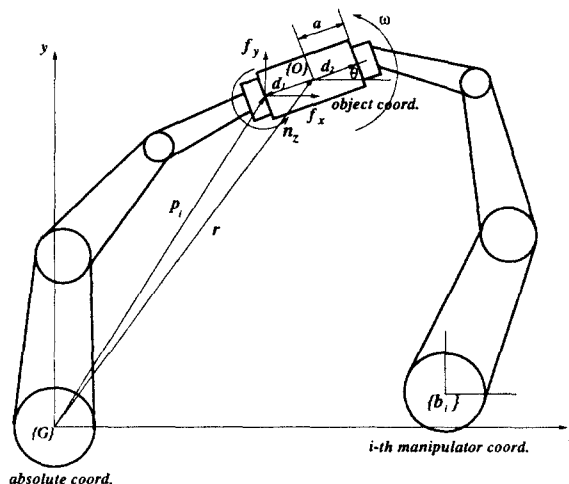


Figure 1: Coordinate Systems for a Dual Manipulator

## 2. DYNAMIC MODELING

In dual manipulation system as seen in Fig. 1 where two robot arms handles an objects with their ends, a closed chain is formed by two robot arms and the object through the ground. To describe the dynamic behavior of the whole dynamic system, we propose to establish equations of motion by considering the system as a closed kinematic chain. By selecting the number of generalized coordinates equal to the degrees of the freedom of the closed chain dynamic of two robot arms is obtained.

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## 2.1. Basic Equations

The dynamic equations for each chain of a dual(multiple)-chain robot system are:

$$\tau_i = \mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{h}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{J}_i^T(\mathbf{q}_i)\mathbf{f}_i, \quad (1)$$

$$i = 1, 2, \dots, N$$

where

$\tau$	input joint torque/force vector, [ $n \times 1$ ],
$\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$	joint displacement, velocity, acceleration vectors of joints, each [ $n \times 1$ ],
$\mathbf{M}_i$	inertia matrix, [ $n \times N$ ],
$\mathbf{h}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$	vector defining nonlinear torques including centrifugal, Coriolis, frictional and gravitational effects, [ $n \times 1$ ]
$\mathbf{J}_i^T(\mathbf{q}_i)$	transpose of Jacobian matrix, [ $n \times m$ ]
$n$	number of degrees of freedom for each manipulator, dimension of joint space,
$m$	dimension of task space,
$N$	number of manipulators.

For kinematically redundant manipulator,  $n > m$  and  $r = n - m$  is called the degree of redundancy.

These  $N$ -dynamic equations are coupled through the terms,  $\mathbf{J}_i^T(\mathbf{q}_i)\mathbf{f}_i$ ; where all the  $\mathbf{f}_i$  terms constitute the required contact force and moment components to give the desired motion specified on the object and the internal force for object grasping.

If we might determine optimal solutions for the contact force/moment vectors,  $\mathbf{f}_i (i = 1, 2, \dots, N)$ , the term  $\mathbf{J}_i^T(\mathbf{q}_i)\mathbf{f}_i$  in Eq. (1) are fixed and known; then the dynamics of each chain are decoupled with these constrained forces and moments.

Motion equations of an object manipulated by robot arms are expressed as follows:

$$\mathbf{I}_o\ddot{\boldsymbol{\phi}} + \mathbf{Q}_o = \mathbf{W}\mathbf{F} \quad (2)$$

where

$$\mathbf{I}_o = \begin{bmatrix} m_o\mathbf{E}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix},$$

$$\ddot{\boldsymbol{\phi}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix},$$

$$\mathbf{Q}_o = \begin{bmatrix} -m_o\mathbf{g} \\ \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega}) \end{bmatrix},$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0} & \mathbf{E}_3 & \mathbf{0} \\ \mathbf{d}_1 \times & \mathbf{E}_3 & \mathbf{d}_2 \times & \mathbf{E}_3 \\ \dots & \mathbf{E}_3 & \mathbf{0} & \\ & \mathbf{d}_3 \times & \mathbf{E}_3 & \end{bmatrix} \in \mathbb{R}^{6 \times 6n},$$

$$\mathbf{d}_i \times = \begin{bmatrix} 0 & -d_{i3} & d_{i2} \\ d_{i3} & 0 & -d_{i1} \\ -d_{i2} & d_{i1} & 0 \end{bmatrix} \in \mathbb{R}^{3 \times 3},$$

$$\mathbf{F} = [\mathbf{f}_1^T \mathbf{n}_1^T \mathbf{f}_2^T \mathbf{n}_2^T \dots \mathbf{f}_n^T \mathbf{n}_n^T]^T \in \mathbb{R}^{6n} \text{ in general.}$$

In order to simplify the problem, we assume the following assumptions on the contact condition:

- (i) Each arm holds an object firmly and exerts both force and moment on the object. That is, a contact point does not move on the object surface unless the manipulator parts from the object or slips on the object.
- (ii) In palm-type grasping or others, the forces and moments exerted to the object can be considered to be applied at a point of action.

Letting  $\mathbf{L} = \mathbf{I}_o\ddot{\boldsymbol{\phi}} + \mathbf{Q}_o$ , for a given motion of the object we can solve the above equation,  $\mathbf{L} = \mathbf{W}\mathbf{F}$  to get  $\mathbf{F}$ :

$$\mathbf{F} = \mathbf{W}^+\mathbf{L} + \mathbf{F}_{int} \quad (3)$$

where  $\mathbf{W}^+$  is a generalized inverse of  $\mathbf{W}$ . The second part of Eq. (3) corresponds to the internal force.  $\mathbf{F}_{int}$  can be any vector in the null of  $\mathbf{W}$  of dimension  $6n$ . Although in some literatures this force is represented as  $(\mathbf{E}_{6n} - \mathbf{W}^+\mathbf{W})\boldsymbol{\xi}$ , it also can be described by the require force space with the proper choice of their basis set as will be done in (5).

## 2.2. Internal Forces

In general, the solution of Eq. (3) is not determined uniquely. It depends on how the load is distributed on among the robot arms. The contact points between the object and the manipulators are also carefully determined to generate arbitrary object motion. Some proposed solutions are presented in [4] and [3].

The internal force component of the applied force,  $\mathbf{F}_{int}$ , produces compression or tensile forces in the object. The interaction force between the object and the manipulator is expressed as the force projected into the line connecting the two contact points.

In addition it is desirable to represent  $\mathbf{F}_{int}$  by  $\mathbf{F}_{int} = \mathbf{B}\mathbf{V}_I$  where  $\mathbf{V}_I$  is the component of  $\mathbf{F}_{int}$  with respect to the basis set  $\mathbf{B}$ . A convenient choice of such basis is suggested in [3].

For example a planar rigid body is formulated as follows:

$$\mathbf{I}_o\ddot{\boldsymbol{\phi}} + \mathbf{Q}_o = \mathbf{W}\mathbf{F}, \quad (4)$$

$$\mathbf{F}_{int} = \mathbf{B}\mathbf{V}_I \quad (5)$$

where

$$\mathbf{I}_o = \begin{bmatrix} m_o & 0 & 0 \\ 0 & m_o & 0 \\ 0 & 0 & I_o \end{bmatrix},$$

$$\ddot{\boldsymbol{\phi}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix},$$

$$\mathbf{Q}_o = \mathbf{0} \text{ when gravity is ignored,}$$

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \text{asin}(\theta) & -\text{acos}(\theta) & 1 \\ & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & -\text{asin}(\theta) & \text{acos}(\theta) & 1 \end{bmatrix},$$

$$\mathbf{F} = [f_{1x} \ f_{1y} \ n_{1z} \ f_{2x} \ f_{2y} \ n_{2z}]^T,$$

$$\mathbf{B} = \begin{bmatrix} \text{cos}(\theta) & -\text{sin}(\theta) & 0 \\ \text{sin}(\theta) & \text{cos}(\theta) & 0 \\ 0 & a & 1 \\ -\text{cos}(\theta) & \text{sin}(\theta) & 0 \\ -\text{sin}(\theta) & -\text{cos}(\theta) & 0 \\ 0 & a & -1 \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix},$$

$$\mathbf{V}_I = [V_{I_x} \ V_{I_y} \ V_{I_\theta}]^T.$$

In the above equation,  $V_I$  consists of a compressing force,  $V_{Ix}$ , a shearing force,  $V_{Iy}$  and bending moment,  $V_{I\theta}$ . Note that  $B$  matrix is not unique. Performing elementary column operation of  $B$  can be another choice of  $B$ .

### 2.3. Complete System

The following constraint equation among the velocities of the manipulator and the object

$$\dot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{J}_o\dot{\phi}. \quad (6)$$

Desired object motion should be transformed to the manipulator coordinate with above relationship and reparametrized so that  $\dot{\mathbf{q}}$  and  $\ddot{\mathbf{q}}$  could be represented with respect to the coordinate composed of  $(\mathbf{p}^T \ \mathbf{n}^T)^T$  which expresses the net motion and null motion of the manipulator[7].

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{R}(\mathbf{J}\mathbf{R})^{-1}\dot{\mathbf{p}} + \mathbf{N}(\mathbf{Z}\mathbf{N})^{-1}\dot{\mathbf{n}} \\ \ddot{\mathbf{q}} &= \mathbf{R}(\mathbf{J}\mathbf{R})^{-1}(\ddot{\mathbf{p}} - \dot{\mathbf{J}}\dot{\mathbf{q}}) + \mathbf{N}(\mathbf{Z}\mathbf{N})^{-1}(\ddot{\mathbf{n}} - \dot{\mathbf{Z}}\dot{\mathbf{q}}) \end{aligned} \quad (7)$$

where  $\mathbf{R} \in \mathbb{R}^{n \times m}$  and  $\mathbf{N} \in \mathbb{R}^{n \times (n-m)}$  constitute  $\mathbf{V} = [\mathbf{R}|\mathbf{N}]$  of  $\mathbf{J} = \mathbf{U}\Sigma\mathbf{V}^T$  and  $\mathbf{Z}$  is the null basis matrix of  $\mathbf{J}$  or  $\mathbf{J}^T\mathbf{J}$  represented by

$$\mathbf{Z} = [\mathbf{J}_{n-m}^T(\text{adj}(\mathbf{J}_m))^T | -\det(\mathbf{J}_m)\mathbf{I}_{n-m}]. \quad (9)$$

Differentiating Eq. (6) with respect of time we get

$$\ddot{\mathbf{p}} = \mathbf{J}_o\ddot{\phi} + \dot{\mathbf{J}}_o\dot{\phi}. \quad (10)$$

The equation describing complete system is obtained by combining (1), (3), (5), (8) and (10):

$$\begin{aligned} \boldsymbol{\tau} &= \mathbf{M}\{\mathbf{R}(\mathbf{J}\mathbf{R})^{-1}(\mathbf{J}_o\ddot{\phi} + \dot{\mathbf{J}}_o\dot{\phi} - \dot{\mathbf{J}}\dot{\mathbf{q}}) \\ &\quad + \mathbf{N}(\mathbf{Z}\mathbf{N})^{-1}(\ddot{\mathbf{n}} - \dot{\mathbf{Z}}\dot{\mathbf{q}})\} + \mathbf{h} \\ &\quad + \mathbf{J}^T\mathbf{W}^+\{I_o\ddot{\phi} + \mathbf{Q}_o\} + \mathbf{J}^T\{\mathbf{B}\mathbf{V}_I\}. \end{aligned} \quad (11)$$

Note that the null motion of manipulator as well as the cartesian output motion is depicted as  $\dot{\mathbf{n}}, \ddot{\mathbf{n}}$  where  $\dot{\mathbf{n}} = \mathbf{Z}\dot{\mathbf{q}}$ .

## 3. COORDINATED CONTROL

### 3.1. Control Objective

The control objective of coordinated motion controller for two robot arms manipulating a rigid object listed as follows

- How to hold an object by robot arms,
- How to control the trajectory of the object, and
- How to control the internal force applied to the object.

### 3.2. Internal Force and Motion Control in the Decomposed Joint Space

Let the desired trajectory of the object be given by  $\phi, \dot{\phi}, \ddot{\phi} \in \mathbb{R}^6$ . Null motion is specified so as to maximize the manipulability and the internal force is represented with respect to the basis set that spans the null space of  $\mathbf{W}$ . Then the required joint torque is supplied to the manipulator as shown below:

$$\begin{aligned} \boldsymbol{\tau} &= \mathbf{J}^T\mathbf{B}[\mathbf{V}_{Id} + \mathbf{K}_{fp}(\mathbf{V}_{Id} - \mathbf{V}_I) \\ &\quad + \mathbf{K}_{fi} \int (\mathbf{V}_{Id} - \mathbf{V}_I) dt] \\ &\quad + \hat{\mathbf{M}}\{\mathbf{R}(\mathbf{J}\mathbf{R})^{-1}(\mathbf{J}_o\ddot{\phi}_d + \mathbf{K}_d\dot{\phi} + \mathbf{K}_p\phi \end{aligned} \quad (12)$$

$$\begin{aligned} &+ \mathbf{K}_i \int e dt + \dot{\mathbf{J}}_o\dot{\phi} - \dot{\mathbf{J}}\dot{\mathbf{q}}) \\ &+ \mathbf{N}(\mathbf{Z}\mathbf{N})^{-1}(\dot{\mathbf{n}}_d + \mathbf{K}_{null}\dot{\mathbf{e}}_n \\ &+ \mathbf{K}_{null,i} \int \dot{\mathbf{e}}_n dt - \dot{\mathbf{Z}}\dot{\mathbf{q}})\} \\ &+ \hat{\mathbf{h}} + \mathbf{J}^T\mathbf{W}^+\{I_o\ddot{\phi}_d + \mathbf{Q}_o\} \end{aligned}$$

where

$$\begin{aligned} \mathbf{e} &= \phi_d - \phi, \quad \dot{\mathbf{e}}_n = \dot{\mathbf{n}}_d - \dot{\mathbf{n}}, \\ \dot{\mathbf{n}}_d &= \kappa\mathbf{Z}\nabla m, \quad \ddot{\mathbf{n}}_d = \kappa\dot{\mathbf{Z}}\nabla m + \kappa\mathbf{Z}\mathcal{H}m, \\ \mathbf{V}_i &= \mathbf{B}^T\mathbf{F}_{ext}. \end{aligned}$$

The desired null motion,  $\dot{\mathbf{n}}_d$  is generated so as to maximize the manipulability measure,  $m$  as  $\dot{\mathbf{n}}_d = \kappa\mathbf{Z}\nabla m$ . The first component in (12) is the internal force control term, the second is the force component acting on the manipulators themselves and the last is the force component acting on the object, *i.e.* the actual load carrying force.

Internal force,  $V_i$  is minimally parametrized such as a normal force, a shearing force and bending moment in planar case. The external force,  $F_{ext}$  which is the reaction force exerted by the object is modeled by the local deformation of the object due to contact as

$$\mathbf{F}_{ext} = K_{stiff}\delta\mathbf{x} \quad (13)$$

where  $\delta\mathbf{x}$  is a deformation normal to the contact surface.

In simulation, a planar 3-DOF manipulator with link length of 0.3m, 0.25m, and 0.2m and mass of 20Kg, 10Kg, and 10Kg is used. All links are assumed as bar-type ones. The mass of object is 2Kg. The simulation results with  $K_p = 140$ ,  $K_d = 70$ ,  $K_i = 50$ ,  $K_{fp} = 5$ ,  $K_{fi} = 30$ ,  $K_{null} = 100$ ,  $K_{null,i} = 10$ , and gradient gain,  $\kappa = 100$  is shown below Fig. 2.

### 3.3. Impedance Control Fashion

Improving the impedance controller, we can decouple the desired impedances in the internal force controlled space, the orthogonal space to the internal force controlled space, and null motion space:

$$\mathbf{f}_{Id} - \mathbf{f}_I = \mathbf{M}_I\dot{\mathbf{e}}_I + \mathbf{B}_I\dot{\mathbf{e}}_I + \mathbf{K}_I\mathbf{e}_I \quad (14)$$

$$\mathbf{f}_O = \mathbf{M}_O\dot{\mathbf{e}}_O + \mathbf{B}_O\dot{\mathbf{e}}_O + \mathbf{K}_O\mathbf{e}_O \quad (15)$$

$$\mathbf{f}_{null} = \mathbf{M}_{null}\dot{\mathbf{e}}_{null} + \mathbf{B}_{null}\dot{\mathbf{e}}_{null} \quad (16)$$

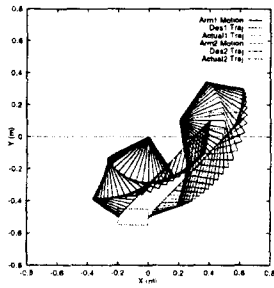
where  $\mathbf{e}_I = \mathbf{B}^T(\mathbf{p}_d - \mathbf{p})$ ,  $\mathbf{f}_O = (\mathbf{B}^\perp)^T\mathbf{f}_{ext}$ ,  $\mathbf{f}_{null} = \mathbf{0}$ ,  $\mathbf{e}_O = (\mathbf{B}^\perp)^T(\mathbf{p}_d - \mathbf{p})$  and  $\dot{\mathbf{e}}_{null} = \dot{\mathbf{n}}_d - \dot{\mathbf{n}}$ . Here  $\mathbf{B}^\perp$  can be any space in the range space of  $\mathbf{W}$ . The desired null motion,  $\dot{\mathbf{n}}_d$  is generated so as to maximize the manipulability measure,  $m$  as  $\dot{\mathbf{n}}_d = \kappa\mathbf{Z}\nabla m$ .

Since internal force is used in the impedance relationship, the object dynamics do not contribute to tracking and steady-state position errors.

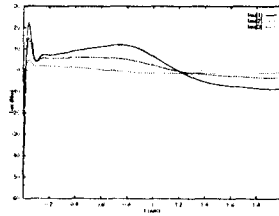
For desired impedances the following control law is used,

$$\begin{aligned} \boldsymbol{\tau} &= \hat{\mathbf{M}}\{\mathbf{R}(\mathbf{J}\mathbf{R})^{-1}\mathbf{B}[\mathbf{B}^T\ddot{\mathbf{p}}_d \\ &\quad + \mathbf{M}_I^{-1}(\mathbf{B}_I\dot{\mathbf{e}}_I + \mathbf{K}_I\mathbf{e}_I - (\mathbf{f}_{Id} - \mathbf{f}_I))] \\ &\quad + \mathbf{R}(\mathbf{J}\mathbf{R})^{-1}\mathbf{B}^\perp\{(\mathbf{B}^\perp)^T\ddot{\mathbf{p}}_d \\ &\quad + \mathbf{M}_O^{-1}(\mathbf{B}_O\dot{\mathbf{e}}_O + \mathbf{K}_O\mathbf{e}_O)\} \\ &\quad + \mathbf{N}(\mathbf{Z}\mathbf{N})^{-1}[\dot{\mathbf{n}}_d + \mathbf{M}_{null}^{-1}\mathbf{B}_{null}\dot{\mathbf{e}}_{null}]\} \\ &\quad + \hat{\mathbf{h}} + \mathbf{J}^T\mathbf{f}_{ext} \end{aligned} \quad (17)$$

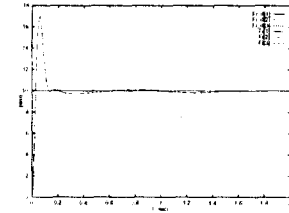
The simulation results with  $M_I = 1$ ,  $B_I = 200$ ,  $K_I = 150$ ,  $M_O = 10$ ,  $B_O = 160$ ,  $K_O = 200$ ,  $M_{null} = 1$ ,  $B_{null} = 100$  and gradient gain,  $\kappa = 100$  is shown below Fig. 3.



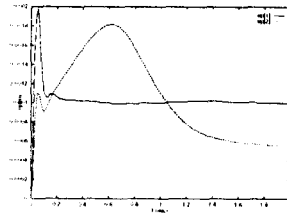
(a) Joint Configurations



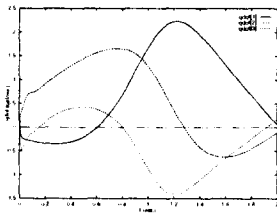
(b) Joint Torque



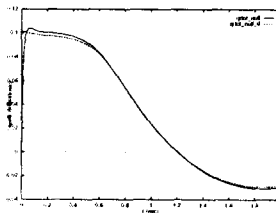
(c) Internal Force  
(Compression Force Only)



(d) Object Deformation &  
position error



(e) Joint Velocity



(f) Joint Null velocity

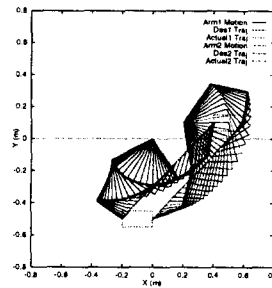
Figure 2: RAC-type Controller in the Joint Decomposed Space

#### 4. CONCLUSION

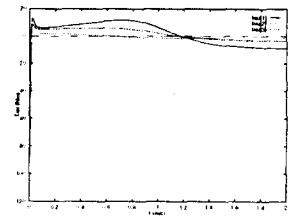
To control the object motion and the internal force interacting between manipulator and object two control methods are proposed in the decomposed joint space. Furthermore the desired null motion is controlled in accordance with given secondary task. All control variables representing the output motion, the null motion and the internal force are minimally parametrized in their space. The proposed control scheme was verified via numerical simulations.

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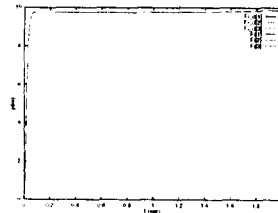
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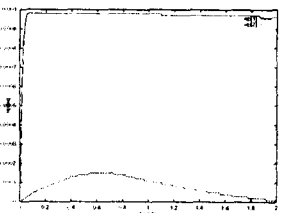
(a) Joint Configurations



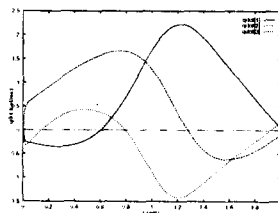
(b) Joint Torque



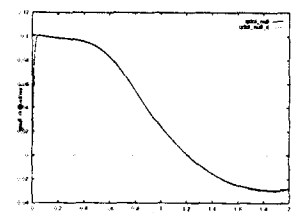
(c) Internal Force  
(Compression Force Only)



(d) Object Deformation &  
position error



(e) Joint Velocity



(f) Joint Null velocity

Figure 3: Impedance Control of Motion and Internal Force

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