Proceedings of the 11th KACC, October 1996

Fuzzy Modeling using Transformed Input Space Partitioning

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Abstract Three fuzzy input space partitoining methods, which are grid, tree, and scatter methtod, are mainly used until now. These partition methods represent good performance in the modeling of the linear system and nonlinear system with independent modeling variables. But in the case of the nonlinear system with the coupled modeling variables, there should be many fuzzy rules for acquiring the exact fuzzy model. In this paper, it shows that the fuzzy model is acquired using transformed modeling vector by linear transformation of the modeling vector.

Keywords Transformed input space partitioning, Linear transformation matrix, Grid partition, Fuzzy modeling, Coupled modeling variables

1. Introduction

Conventional input space partitioning methods in fuzzy modeling seperate modeling variables into each independent variable and compose the total output by the combination of local input space outputs. These methods are grid, tree, and scatter partition. Using such fuzzy input space partition methods, many partition numbers are needed for exact fuzzy modeling of nonlinear system. And fuzzy rules, modeling time, and controller time are exponentially incleased by the incleasing of the input partition numbers. So in recent years, some researchers proposed fuzzy modeling methods using input partition methods by neural network and genetic algorithm[1-4], analyzed partition spaces in view of a varable structure system[5-6], a frequency shaping control[7], a facet function[8].

This paper objectes to decrease the number of the fuzzy rules and to get the exact fuzzy model using linear transformation matrix with multi-dimensional modeling variables (input and state varialbes). But the previous researched fuzzy modeling methods were applied to only less 2-dimensional modeling variable spaces for application of the proposed partitioning algorithm. So, available variables for modeling are e and $\dot{e}$[3, 5]. These make impossible the modeling of complex nonlinear system, which has many modeling variables. Because Wang-type fuzzy modeling method[4] makes possible the application of the proposed partitioning algorithm for multi-dimensional modeling variables, we compose the fuzzy model with Wang-type fuzzy modeling method and input partition method using the linear transformation matrix.

Unknown nonlinear system is give in section 2. Wang-type fuzzy modeling method with grid partition method are introduced in section 3. The fuzzy model and parameter adaptation methods using input partition with linear transformation matrix are proposed in section 4. Simulation results for modeling and concluding remarks follow in section 5 and 6, respectively.

2. System for modeling

General nonlinear system for fuzzy modeling is represented in eq.(1).

$$y = f(X) \tag{1}$$

where, variables are defined by

$y \in R$: System output
$f(\cdot) \in R$: Nonlinear function
$X \in R^n$: Modeling vector (state and input vector), $X = [\ x_1\ x_2 \cdots x_n\ ]^T$

3. Wang-type Fuzzy Model

In this section, the fuzzy modeling method proposed by L. X. Wang[4] is presented. Because Wang-type fuzzy modeling method uses back-propagation algorithm for the training of the membership function parameters and each rule output, it makes the parameter tuning of multi-dimensional input space using the transformed fuzzy input space partitioning algorithm.

3.1 Fuzzy Rule

i-th fuzzy rule of Wang-type fuzzy model for the modeling of the nonlinear system is presented.

$$L^i: \text{ IF } x_1 \text{ is } A_1^i,\ x_2 \text{ is } A_2^i,\ \cdots,\ x_j \text{ is } A_j^i,\ \cdots,\ \cdots,\ x_n \text{ is } A_n^i, \text{ THEN } y^i = C^i \tag{2}$$

where, varialbes are defined by

L^i : i-th fuzzy rule

A_j^i : the fuzzy membership function of i-th fuzzy rule related to j-th state variable x_j

y^i, C^i : the output of i-th fuzzy rule

3.2 Membership Function

The gaussian function is adopted as the membership function for the differentiability.

$$A_j^i(x_j) = \exp\left[-\left(\frac{x_j - \overline{x}_j^i}{(\sigma_X)_j^i}\right)^2\right] \tag{3}$$

where, each variable is defined by

$A_j^i(x_j)$: contributionability of the state varible x_j related to the i-th fuzzy rule output

3.3 Model Output

$$\hat{y} = \hat{f}(X) = \frac{\sum_{i=1}^{M}\left\{y^i\left(\prod_{j=1}^{n} A_j^i(x_j)\right)\right\}}{\sum_{i=1}^{M}\left(\prod_{j=1}^{n} A_j^i(x_j)\right)} = \frac{\sum_{i=1}^{M} y^i w^i}{\sum_{i=1}^{M} w^i} \tag{4}$$

where, w^i : contributionability of the i-th fuzzy rule output related to the model output

3.4 Parameter adaptation algorithm

Wang-type algorithm for estimation of the model parameter (e.g. output of each model rule, mean and variance of the membership functions) uses back-propagation algorithm. The cost function e is defined like eq. (5)

$$e = \frac{1}{2}\left[\hat{f}(X^t) - f(X^t)\right]^2 \tag{5}$$

where, each variable is defined by

X^t : system state vector for t-th time interval

U^t : system inptu vector for t-th time interval

$f^t(\cdot)$: system output for t-th time interval .

Back-propagation fuzzy modeling algorithm by Wang is presented.

(1) Output of each Fuzzy Rule

$$y^i(K+1) = y^i(K) - \alpha \left.\frac{\partial e}{\partial y^i}\right|_K = y^i(K) - \alpha \frac{\hat{f} - f^t}{b} z^i \tag{6}$$

where,

$$b = \sum_{i=1}^{M} z^i$$

$$z^i = \prod_{j=1}^{n} \exp\left[-\left(\frac{x_j - \overline{x}_j^i}{(\sigma_X)_j^i}\right)^2\right]$$

(2) Mean of each membership function related to state and input variables

$$\overline{x}_j^i(K+1) = \overline{x}_j^i(K) - \alpha \left.\frac{\partial e}{\partial \overline{x}_j^i}\right|_K = \overline{x}_j^i(K) - \alpha \frac{\hat{f} - f^t}{b}(y^i - \hat{f})\, z^i\, \frac{2\left(x_j - \overline{x}_j^i(K)\right)}{[(\sigma_X)_j^i]^2} \tag{7}$$

(3) Variance of each membership function related to state and input variables

$$(\sigma_X)_j^i(K+1) = (\sigma_X)_j^i(K) - \alpha \left.\frac{\partial e}{\partial (\sigma_X)_j^i}\right|_K = (\sigma_X)_j^i(K) - \alpha \frac{\hat{f} - f^t}{b}(y^i - \hat{f})\, z^i\, \frac{2\left(x_j - \overline{x}_j^i(K)\right)^2}{[(\sigma_X)_j^i]^3} \tag{8}$$

4. Proposed Fuzzy Model

Using wang-type fuzzy modeling method and partition by linear transformation matrix, new fuzzy modeling algorithm is proposed in this section.

4.1 Fuzzy Rule

The composed fuzzy rule is presented.

$$L^i: \text{ IF } q_1^i = (T_1^i)^{\mathrm{T}} X \text{ is } F_1^i,\ q_2^i = (T_2^i)^{\mathrm{T}} X \text{ is } F_2^i,\ \cdots,\ q_l^i = (T_l^i)^{\mathrm{T}} X \text{ is } F_l^i,\ \cdots,\ q_p^i = (T_p^i)^{\mathrm{T}} X \text{ is } F_p^i, \text{ THEN } y^i = C^i \tag{9}$$

where, each variables is defined by

L^i : i-th ruzzy rule

q_l^i : j-th input variable of i-th fuzzy rule

$T_l^i \in R^n$: The linear transformation vector related to l-th state variable of i-th fuzzy rule ($l \in [1, p]; p > 1$) ($\therefore \|T_l^i\|_2 = 1$)

$$q_l^i = (T_l^i)^{\mathrm{T}} X = \left[\, t_{1,l}^i\ t_{2,l}^i \cdots t_{n,l}^i \,\right] \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{10}$$

$T^i \in R^{n \times p}$: Linear transformation matrix of i-th fuzzy rule

$$Q^i = (T^i)^T X$$

$$= \begin{bmatrix} q_1^i \\ q_2^i \\ \vdots \\ q_l^i \\ \vdots \\ q_p^i \end{bmatrix} = \begin{bmatrix} t_{1,1}^i & t_{2,1}^i & \cdots & t_{n,1}^i \\ t_{1,2}^i & t_{2,2}^i & \cdots & t_{n,2}^i \\ \vdots & \vdots & \vdots & \vdots \\ t_{1,l}^i & t_{2,l}^i & \cdots & t_{n,l}^i \\ \vdots & \vdots & \vdots & \vdots \\ t_{1,p}^i & t_{2,p}^i & \cdots & t_{n,p}^i \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad (11)$$

F_l^i : the fuzzy membership function related to l-th state variable q_l^i of i-th fuzzy rule

y^i, C^i: the output of i-th fuzzy rule

Input space is divided into independent modeling variables x_j at the inference part (IF part) using the conventional fuzzy input partition methods (e.g. grid, scatter, tree). Because the proposed fuzzy modeling algorithm divides input space by the linear combination of the modeling variables, it shows good performance for nonlinear system modeling.

The simple example is presented. Fig.1 shows the conventional fuzzy input partition method for 2-dimensional space. Fig.2 shows the proposed fuzzy input partition method for 2-dimensional space.

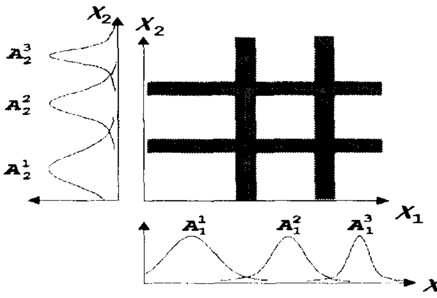

Fig. 1 The conventional fuzzy modelling method

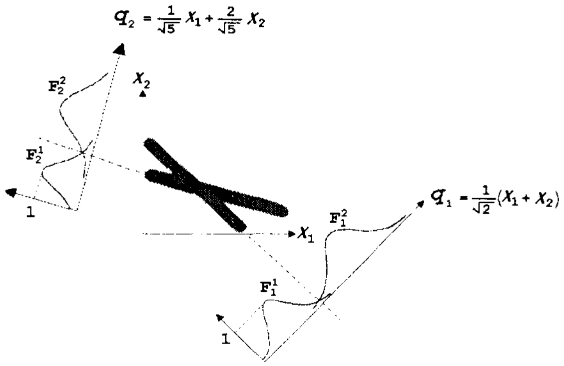

Fig. 2 The proposed fuzzy modelling method

4.2 Membership Function

The gaussian function is adopted for the differentiability of the membership function.

$$F_l^i(q_l^i) = \exp[-(\frac{q_l^i - \overline{q_l^i}}{(\sigma_Q)_l^i})^2] \quad (12)$$

where, $F_l^i(q_l^i)$: contributionability of l-th modeling variable q_l^i related to the i-th fuzzy rule output

4.3 Model Output

The final model output $\hat{y}$ is calculated by weighted sum of each rule output.

$$\hat{y} = \hat{f}(X, U) = \frac{\sum_{i=1}^{M}\left\{ y^i \left[\prod_{l=1}^{p} F_l^i(q_l^i) \right] \right\}}{\sum_{i=1}^{M}\left[\prod_{l=1}^{p} F_l^i(q_l^i) \right]} = \frac{\sum_{i=1}^{M} y^i w^i}{\sum_{i=1}^{M} w^i} \quad (13)$$

where, w^i : contributionability of the i-th fuzzy rule output related to the model output

4.4 Parameter adaptation algorithm

The proposed algorithm calculates the model parameter (e.g. output of each model rule, mean and variance of the memebership functions, and linear transformation matrix elements) by back-propagation algorithm and linear transformation matrix. The cost function e is defined by eq.(14).

$$e = \frac{1}{2}\left[\hat{f}(Q^t) - f^t(X^t)\right]^2 \quad (14)$$

where,

X^t : system state vector for t-th time interval

$f^t(\cdot)$: system output for t-th time interval .

The proposed algorithm is presented.

(1) Output of each Fuzzy Rule

$$\begin{aligned} y^i(K+1) &= y^i(K) - \alpha \left.\frac{\partial e}{\partial y^i}\right|_k \\ &= y^i(K) - \alpha \frac{\hat{f} - f^t}{b} z^i \end{aligned} \quad (15)$$

where,

$$b = \sum_{i=1}^{M} z^i$$

$$z^i = \left\{ \prod_{l=1}^{p} \exp\left[-\left(\frac{q_l^i - \overline{q_l^i}}{(\sigma_Q)_l^i} \right)^2 \right] \right\}$$

(2) Mean of each membership function related to state and input variables

Eq. (16) shows the adaptation algorithm of the mean values using the linear transformation of modeling variables.

$$\overline{q}_l^i(K+1) = \overline{q}_l^i(K) - \alpha \frac{\partial e}{\partial \overline{q}_l^i}\bigg|_K$$
$$= \overline{q}_l^i(K) - \alpha \frac{\hat{f}-f}{b}(y^i-\hat{f})\, z^i \frac{2\left(q_l^i - \overline{q}_l^i(K)\right)}{[(\sigma_Q)_l^i]^2} \tag{16}$$

(3) Variance of each membership function related to state and input variables

Eq. (17) shows the adaptation algorithm of the variance values using the linear transformation of modeling variables.

$$(\sigma_Q)_l^i(K+1)$$
$$= (\sigma_Q)_l^i(K) - \alpha \frac{\partial e}{\partial (\sigma_Q)_l^i}\bigg|_K$$
$$= (\sigma_Q)_l^i(K) - \alpha \frac{\hat{f}-f}{b}(y^i-\hat{f})\, z^i \frac{2\left(q_l^i - \overline{q}_l^i(K)\right)^2}{[(\sigma_Q)_l^i]^3} \tag{17}$$

(4) Linear Transformation Matrix

Eq. (18) shows the adaptation algorithm of linear transformation matrix values using the linear transformation of modeling variables.

$$t_{r,l}^i(K+1) = t_{r,l}^i(K) - \alpha \frac{\partial e}{\partial (t_{r,l}^i(K))}\bigg|_K$$
$$= t_{r,l}^i(K) + \alpha \frac{\hat{f}-f}{b}(y^i-\hat{f})\, z^i \frac{2\left(q_l^i - \overline{q}_l^i(K)\right)}{[(\sigma_Q)_l^i]^2} x_r \tag{18}$$

where, $r = [1, n]$.

5. Simulatioin Results

This section shows fuzzy modeling example using linear transformation matrix. And each example shows that the proposed algorithm presents better performance than the conventional input partition algorithm.

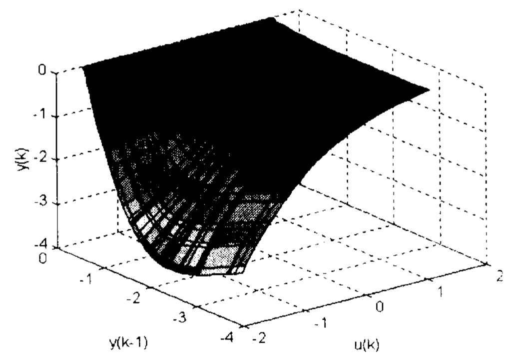

Fig 3. nonlinear system input-output relation of example 1

5.1 Example 1.

Eq. (19) is nonlinear system for estimating.

$$y(k+1) = g[y(k), u(k)]$$
$$g[y(k), u(k)] = 3\frac{y(k)e^{u(k)}}{1+y(k)^2+u(k)^2} \tag{19}$$

where, input is $u(k) = 0.8\left(\sin(\frac{2\pi k}{25}) + \sin(\frac{5\pi k}{25})\right)$.

And, eq.(20) shows the fuzzy model using series-parallel scheme.

$$\hat{y}(k+1) = \hat{f}[y(k), u(k)] \tag{20}$$

where, $\hat{f}[y(k), u(k)]$ is defined like eq.(4).

(1) Fuzzy modeling by Wang-type

In the conventional algorithm, fuzzy rules M are 40, adaptation rete α is 0.1, state number n is 2, and input number m is 0. The ranges of the input variables are [-2, 2] as fuzzy rule output, [-1, 1] as mean of membership function, [0 0.3] as variance of membership function. All initial values are distributed with linear relation. The output of the fuzzy model is presented in fig. 4.

(2) Fuzzy modeling by the proposed algorithm

Adaptation of the linear transformation matrix elements are added in the proposed algorithm. The number p of the linear transformation vector are setted by p=1, 2, 3.

A. Case 1: p=1

In the case of p=1, each fuzzy rule has one linear transformation variable q. Simulating after addition of q in the conventional algorithm, the output of the fuzzy model is presented by fig. 5. In this case, initial linear transformation matrix is $[1\ 0]^T$.

B. Case 2: p=2

In the case of p=2, linear transformation matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

C. Case 3: p=3

In the case of p=3, linear transformation matrix is $\begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \\ 0 & 1 \end{bmatrix}$.

Wang-type modeling method using the grid partition algorithm has the independent elements of each input variable. But as the method using the proposed partition algorithm has the several maximum axis elements, exact modeling is possible.

6. Conclusion Remarks

This paper showed that the fuzzy modeling method applying linear transformaiton matrix to modeling variables is better than the conventional modeling method. Conventional fuzzy modeling methods are included by identity linear transformation matrix. If the direction of linear transformation matrix is known, expert knowledge will be used.
The future research areas are the development of the common linear transformation matrix and the proof of the good robustness.

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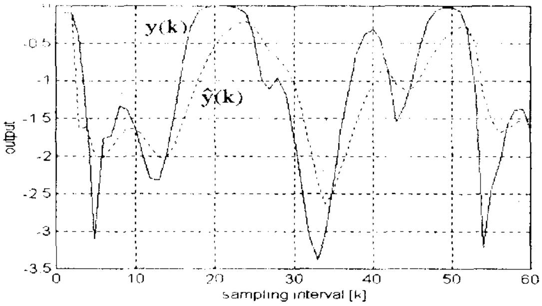

Fig 4. nonlinear system output(-) and wang-type fuzzy model output( )

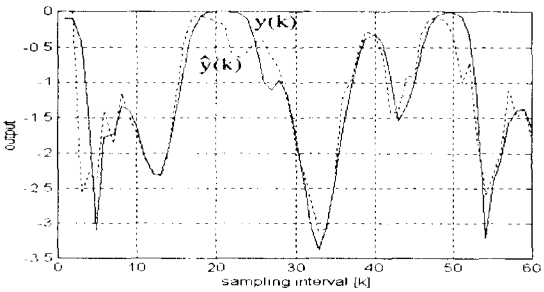

Fig 5. nonlinear system output(-) and proposed fuzzy model output with p=1 ( )

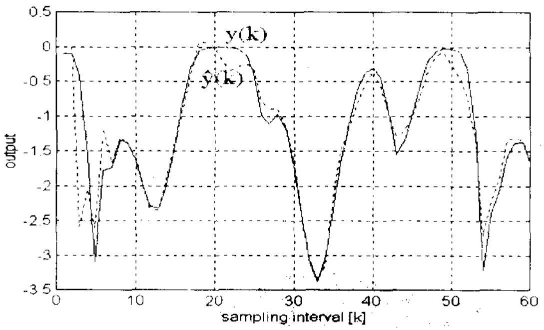

Fig 6. nonlinear system output(-) and proposed fuzzy model output with p=2 ( )

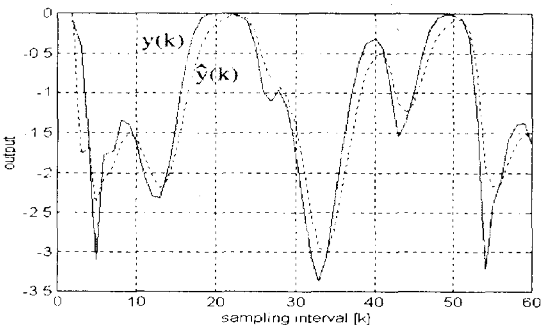

Fig 7. nonlinear system output(-) and proposed fuzzy model output with p=3 ( )