Proceedings of the 11th
KACC, October 1996

H∞노음조건을 만족하는 강인한 일반형예측제어기

A Robust Genaralized Predictive Control which Guarantees H∞ Norm Bounds

°이 영 일*, 김 용 호**, 권 욱 현**

*경상대학교 제어계측공학과(Tel:0591-751-5370, Fax:0591-57-3974, e-mail:ciman@nongae.gsnu.ac.kr)
**서울대학교 제어계측공학과(Tel:02-873-2279, Fax:02-871-7010, e-mail:yongho@cisl.snu.ac.kr)

Abstract : In this paper, we suggest a H∞ generalized predictive control(H∞GPC) which guarantees H∞-norm bounds. The suggested control is obtained by solving the min-max problem in nonrecursive forms. The stability conditions of the suggested control are derived in a somewhat simple form and it is not required for the derived solution to be a saddle point solution. It is also shown that the suggested control guarantees the H∞-norm bounds under the same conditions of stability.

keywords : GPC, receding horizon, stability, H∞-norm

1. Introduction

The control methods of GPC[1] has received a wide acceptance in the process industries because of their good performance and insensitiveness to model-plant mismatch. In these days, the equivalence of GPC to some state-space model based design methods such as receding horizon control(RHC) is well known[3]. Using the terminal equality constraint in RHC[2], some stability results of GPC are obtained[4]. In some cases, the stability properties were investigated using the cost monotonicity[5].

Robustness is another important issue of GPC. There are some results on improving the robustness of GPC by parameter tuning via closed loop system analysis[6]. This approach mainly utilize simulation results to analyze the robustness properties. For the case of RHC, in the very recent years, there have been an attempt to construct a receding horizon H∞ control for discrete linear systems and analyze its properties[9].

In this paper, we suggest a robust GPC(RGPC) which guarantees H∞-norm bounds for SISO systems and obtain some stability conditions of it. The original form of this RGPC was first proposed in [8]. In [8], however, there is no analysis on stability and norm bounds. The solution of the suggested H∞ control is given in one-shot forms, and its stability property of the suggested control is examined by checking the monotonicity of the cost function. The fact that the $H\infty$ norm bound from disturbance to output is guaranteed with this control also can be shown based on the cost monotonicity.

2. Problem Formulation

Consider a linear time invariant system described by

$$A(q^{-1})y(t) = B(q^{-1})\Delta u(t-1) + C(q^{-1})\omega(t)$$

where

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \\ C(q^{-1}) &= 1 + c_1 q^{-1} + \cdots + c_p q^{-p} \end{aligned} \tag{1}$$

$q^{-1}$is the unit delay operator, $\Delta u(t)$ and $y(t)$ are the change of the control input and the system output at time t, respectively, $\omega(t)$ is an uncorrelated and zero mean disturbance on the system which must be taken into account. In this paper, $C(q^{-1})$ is assumed to be 1 and $n \geq m$.. In oder to derive a GPC formulation for the system (1) which guarantees H∞-norm bounds, we consider the following cost function:

$$\begin{aligned} J_t(\bar{u}_t, \bar{\omega}_t) = & \sum_{i=1}^{N} \{(y(t+i) - y_r(t+i))^2 \\ & + \lambda \Delta u(t+i-1)^2 - \gamma^2 \omega(t+i)^2\} \\ & + \sum_{i=N+1}^{N+N_p} Q(\hat{y}(t+i|t+N) - y_r(t+i))^2 \end{aligned} \tag{2}$$

where

$\bar{u}_t = [\Delta u(t) \cdots \Delta u(t+N-1)]'$, $\bar{\omega}_t = [\omega(t+1) \cdots \omega(t+N)]'$ and $\hat{y}(t+i|t+N)$is the prediction of the output $y(t+i)$ which is made at time t+N. It is assumed that $\Delta u(t+j) = 0, j = N, N+1, \cdots$. We would like to find the optimal $\overline{u}_t$ which minimizes J_t while ω_t tries to maximize it as the following equation:

$$\min_{\overline{u}_t} \max_{\overline{\omega}_t} J_t(\overline{u}_t, \overline{\omega}_t) \quad (3)$$

given the command signals $y_r(t+1), y_r(t+2), \cdots y_r(t+N+N_F)$. Assuming that $\overrightarrow{u}^*_t$, $\overrightarrow{\omega}^*_t$ are minimizing and maximizing solutions, respectively, the following inequality is satisfied for any $\overline{\omega}_t$:

$$J_t(\overrightarrow{u}^*_t, \overline{\omega}_t) \le J_t(\overrightarrow{u}^*_t, \overrightarrow{\omega}^*_t), \quad (4)$$

then feeding the optimal control sequence $\overrightarrow{u}^*(t)$ to the system during [t, t+N-1] guarantees

$$\sum_{i=1}^{N}(y(t+i)-y_r(t+i))^2 \le \sum_{i=1}^{N}\gamma^2\omega(t+i)^2 + \hat{J}_t(\overrightarrow{u}^*_t, \overrightarrow{\omega}^*_t) \quad (5)$$

where

$$\hat{J}_t(\overrightarrow{u}^*_t, \overrightarrow{\omega}^*_t) = J_t(\overrightarrow{u}^*_t, \overrightarrow{\omega}^*_t) - \lambda\, \overrightarrow{u}^{*\prime}_t\, \overrightarrow{u}^*_t - \sum_{i=N+1}^{N+N_F} Q\{\hat{y}(t+i|t+N) - y_r(t+i))\}^2.$$

It can be easily shown that $J_t(\overrightarrow{u}^*_t, \overrightarrow{\omega}^*_t)$ is zero with zero initial condition, and the inequality (5) says that the induced norm from ω to $y-y_r$ is bounded by γ^2 during the finite horizon with the control $\overrightarrow{u}^*_t$.

In this paper, we will consider the case in which only the first element of $\overrightarrow{u}^*_t$ is applied at time t, at the next time t+1, $\overrightarrow{u}^*_{t+1}$ is obtained for the receded future horizon and the same procedure is repeted. This strategy is the same as that of GPC, which is called the receding horizon strategy.

3. H∞ GPC

At the present time t, the future system output $y(t+j)$, $j=1,2,\cdots,N$ can be calculated as follows under the assumption that the future values of $\Delta u(\cdot)$ and $\omega(\cdot)$, during the horizon [t, t+N], are determined as follows,

$$y(t+j) = g_j(q^{-1})\Delta u(t+j-1) + d_j(q^{-1})\omega(t+j) + f_{t,j} \quad (6)$$

where $g_j(q^{-1})$ and $d_j(q^{-1})$ are polynomials of q^{-1} of order j-1, $f_{t,j} = a_j(q^{-1})y(t) + b_j(q^{-1})\Delta u(t)$ and $a_j(q^{-1})$, $b_j(q^{-1})$ are polynomials of q^{-1} of orders n and m, respectively. The coefficients of the above polynomials can be obtained recursively or by solving a diophantine equation[1][7]. Thus, for the horizon [t+1, t+N], the future system outputs can be written in a vector form as follows.

$$\overline{y}_t = G_1\overline{u}_t + D_1\overline{\omega}_t + F_{1y}\,\underline{y}_t + F_{1u}\,\underline{u}_t \quad (7)$$

where

$\overline{y}_t = [y(t+1)\cdots y(t+N)]'$, $\underline{y}_t = [y(t)\ \cdots\ y(t-n+1)]'$, and $\underline{u}_t = [\Delta u(t-1)\ \cdots\ \Delta u(t-m)]'$ and

$$G_1 = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ g_{N-1} & g_{N-2} & \cdots & g_0 \end{bmatrix} D_1 = \begin{bmatrix} d_0 & 0 & \cdots & 0 \\ d_1 & d_0 & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ d_{N-1} & d_{N-2} & \cdots & d_0 \end{bmatrix}$$

$$F_{1y} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,n} \end{bmatrix} F_{1u} = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,m} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,m} \\ \vdots & \vdots & \cdots & \vdots \\ b_{N,1} & b_{N,2} & \cdots & b_{N,m} \end{bmatrix}.$$

The elements g_i, d_i, $a_{j,i}$ and $b_{j,i}$ are the i^{th} coefficients of $g_i(q^{-1})$, $d_i(q^{-1})$, $a_j(q^{-1})$ and $b_j(q^{-1})$, respectively. The output prediction $\hat{y}(t+j|t+N)$, $j=N+1,\cdots,N+N_F$ is obtained from Equation (6), assuming that $\omega(t+N+j)=0$, $\Delta u(t+N+j-1)=0$ for $j=1,2,\cdots$, and it can be written in a vector form as follows:

$$\hat{\overline{y}}_{t,N} = G_2\overline{u}_t + D_2\overline{\omega}_t + F_{2y}\,\underline{y}_t + F_{2u}\,\underline{u}_t \quad (8)$$

where $\hat{\overline{y}}_{t,N} = [\hat{y}(t+N+1|t+N)\cdots \hat{y}(t+N+N_F|t+N)]'$, and

$$G_2 = \begin{bmatrix} g_N & g_{N-1} & \cdots & g_1 \\ g_{N+1} & g_N & \cdots & g_2 \\ \vdots & \vdots & \cdots & \vdots \\ g_{N+N_F-1} & g_{N+N_F-2} & \cdots & g_{N_F} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} d_N & d_{N-1} & \cdots & d_1 \\ d_{N+1} & d_N & \cdots & d_2 \\ \vdots & \vdots & \cdots & \vdots \\ d_{N+N_F-1} & d_{N+N_F-2} & \cdots & d_{N_F} \end{bmatrix}$$

$$F_{2y} = \begin{bmatrix} a_{N+1,1} & a_{N+1,2} & \cdots & a_{N+1,n} \\ a_{N+2,1} & a_{N+2,2} & \cdots & a_{N+2,n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N+N_F,1} & a_{N+N_F,2} & \cdots & a_{N+N_F,n} \end{bmatrix}$$

$$F_{2u} = \begin{bmatrix} b_{N+1,1} & b_{N+1,2} & \cdots & b_{N+1,m} \\ b_{N+2,1} & b_{N+2,2} & \cdots & b_{N+2,m} \\ \vdots & \vdots & \cdots & \vdots \\ b_{N+N_F,1} & b_{N+N_F,2} & \cdots & b_{N+N_F,m} \end{bmatrix}.$$

Equations (7) and (8), can be combined as follows:

$$\overline{Y}_t = G\,\overline{u}_t + D\,\overline{\omega}_t + F_y\,\underline{y}_t + F_u\,\underline{u}_t \quad (9)$$

where $\overline{Y}_t = [\overline{y}_t'\ \ \hat{\overline{y}}'_{t,N}]'$, $G=[G_1'\ G_2']'$, $D=[D_1'\ D_2']'$, $F_y=[F_{1y}'\ F_{2y}']'$, $F_u=[F_{1u}'\ F_{2u}']'$. Using Equation (9), the cost (2) can be rewritten as:

$$J_t(\bar{u}_t, \bar{\omega}_t) = (\bar{Y}_t - \bar{Y}_{r,t})'\bar{Q}(\bar{Y}_t - \bar{Y}_{r,t}) + \lambda \bar{u}_t' \bar{u}_t - \gamma^2 \bar{\omega}_t' \bar{\omega}_t \quad (10)$$

where

$\bar{Y}_{r,t} = [\bar{y}_{1r,t}' \ \bar{y}_{2r,t}']'$, $\bar{y}_{1r,t} = [y_r(t+1) \cdots y_r(t+N)]'$, $\bar{y}_{2r,t} = [y_r(t+N+1) \cdots y_r(t+N+N_F)]'$, and $\bar{Q} = diag\{I_N \ QI_{N_F}\}$ Then, the maximizing disturbance $\bar{\omega}_t^*(\bar{u}_t)$ is obtained by differentiating J_t in Equation (10) with $\bar{\omega}_t$ as follows:

$$\bar{\omega}_t^*(\bar{u}_t) = (\gamma^2 I - D'\bar{Q}D)^{-1}D'\bar{Q} \cdot (G\bar{u}_t + F_y y_t + F_u u_t - \bar{Y}_{r,t}), \quad (11)$$

provided $\frac{\partial^2 J_t}{\partial \bar{\omega}_t^2} = 2(D'\bar{Q}D - \gamma^2 I) < 0$. When the disturbance $\bar{\omega}_t$ takes the value as Equation (11), the minimizing control sequence $\bar{u}_t$ can be obtained by differentiating $J_t(\bar{u}_t, \bar{\omega}_t^*(\bar{u}_t))$ with $\bar{u}_t$ as follows:

$$\bar{u}_t^* = \{\lambda I + G'(\bar{Q} + \bar{Q}D\Omega^{-1}D'\bar{Q})G\}^{-1} \cdot G'(\bar{Q} + \bar{Q}D\Omega^{-1}D'\bar{Q}) \cdot (\bar{Y}_{r,t} - F_y y_t + F_u u_t), \quad (12)$$

where $\Omega = \gamma^2 I - D'\bar{Q}D$. It should be noted that $\frac{\partial^2 J_t}{\partial \bar{u}_t^2} > 0$, which means that (12) is the minimizing solution. Equation (12) contains control inputs for the future N times, and only the first one is fed to the system. At the next time t+1, the same procedure is repeated. Manipulating (12) with the matrix inversion lemma, a minimizing solution with infinity Q can be obtained. In this case, however, the condition $(D'\bar{Q}D - \gamma^2 I) < 0$ can not be met with finite γ, and it has no practical meaning.

4. Stability and H∞-norm bound

In this section, we consider the conditions of stability and H∞-norm bound of the system when the control (12) is applied to the system in the receding horizon manner. First, we will examine the stability of this control by checking the monotonicity of the cost function. Although the optimal values (11) and (12) may not be a saddle point solution, we can obtain the following inequalities:

$$J_t(\bar{u}_t^*, \bar{\omega}_t) \le J_t(\bar{u}_t^*, \bar{\omega}_t^*(\bar{u}_t^*)) \le J_t(\bar{u}_t, \bar{\omega}_t^*(\bar{u}_t)). \quad (13)$$

Before we propose a theorem regarding the stability of the control (12), using relation (13), we define some notations for briefness of description. From now on, we assume that $N = N_F = n$ and $\hat{y}_{t,N} = [\hat{y}(t+N+1|t+N) \cdots \hat{y}(t+N+N_F|t+N)]'$ is made under the assumption that $\Delta u(t+j) = 0$, $j \ge N$ while $\hat{y}_{t+1,N} = [\hat{y}(t+N+2|t+N+1) \cdots \hat{y}(t+N+N_F+1|t+N+1)]'$ is made under the assumption that $\Delta u(t+j) = 0$, $j \ge N+1$ with $\Delta u(t+N) = -K\hat{y}_{t,N}$ for some vector K of adequate dimension. First, consider the relations between $\hat{y}_{t,N}$ and $\hat{y}_{t+1,N}$. Under the conditions above mentioned, we can obtain:

$$\hat{y}_{t+1,N} = A\hat{y}_{t,N} + B\Delta u(t+N) = (A - BK)\hat{y}_{t,N} \quad (14)$$

where

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} B = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix}.$$

Note that the inverse of A exists if $a_n \ne 0$. If a disturbance vector $\bar{\omega}_{t+1}$ is applied to the system during the horizon [t+1, t+N+1], the system output y(t+1+N) is given as:

$$y(t+1+N) = \hat{y}(t+1+N|t+N) + g_1\Delta u(t+N) + d_0\omega(t+N+1) = \bar{g}\hat{y}_{t,N} + \omega(t+N+1) \quad (15)$$

where $\bar{g} = [1 \ 0 \cdots 0] - g_1 K$. Based on the relations above mentioned, we can state a theorem about the stability of the control (12).

Theorem 1. Consider the system (1). For $N = N_F = n$the control (12) stabilize the closed loop system when it is applied in the receding horizon manner provided:

$$\gamma^2 I - D_1' D_1 > 0 \quad (16)$$

$$(I - (A - BK)'(A - BK)) - Q^{-1}\{\lambda K'K + \bar{g}'(1 + (\gamma^2 - 1)^{-1})\bar{g}\} \ge 0. \quad (17)$$

Proof: Since we are concerning the closed loop stability, we assume that $y_r(\cdot) = 0$ and the control (12) is applied to the system (1) with $\omega(t) = 0$. Let $\bar{u}_t^* = [u_{t,0}^* \ u_{t,1}^* \cdots u_{t,N-1}^*]'$ and $\bar{\omega}_t^*(\bar{u}_t^*) = [\omega_{t,1}^*(\bar{u}_t^*) \cdots \omega_{t,N}^*(\bar{u}_t)]$ are the minimizing and maximizing solutions of J_t, respectively, and $\hat{u}_{t+1} = [\Delta u_{t,1}^* \ \Delta u_{t,2}^* \cdots \Delta u_{t,N-1}^* \ -K\hat{y}_{t,N}]'$, $\hat{\omega}_{t+1} = [0 \ \ \omega_{t+1,1}^* \ \omega_{t+1,2}^* \cdots \omega_{t+1,N-1}^*]'$. Then, from the properties of Equation (13), we get:

$$\begin{aligned} J_t(\bar{u}_{t^*}, \bar{\omega}_{t^*}(\bar{u}_{t^*})) &\ge J_t(\bar{u}_{t^*}, \hat{\omega}_{t+1}(\hat{u}_{t+1})) \\ &\ge y(t+1)^2 + \lambda\Delta u_{t,0}^{*2} \\ &\quad + J_{t+1}(\bar{u}_{t+1^*}, \bar{\omega}_{t+1^*}(\bar{u}_{t+1^*})) + R_t \end{aligned} \quad (18)$$

where $R_t = Q(\hat{y}_{t,N}'\hat{y}_{t,N} - \hat{y}_{t+1,N}'\hat{y}_{t+1,N}) - (\bar{g}\hat{y}_{t,N} + \omega^*_{t+1,N}(\hat{u}_{t+1}))^2 + \gamma^2 \omega^*_{t+1,N}(\hat{u}_{t+1})^2 - \lambda \hat{y}_{t,N}'K'K\hat{y}_{t,N}$.

If R_t is shown to be nonnegative, then inequality (18) implies that y(t+1) and $\Delta u(t)$ converges to zero as $t \to \infty$, since $J_t(\cdot)$ is nonnegative and bounded below. Thus, it is left to show that under the condition (17), R_t is guaranteed to be nonnegative.

From relation (14):

$$R_t = \hat{y}_{t,N}'\{Q(I - \Phi'\Phi) - \lambda K'K\}\hat{y}_{t,N} + \gamma^2 \omega^{*2}_{t+1,N} - \hat{y}_{t,N}'\bar{g}'\{1 + (\gamma^2 I - 1)^{-1}\}\bar{g}\hat{y}_{t,N} - \gamma^2 \omega^{*2}_{t+1,N} + \Omega_1'(\gamma^2 - 1)\Omega_1,$$

where $\Omega_1 = \omega^*_{t+1,N} - (\gamma^2 I - 1)^{-1}\bar{g}\hat{y}_{t,N}$ and $\Phi = A - BK$.

From the above relation, we can conclude that $R_t \geq 0$ provided the conditions (16)-(17). ■

It is left for us to show that the H∞-norm(=induced 2-norm) is bounded when control (12) is used in the receding horizon manner. The following theorem says that RGPC guarantees the H∞-norm bounds under the same condition of Theorem 1. The proof is also based on the monotonicity of cost function and is omitted here.

Theorem 2. When the control (12) is applied to the system (1) in the receding horizon manner and the conditions (16) and (17) are met, the H∞-norm(induced 2-norm) from ω to y is bounded by γ^2. ■

From the above two theorems, we can say that RGPC stabilizes the closed loop system while guaranteeing the H∞ norm bounds under the conditions (16) and (17). These conditions are sufficient conditions for closed loop stability and norm bounds.

5. Conclusions

In this paper, we obtained H∞ GPC solution in one shot form for SISO systems using game theoretic approach. Some conditions for guaranteed stability and H∞-norm bounds of RGPC were derived. The analysis of stability and norm bound was carried out by examining the monotonicity of cost functions and it is not required for the solutions to be saddle point solutions. Although SISO systems are considered in this paper, it is expected that the methology of analysis and synthesis of this paper can be easily extended to MIMO systems and state-feedback cases. It is also expected that the conditions (16) and (17) can be changed in another form so that stabilizing pairs of λ and Q can be obtained systematically via LMI approach.[11].

References

[1] D.W. Clarke, C. Mohtadi and P.S. Tuffs, "Generalized predictive control-Part I. The basic algorithm," Automatica, vol.23, pp.137-148, 1987.

[2] W.H. Kwon, and A.E. Pearson, "A modified quadratic cost problem and feedback stabilization of a linear system," IEEE Trans. on AC, vol.22, pp.838-842, 1977.

[3] Y.I. Lee, W.H. Kwon, and S. Noh, "Receding horizon predictive control and its related GPC with stability properties," Control Theory and Advanced Technology, vol.10, N0.3, pp.523-537, 1994.

[4] D.W. Clarke, and R. Scattolini, "Constrained receding horizon predictive control," IEE proceedings part D, vol.138, No.4, pp.347-354, 1991.

[5] J.B. Rawlings, and K.B. Muske, "The stability of constrained receding horizon control," IEEE Trans. on AC, vol.38, No. 10, pp.1512-1516, 1993.

[6] T.W. Yoon and D.W. Clarke, "Receding horizon predictive control with exponential weighting," Int. J. of Control, vol.24, No.9, pp.1745-1757, 1993.

[7] W.H. Kwon, H. Choi, D.G. Byun, and S. Noh, "Recursive solution of generalized predictive control and its equivalence to receding horizon tracking control," Automatica, vol.28, No.6, pp.1235-1238, 1992.

[8] S. Noh, Y.H. Kim, Y.I. Lee, and W.H.Kwon, "A robust generalized predictive control with terminal output weightings," vol.6, In 2-3, pp137-144, 1996.

[9] J.W. Lee, W.H. Kwon and J.H. Lee, "Receding horizon H∞ control for time-varying discrete linear systems," submitted to Control and Syatem letters.

[10] W.H. Kwon, S. Noh, and Y.I. Lee, "A stability guaranteed GPC and its equivalence to receding horizon tracking control." Proc. of ACC'92, pp.2037-2041, 1992.

[11] Stephen Boyd, Laurent El Ghaoui, Eric Feron, and Venkataramanan Balakrishnan, 'Linear Matrix Inequalities in System and Control Theory,' SIAM, 1994.