

# Force Control of an Asymmetric Hydraulic Cylinder for Active Suspensions

Wanil Kim, Byung-Youn Lee, Sang-Woo Kim, Sang-Chul Won

Dept. of Electr. Engr., Postech, 279-5588; Fax: 279-2903; E-mail: wikim@jane.postech.ac.kr

**Abstract** Asymmetric cylinders are usually used as an actuator of active suspensions. Since the force is influenced not only by the control but by the road roughness, force control is needed to track the desired force. But the conventional error feedback control treats the valve-cylinder dynamics at its operating point and many use the symmetric model which differs in all respects. We adopt an asymmetric cylinder model and apply a feedback linearization method for the force control to compensate both the valve nonlinearities and the effects of the road roughness.

**Keywords** Active suspension, Asymmetric cylinder, Force control, Feedback linearization

## 1. Introduction

To achieve the goals of active suspensions (e.g. ride comfort, road holding, suspension constraints), hydraulic cylinder and servo valve are commonly used as an active force generator. Various sensor data from accelerometers, position and pressure transducers are used in a controller to calculate the desired force to be tracked by the actuator. Final control input is usually voltage or current causing the servo valve spool movement. Until now most researches are concerned about what is the best desired force and how we can calculate it. Relatively less attention was paid to how we can control the servo valve to follow the desired force. In this study we call the former the outer loop control, the latter the inner loop or the force control. Our main concern is the inner loop control.

Two things make the inner loop complex. First the flow-pressure relationship of a hydraulic cylinder and a servo valve is nonlinear, time-varying. Second as in Fig. 1. the actuator is located between the sprung and the unsprung mass, which causes the direct force transmission induced by the road roughness. In [2], it is shown that the piston velocity is a disturbance input to the inner loop. But their simple error feedback scheme requires tremendous open loop gain to cancel this disturbance.

Feedback linearization method uses the knowledge of actuator dynamics to make the resultant outer loop look linear. In [1] with the symmetric cylinder model, they applied feedback linearization method to track the desired sky-hook force. Some parameters are recursively identified whose structure is known. But strange enough, they identified the parameters having

two values dependent on the sign of the input while the parameters are global constants. The reason of their inconsistency is due to their misuse of the cylinder model. They use the symmetric model even though car suspension structure permits only asymmetric cylinders that have one piston rod on one side. The important fact that asymmetric cylinders have different piston areas explains all the incompatible behaviors and indicates totally different controller design procedures [3-4].

A quarter car and an asymmetric cylinder model are given and analyzed in section 2. Feedback linearization method for the inner loop and output feedback method for the outer loop are applied in section 3. Step-up road input simulation results and concluding remarks follow in section 4 and 5, respectively.

## 2. SYSTEM MODELING

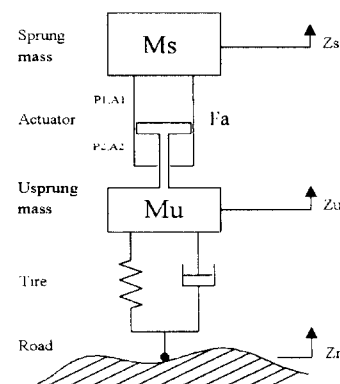


Fig. 1. Quarter Car Suspension

## 2.1 Quarter Car Model

A fully active quarter car model is used in this study and all the data are based on the Test Rig in Control Lab., POSTECH. An asymmetric actuator replaces with passive elements. Tire is modeled as a stiff spring parallel with a small damper. Applying the Newton's law, we get

$$M_s \ddot{z}_s = F_a \quad (1)$$

$$M_u \ddot{z}_u = -F_a - K_t(z_u - z_r) - B_t(\dot{z}_u - \dot{z}_r) \quad (2)$$

Road displacement and velocity form a disturbance input. Measured outputs are the sprung and unsprung acceleration, the suspension space ( $z_s - z_u$ ), the actuator force, and the pressures of both sides of the cylinder. We don't assume the tire deflection ( $z_u - z_r$ ), the absolute position of each mass are measurable because it is impractical.

The actuator force  $F_a$  should not be thought as a control input, which is rather an internal state. But when we design the outer loop we consider it as a virtual input.

## 2.2 Symmetric vs. Asymmetric Cylinder

We compare both types of cylinders in 3 aspects: valve's flow-pressure equation, cylinder continuity equation, and force equation.

The flow rate of a servo valve is proportional both to the spool displacement and the square root of the pressure drop across it.

$$Q_i = K_{eq} X_{sp} \sqrt{\Delta P_i} \quad (3)$$

For symmetric valves when the piston slides at a constant speed,

$$v_{pis} = Q_1/A_1 = Q_2/A_2 \quad (4)$$

For a symmetric cylinder, the piston areas,  $A_1$  and  $A_2$  are equal, therefore so are the pressure drops,  $\Delta P_1$  and  $\Delta P_2$ , flow rate,  $Q_1$  and  $Q_2$ . Then the following useful identity holds.

$$P_1 + P_2 = P_s \quad (5)$$

However as for an asymmetric cylinder, the pressure drops and flow rates of side 1 and 2 are unequal due to the difference in area. (5) does not hold either.

Following the notations in [3], we get the cylinder continuity equations.

$$\dot{P}_1 = \frac{1}{C_1(y)} (Q_1 - \dot{y}A_1), \quad C_1(y) = \frac{A_1 \lambda_1(y)}{\beta} \quad (6)$$

$$\dot{P}_2 = \frac{1}{C_2(y)} (\dot{y}A_2 - Q_2), \quad C_2(y) = \frac{A_2 \lambda_2(y)}{\beta} \quad (7)$$

Leakage terms are excluded in (6),(7), because their effects are negligible. The role of the piston areas in the above equations are apparent. For a symmetric cylinder, defining  $P_L$  as  $P_1 - P_2$  and with (3), (5-7), we

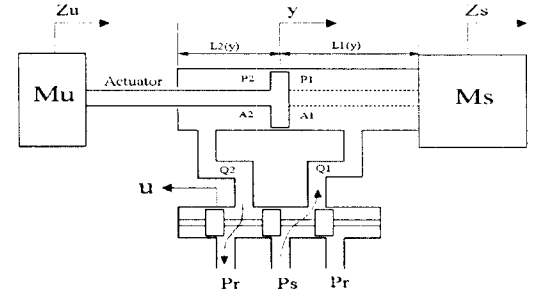


Fig.2. Asymmetric cylinder & servo valve

get the following[1].

$$\dot{P}_L = K_{eq} u \sqrt{P_s - \text{sign}(u)P_L} - T \dot{y} \quad (8)$$

Note that the representation is unifor and  $T$  in (8) is constant.

Next consider the actuator's force equations. For symmetric case, (9) is reduced to

$$F_a = P_1 A_1 - P_2 A_2 - M_s g = P_L A_1 - M_s g \quad (9)$$

For our asymmetric cylinder, combing (3), (6-7), (8).

$$\dot{F}_a = Q(u, y, P_1, P_2)u - T(y)\dot{y} \quad (10)$$

where

$$Q(u, y, p_1, p_2) = K_{eq} \beta \left( \frac{\sqrt{\Delta p_1}}{\lambda_1(y)} + \frac{\sqrt{\Delta p_2}}{\lambda_2(y)} \right) \quad (11)$$

$$T(y) = \beta \left( \frac{A_1}{\lambda_1(y)} + \frac{A_2}{\lambda_2(y)} \right) \quad (12)$$

Note that both the pressure,  $P_1$  and  $P_2$ , and piston displacement,  $y$ , are needed whie the difference of the pressures is needed only in (8).

## 3. Controller Design

### 3.1 Inner Loop Control: Feedback linearization

The purpose of inner loop control is to provide corresponding voltage,  $u$ , to the servo valve in (10) making the real force,  $F_a$ , follow the desired force,  $F_d$ . In [2],  $u$  is simply a constant gain times the force error ( $F_d - F_a$ ). In this manner they suffer from the nonlinear, time-varying features of  $Q$  and  $T$ , and without a huge gain the "lock-up" phenomena occurs.

We apply the well-known feedback linearization method for the inner loop force control as in [1], [3]. But our dynamic model equation is (10) representing an asymmetric features well. Provided that  $Q$  in (10) is invertible, then defining the control input  $u$  such that

$$u = Q^{-1}(u, y, p_1, p_2) (\dot{v} + T(y)\dot{y}) \quad (13)$$

results in the linearized system

$$\dot{F}_a = v \quad (14)$$

where  $v$  is the new control input given by the outer

loop. The only case  $Q$  is not invertible is when  $y=0$  or  $y=L$ , that is, the rod reaches its maximum stroke. But if we rule out this undesirable and rare situation with a very rough road,  $Q$  is always invertible.

$T(y)$  in (13) can be viewed as a feedforward compensator using the measured disturbance  $\dot{y}$ .

### 3.2 Outer Loop Control: Measurement Feedback

Outer loop control decides what is the most profitable force to achieve the best performance against the road roughness. To evaluate and compare the performances, quadratic criterion functions are usually used and the relative weights of each terms compromise the competing goals. Since many important results are available already, we review some respects of performance goal and give our heuristic but quite meaningful output feedback controller.

As for the ride quality, net force  $F_a$  by the actuator should be minimized. Ideally if  $F_a$  equals zero, the passenger will not feel any force. But this does not imply  $u$  equals zero because  $\dot{y}$  is not zero. In this case since  $F_a$  equals zero, we have no way to control the unsprung motion, road holding is impossible. As for road holding, wheel-hop occurs if the road to unsprung transmissibility is greater than 1.5. So the our goal for the road holding is sustaining its maximum less than 1.5. Suspension space should be always less than the maximum stroke according to our assumption in the previous section. For the various operating conditions of a car, proper suspension level should be maintained (levelling control). We assume the normal position is half the the maximum stroke( $L$ ) and the goal is to maintain it within  $\pm L/2$ .

Based on the measured output our desired force is

$$F_d = -k_1 \dot{z}_s - k_2 \dot{z}_u - k_3 (z_s - z_u) \quad (15)$$

wher  $k_1$  and  $k_2$  can be viewed as the terms providing damping forces to the sprung and unsprung mass, respectively.  $k_3$  looks like a suspension spring and provides levelling control. If  $k_1 = -k_2$ ,  $F_d$  is equal to the force of a passive suspension, but choosing  $k_1$  and  $k_2$  differently we can control each body motion respectively. So  $k_1$  looks like a so-called sky-hook damper.

Finally new control input  $v$  in (14) is given by

$$v = k_c (F_d - F_a) \quad (16)$$

resulting in

$$\dot{F}_a = k_c (F_d - F_a) \quad (17)$$

The relationship between the real and the desired forces is

$$F_a = \frac{k_c}{s + k_c} F_d \quad (18)$$

Combining the quarter car equation (1), (2) we get

$$M_s \ddot{z}_s = \frac{k_c}{s + k_c} F_d \quad (19)$$

$$M_u \ddot{z}_u = -\frac{k_c}{s + k_c} F_d - K_t (z_u - z_r) - B_t (\dot{z}_u - \dot{z}_r) \quad (20)$$

So if we choose  $k_c$  sufficiently large the actuator can provide the desired force  $F_d$ . And  $k_c$  can determine the bandwidth of the inner loop force control.

The overall control block diagram including the internal and the external loop is given in Fig. 3.

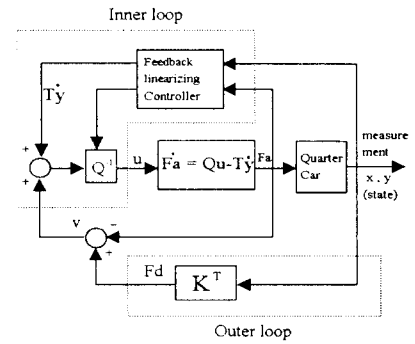


Fig. 3. Block diagram of controller

## 4. Simulation Results

Using the control laws acquired in the last section, we perform simulation study for a step-up road disturbance input. We compare the results of a passive suspension, conventional force control, and feedback linearization based force control in time domain. Passive suspension has fixed spring and damper rates. In conventional force control, servo valve input  $u$  is directly proportional to the force error.

$$u = k_c (F_d - F_a) \quad (21)$$

leaving the velocity term as a disturbance input. For both force control schemes we do not find outer loop controller parameters analytically rather we tune the parameters heuristically until we get satisfactory results in view of ride comfort and road holding.

Step or impulse response of a system shows its fundamental characteristics well and from it we can evaluate the performance of a proposed control law. This is also true for car suspensions and we see a lot of step-like road shapes around us. In this simulation it is assumed that a car is moving forward at a constant speed and meeting a step-up road of height 10mm at time zero. We can see in Fig. 4. that the sprung masses transit smoothly for the both kinds of

active suspensions while the duration of fluctuation is long for the passive suspension. All the unsprung masses track the road shape quickly within 0.1 sec. As for the sprung mass acceleration, the indicator of ride quality, rms values of active suspensions are reduced but the peak values are slightly larger than that of passive one(Fig. 5.). It is not easy to decide which is better. For the internal loop control of the active suspensions, the desired force and the actual force of the actuator coincide well for the proposed feedback linearization method while those of the conventional force feedback are totally different due to the nonzero velocity term in (11)(Fig. 6.). It is interesting to observe that for both cases the input wave forms look quite similar as in Fig. 7 Maybe it is because tune the outer loop controller until we get the acceptable results. This means that the conventional force feedback scheme can't guarantee the actual force follows the desired force in the presence of velocity term in (11) and tuning the controller gain is not easy. And trying to increase the proportional gain  $k_c$  in (22) can lead to the servo valve input saturation.

## 5. Concluding Remarks

We adopt a dynamic model of asymmetric cylinders for the actuator of active suspensions and compare with that of symmetric cylinders. Since they behave quite differently, it is indicated that the use of a symmetric model of an asymmetric cylinder do not agree with the real system. The proposed feedback linearization controller appears to follow the calculated desired force well, while the conventional force feedback do not.

Currently we are planning to implement the control law into our test rig and prove its effectiveness. Since feedback linearization method needs the true parameter values, we consider using a parameter adaptation or a robust control methods. And a more systematic way of outer loop design should be considered.

## Reference

- [1] A. Alleyne and J.K. Hedrick, "Nonlinear Adaptive Control of Active Suspensions", *IEEE Trans. on Control System Technology*, vol. 3, pp. 94-101, 1995
- [2] A.G. Thompson and P.M. Chaplin, "Force Control in Electrohydraulic Active Suspensions", *Vehicle System Dynamics*, vol. 25, pp.185-202, 1996
- [3] G. Vossoughi and M. Donath, "Dynamic Feedback

- Linearization for Electrohydraulically Actuated Control Systems", *Trans. of the ASME*, vol.117, pp. 468-477, 1995
- [4] T.J. Viersma, *Analysis, Synthesis and Design of Hydraulic Servosystems and Pipelines*, Elsevier Scientific Publishing Company, Amsterdam, 1980

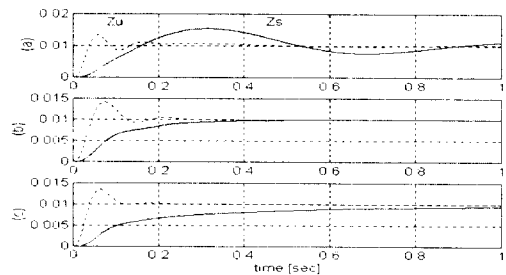


Fig. 4 sprung(-) and unsprung displacement(○)[m] (a) passive suspension (b) conventional force feedback (c) feedback linearization

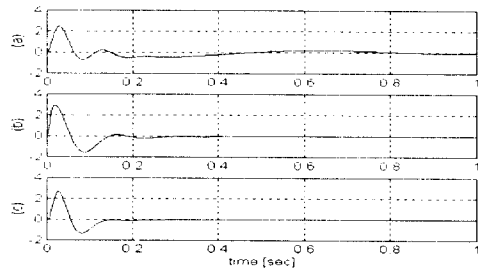


Fig. 5 sprung mass acceleration[m/sec<sup>2</sup>] (a) passive suspension (b) conventional force feedback (c) feedback linearization

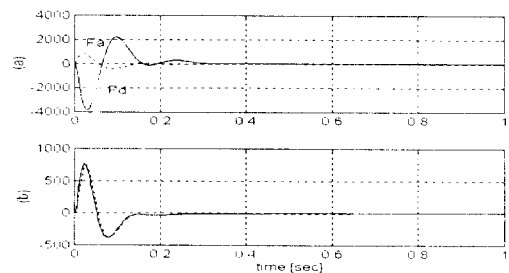


Fig. 6 desired(-) vs. actual force(○)[N] (a) conventional force feedback (b) feedback linearization

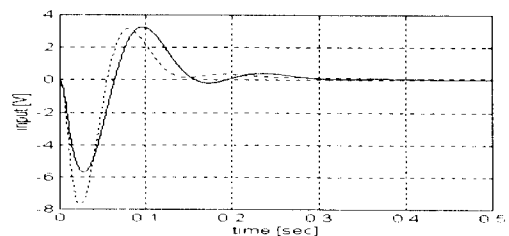


Fig. 7 Servo valve input  $u$ : conventional force feedback(-), feedback linearization(○)