

## 비선형 PI 제어기의 최적 조율법

### Optimal Tuning Method for Nonlinear PI Controllers

이동권, 박철규, 이문용

영남대학교 화학공학부 (Tel: 810-2586; Fax: 814-8790; E-mail: mynlee@yucc.yeungnam.ac.kr)

**Abstracts** : Nonlinear PID controllers have increasingly used in current industrial practice because it is robust and is easy to operate. Little guideline and tuning method, however, has been recommended for the nonlinear PID controllers while a lot of result is available for the linear PID controllers. Application guideline and tuning formulae are presented for error square type nonlinear controllers, which are most popular currently, are presented.

**Keywords** : Nonlinear PID Controller, Optimal Tuning, Error square Controller, On-line identification

#### 1. Introduction

Most industrial processes are controlled using proportional-integral-derivative (PID). The popularity of PID controllers can be attributed to their robust performance in a wide range of operating conditions and to their functional simplicity, which allows process engineers to operate them in a simple and straightforward manner. To implement such a controller, three parameters must be determined for the given process: proportional gain, integral time constant, and derivative time constant. For this reason, extensive research have been done in relation to overcome this problem. The most well-known method is that of Ziegler and Nichols (1953). Their method determines the parameters by observing the gain at which the plant becomes oscillatory and the frequency. In Cohen and Coon (1953) reported design relations that were developed empirically to provide closed-loop responses with a decay ratio of 1/4. A more useful extension of the method allows the determination of the parameters from the observation of the open-loop response of the plant to a step input change. In Lopez et al.(1967) are develop controller design relations based on a performance index that considers the entire closed-loop response. The IMC-PID tuning method (Rivera et al.(1986), Morari et al.(1989)) and the direct synthesis method (Smith et al.(1975)) are the typical ones based on the criterion to keep the controlled variable closed to the desired closed-loop response. Sometimes, a process is too nonlinear to be dealt with the PID controller. In order to overcome this problem, many attempts have been done to alter the nonlinear behavior of a system to favor one operating objective or condition over another. Its purpose is to linearize a nonlinear loop by incorporating nonlinear action to linear PID controllers, so called the nonlinear PID controllers. However, the nonlinear controllers are often used even linear processes in order to improve control performance of the

system. Also, the development in the design and implementation of control algorithm makes it possible to implement the nonlinear control algorithm very easy. As a result, the number of the processes in which nonlinear PID controllers are introduced have been rapidly increased. The nonlinear PID controllers are widely used in actual fields because they show high control performance while simplicity in the algorithms remains. However, in the case of the nonlinear PID controller, the tuning and analysis of the controller becomes much more difficult compared with that of the linear PID controller because the optimal tuning parameter depends on the magnitude of setpoint or disturbances as well as their types. Furthermore, so far little guideline for the application of the controller is available.

In this work, we present the application guideline and tuning rules for the error square type nonlinear controllers which are currently most popular in process industries.

#### 2 Theory

##### 2.1 PID controller

The ideal PID controller algorithm is given by

$$m(t) = K_c \left[ e + \frac{1}{\tau_I} \int e dt + \tau_D \frac{de}{dt} \right] \quad (1)$$

where  $m(t)$  = the controller output

$K_c$  = the proportional gain

$\tau_I$  = the integral time constant

$\tau_D$  = the derivative time constant

$e$  = the process error

Proportional action speeds up the control response and reduces the offset. The addition of integral control action eliminates offset but tends to make the response more oscillatory. Adding derivative action reduces the degree of

oscillation and the response time. However, one could argue that for pure first order systems the derivative term is not required (Miller et al., 1967), the main reason for the avoidance of the derivative term is difficulty of practical implementation. Also the inherent noise in the process measurement is amplified by the derivative term. To alleviate of this shortcoming, filtering algorithms has been introduced. These filtering algorithms reduce not only the sensitivity of control logic to inherent noise but also the control performance. One method of dealing with this defect is to remove the derivative term, but compensate for the loss of the responsiveness by using the controller to increase its proportional action according to the magnitude of the error  $e(t)$ . Clark proposed replacing  $K_c$  by

$$K_c = K(1 + \beta |e|) \quad (2)$$

Structure 1

$$m(t) = K(1 + \beta |e|) \left[ e + \frac{1}{\tau_I} \int e dt \right] \quad (3)$$

This type controller would permit us to use a low value of gain so that the system is stable near the setpoint over a broad range of operating levels with changing process gains. Another advantage of this kind of nonlinear controller is that error square at setpoint reduces the effects of noise. This error square controller is suitable for average level control because the deviation never comes to rest at zero in it. However it might not be reasonable error-square controller whose integral time constant changes with deviation. Therefore, the controller can be modified as

Structure 2

$$m(t) = K(1 + \beta |e|) e + \frac{1}{\tau_I} \int e dt \quad (4)$$

The above controller can be intended to maintain a constant damping factor.

### 2.2 Error square controller tuning

In order to develop tuning formulae for these two algorithms, a similar approach to Lopez et al. (1967) was used. They proposed that the simplest process model that can be reliably determined from experimental identification tests on a process is a first order plus deadtime :

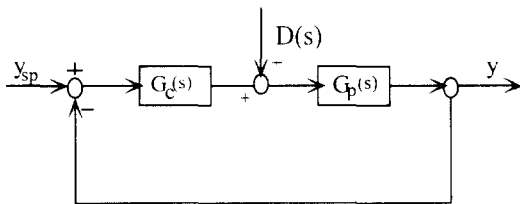


Figure 1. Block diagram for feedback control system

$$G_p(s) = \frac{K e^{-\theta s}}{\tau s + 1} \quad (5)$$

where  $s$  : the Laplace variable

$K$  : the process gain

$\tau$  : the time constant

$\theta$  : the deadtime

They considered the control loop shown in Figure 1. The disturbance  $D(s)$  enters directly into the process  $G(s)$ . However, the controller  $G_c(s)$  is two kinds of error square controller and thus refers qualitatively to the transfer function relating the controller output  $m(s)$  to the error  $e(s)$ . For a given process  $(\theta/\tau)$  is fixed. We seek correlation among  $(KK_c)$ ,  $(\tau_I/\tau)$ , and  $(\beta \times y_{sp})$ , for various values of  $(\theta/\tau)$  which occur for optimal tuning. Miller et al.'s (1967) recommendation of using ITAE optimal tuning criteria was followed here, where

$$\min ITAE = \int t |e(t)| dt \quad (6)$$

A unit step change for load or set point was introduced and their corresponding value for ITAE calculated. This criterion penalizes errors that persist for long period of time while intuitively we know early errors are inevitable. As a result, this criteria demand as little oscillation as possible. The simulation was combined with a simplex search optimization algorithm, using random initial guesses.

## 3. ITAE Tuning for Nonlinear PI Controller

### 3.1 Load change

The optimal tuning formulae were obtained using linear regression of the dimensionless groups  $(KK_c)$ ,  $(\tau_I/\tau)$ , and  $(\beta \times D_{max})$ . The nonlinear PI tuning parameters are very well described by the following equations.

Structure 1 :

$$KK_c = 0.152 (\theta/\tau)^{-1.11} \quad (7)$$

$$\tau_I/\tau = 1.096 (\theta/\tau)^{-0.4833} \quad (8)$$

$$\beta \times D_{max} = 5.335 (\theta/\tau)^{-0.4013} \quad (9)$$

Structure 2 :

$$KK_c = 0.1682 (\theta/\tau)^{-1.164} \quad (10)$$

$$\tau_I/\tau = 1.49 (\theta/\tau)^{1.308} \quad (11)$$

$$\beta \times D_{max} = -6.164 (\theta/\tau) + 10.61 \quad (12)$$

A significant danger in the nonlinear controller is that the controller will be tuned only under conditions of small deviations, in which case the loop may not be stable for large upsets. Conceivably, a large disturbance could produce a deviation large enough to cause the loop gain to exceed 1. This begin an expanding cycle that could continue to expand because of increasing controller gain until output limits are reached or some damage is done to the process.

We can see the error square controller has serious problem in  $(\beta \times D_{max})$ . Since it is hard to know the magnitude of the

disturbance exactly, we should clearly understand the system or retune the parameter  $\beta$ . It is feasible that this type of controllers have several near optimal point.

### 3.2 Set point changes

A similar approach was used for set point tuning formulae.

Structure 1 :

$$KK_c = 0.02586 + 0.0982 (\theta/\tau)^{-1.049} \quad (13)$$

$$\tau_I/\tau = 0.997 + 0.08 (\theta/\tau)^{2.351} \quad (14)$$

$$\beta \times y_{sp} = 4.259 + 0.5362 (\theta/\tau)^{0.667} \quad (15)$$

Structure 2 :

$$KK_c = 0.1245 (\theta/\tau)^{-0.965} \quad (16)$$

$$\tau_I/\tau = 1.57 (\theta/\tau) + 0.09 \quad (17)$$

$$\beta \times y_{sp} = 1.05 (\theta/\tau)^{0.763} + 4.12 \quad (18)$$

## 4. Simulation Results

Several different sets of responses were simulated for both disturbance and set point changes. Figure 2 and 3 show the control results which is the comparison of the ITAE-PI and nonlinear PI. It can be seen from these figures that the nonlinear PI controllers use more proportional action and relatively less integral action than ITAE-PI and nonlinear PI was always able to do better control performance than the ITAE-PI controller.

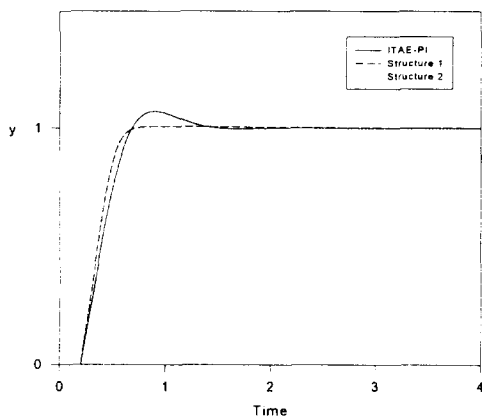


Figure 2 Set point change for  $(\theta/\tau=0.2)$

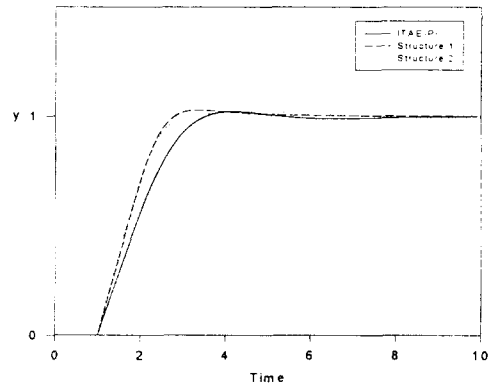


Figure 3 Set point change for  $(\theta/\tau=1.0)$

Consider the process  $G_p(s) = \frac{e^{-3s}}{(s+1)^2(2s+1)}$ . The process model is obtained  $G_m(s) = \frac{e^{-4.48s}}{2.95s+1}$  based on the on-line identification method by Chen (1989).

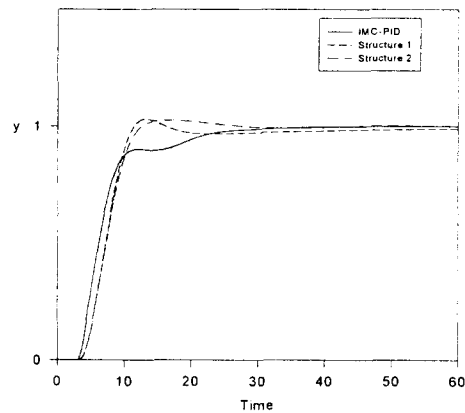


Figure 4. Autotuning result

Figure 4 compares the responses of the PID controller tuned by the Internal Model Control (IMC) tuning rule and the nonlinear PI controller tuned by the proposed tuning rule. The nonlinear PI controller shows similar control performance to that of the PID controller.

## Acknowledgement

Financial support from the Korea Science and Engineering Foundation 1996 core research grant is gratefully acknowledged. We also acknowledge the Automation Research Center at Postech.

## References

- [1] Arthur Jutan, "Anonlinear PI(D) Controller", *The Can. J. of Chem. Eng.*, vol 67, 485-493, 1989
- [2] Chen C., "A Simple Method for On-line Identification and Controller Tuning", *AIChE J.*, 35, 2037, 1989
- [3] Rivera D. E., Morari M., and Skogestad, "Internal Model Control : 4. PID controller design", *Industrial Engineering Chemical process Design and Development*, 25, 252, 1986
- [4] Lopez, A. M., J. A. Miller, C. L. Smith, and P. W. Murrill, "Tuning Controllers with Error Integral Criteria", *Instrumentation Techonology*, 14(12), 57-62, 1967
- [5] Cohen G. H. and Coon G. A., " Theoretical Considerations of Retarded Control", *Trans. ASME*, 75,827, 1953
- [6] Ziegler J. G. and Nichols N. B., "Optimum Setting for Automatic Controllers", *Translation ASME*, 64, 759, 1942

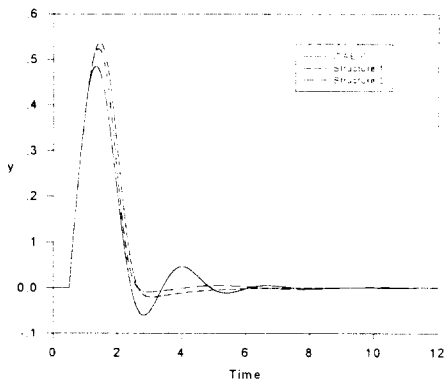


Figure 5 Load change for  $(\theta/\tau=0.5)$

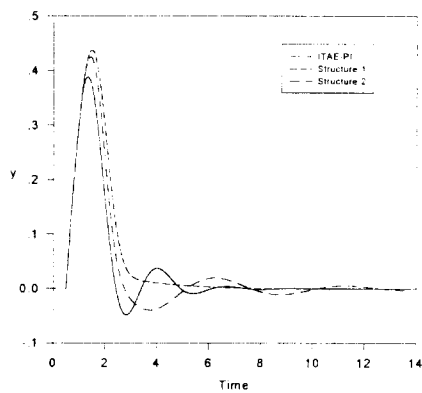


Figure 6 Load change for  $(\theta/\tau=0.5)$  in 20% mismatches

Figure 5 and 6 compare the output responses to step disturbance of the ITAE-PI controller and the nonlinear PI controller. The maximum peak of the nonlinear PI controller is higher than that of the ITAE-PI controller. This cause closed-loop unstable at that point. In figure 6, it is assumed that the error of  $D_{\max}$  is 20%.

## 5. Conclusions

We have presented the application guideline and tuning formulae based on ITAE criterion for error square type nonlinear PI controller. The nonlinear PI controller with the tuning rule provides a better control performance for setpoint change. However, ITAE tuning criteria for the nonlinear PI controller seems to be not good in disturbance rejection.