Optimal Pricing under Uncertain Product Lifetime Conditions and A Simulation Study

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Introduction

Optimal pricing research in general has been focused on profit maximizing strategy under the given product life-time T. Here we have tried to study the effect of uncertain product life-time on dynamic optimal pricing strategy. In reality, the life-time of product is more likely to be uncertain and not known as well. In terms of approximating the model to the concerned reality, so-called model validity, it seems to be more desirable to consider the uncertainty of product life-time into the optimal pricing strategy model.

For this purpose, we tried two different approaches. One is to consider diverse product life-time probability functions under fixed life-time T. In this case, we might have the same product life-time as the previous study, but the process could be different in the expectation of product's discontinuity. The other is that life-time itself is not determined and thus it is the situation in which we can only decide optimal price on incremental basis. The former is the situation in which although we got some strong guess on life-time of a certain product, the pattern of expected life-time probability could be different. The question is what could be optimal pricing strategies on such different product life-time situations. But since in the latter, we don't assume any idea on the life-time of product, proper optimal pricing could be derived only from the past prices and diffusion information. While the latter seems to be safer in the aspect of model assumption, the former could be more realistic because we might have more or less *a prior* knowledge on the product life-time itself.

Background and Process

The paper starts with mathematical approach to build the optimal pricing model under general life-time function. The goal is to construct dynamic optimal pricing model, in which optimal pricing maximizes total discounted profit throughout life-time of the product. This work occupies the first part of the paper. In this part, we begin with a simple profit maximization function.

$$Max_{p(t)}\pi(t) = \int_0^T e^{-rt}\left[p(t) - c(x(t))\right]s(t)\,dt$$

$$s.t.\ x(t) = s(t) = f(x(t), p(t));\ x(0) = x_0.$$

where s(t) incremental sales could be given in a more concrete form.

Now we include the product lifetime probability function into the above model.

$$Max_{p(t)}\pi(t) = Max_{p(t)}\int_0^T\left[1 - F(t)\right]e^{rt}\,(p(t) - c(x(t)))\,s(t)\,dt$$

$$s.t.\ x(t) = s(t) = f(x(t), p(t));\ x(0) = x_0\,.$$

Using Hamiltonian equation with a concept of shadow price, we get the dynamic optimal pricing equation in a integration format as follows

$$H(p, x, \lambda; t) = e^{rt}\,[1 - F(t)][(p(t) - c(x(t)) + \lambda(t)]f(x(t), p(t))$$

$$\frac{\partial H}{\partial P} = 0 \Rightarrow p(t) = c(x(t)) - \lambda(t) - \frac{f(x, p)}{\partial f(x, p)/\partial p(t)}$$

$$= \frac{\eta}{\eta - 1}[c(x(t)) - \lambda(t)].$$

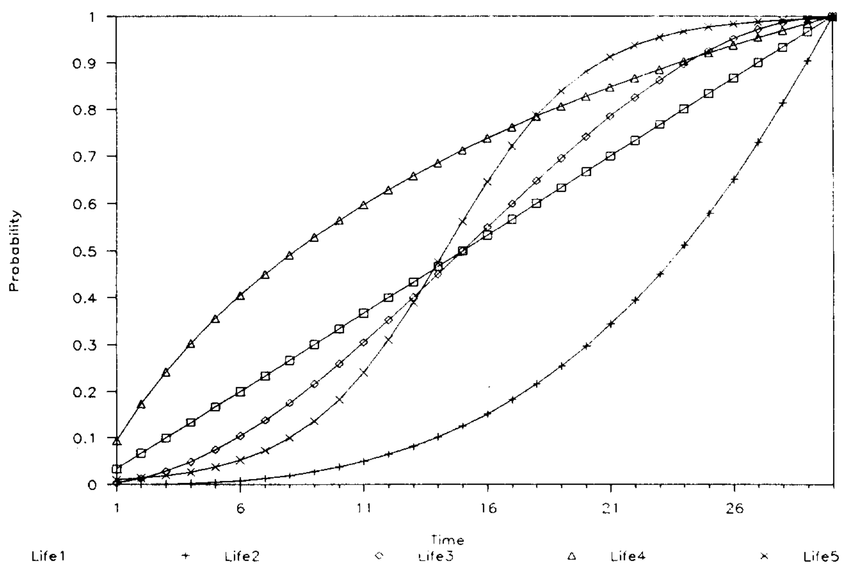
$$p^*(t) = \frac{\eta}{\eta - 1}\left[c(x(t)) - e^{\int_t^T\left(r + \frac{F'(v)}{1 - F(v)}\right)dv}\int_t^T\left(c'f + \frac{ff_x}{f_p}\right) - e^{-\int_t^T\left(r + \frac{F'(v)}{1 - F(v)}\right)dv}\right]$$

where eta is elasticity of demand and C is constant.

Although we have the solution of optimal pricing, we didn't have quite solution under specific situation such as under specific sales function s(t) or product life-time cumulative probability function F(t).

The second part is assigned to compare concrete optimal pricing strategies with different life-time functions and hopefully to get some generalized results. For the latter, we approach to simulation study with specific model parameters to get concrete outputs. The purpose of simulation study is to understand time-transitional dynamics under more concrete situations in our optimal pricing model. Especially to compare diverse functional situations, we need to substitute our general function s(t) and F(t) into specific functions with numeric parameters.

Graph 1 : Probabilistic Product Life Functions



To compare different product life-time situations, first we chose five different cumulative life-time probability functions, as shown in the <Graph 1>. Each represents different product life-time situation. However, 1), 2) and 4) construct one group and 3), 5) do the other. They can be classified according to the consistency of sales trend. More specifically, 1) shows the case of constant expected probability for product discontinuity, 4) that of initially rapid die-away product life-time, and 5) that of initially slow but at later stage rapidly increasing of product's discontinuity. Also 3) is a typical logistic function, such that the incremental sales only depends on remaining market size, M-x(Market saturation-cumulative sales). In fact, 3) and 5) are similar situations in which they shows transitional trend of the sales (first increasing and then later decreasing), except the difference of its degree.

Simulation Study

In the simulation approach like as the previous model approach, we began with sales function f(P, x; t), a dynamic function of price P and cumulative sales x. As mentioned in the model approach, we have four different sales situations; where it depends 1) only on price, 2) only on cumulative sales (diffusion effect), 3) multiplicatively on price and diffusion and 4) combinedly on both. In fact, no pricing effect case such as 2) is not necessary in optimal pricing study and only price dependent such as 1) can be included in more general 3) or 4). For analytic purpose of our simulation study, the third situation where price and cumulative sales are multiplicatively dependent on the incremental sales will be mainly reviewed, only distinctions will be indicated in the combined case.

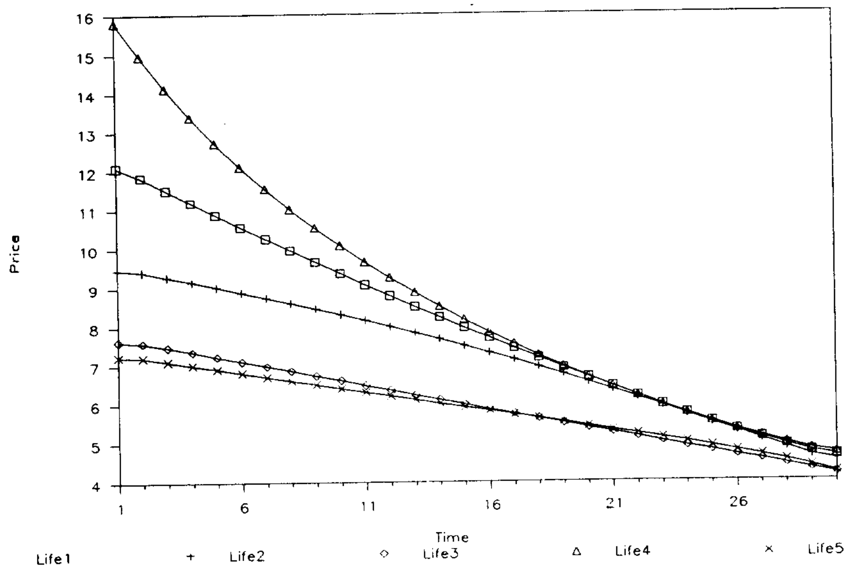
Once we have sales function s(t) at time t, its cumulative becomes x(t). Since the cost is assumed to depend only on the cumulative sales (learning effects only), the incremental sales also determines the cost. For the simulation purpose, we first chose arbitrary initial sales x0 and initial price P0, which determine the first-stage cumulative sales x1 and optimal price P1. They, however, can be different according to different product life-time situations. Those again become the inputs to determine optimal pricing and sales amount at the second stage. And this process goes on and on until the end of life-time of the product to construct dynamic model simulation.

The sales function in the model includes diffusion effect, which is summarized as such way that current sales depends on the previous cumulative sales. Thus if the past cumulative sales positively affects the present incremental sales, we call the product has a positive diffusion effect and vice versa. Our simulation study focus on the comparison of optimal pricing strategies in different diffusion situations, in which different product life-time expectations cases are considered.

The followings are some noticeable results derived from the model simulation. First of all, there is a distinctive group difference on optimal pricing strategy due to the different type of product's life-time probability functions. The first group is classified with the cases that the incremental sales, as a derivative of cumulative sales, shows the same direction(say, consistent) as being increasing or decreasing. The second are the cases that show transitional change in sales trend, exemplified as that of logistic function. While within each group we could compare the effect of different product life-time expectation, it might not be quite intuitive to compare between situations in different groups. Under each group of the product life-time functions, we could get some meaningful results as follows.

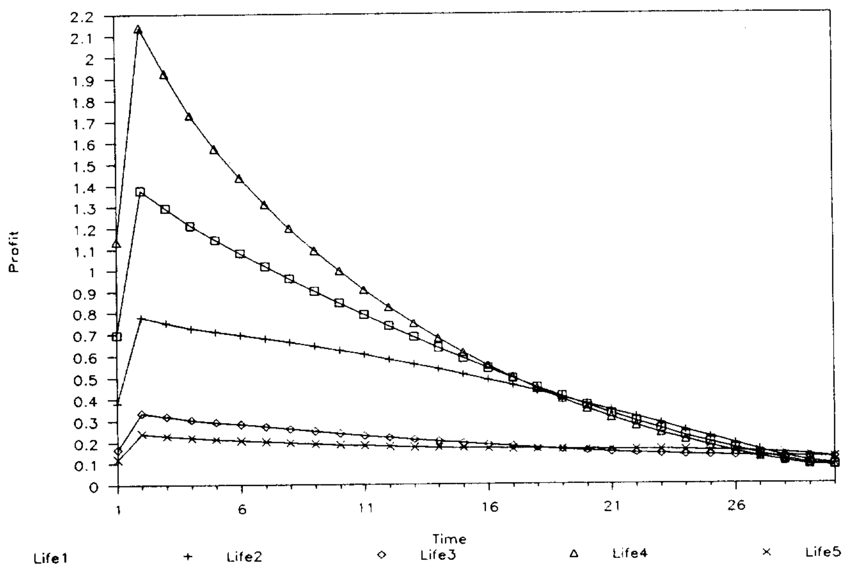
As mentioned earlier, let us focus on the case that the current sales depends only on price and cumulative sales in a multiplicative functional way, f(P, x; t)= g(P(t))*h(x(t)). For the purpose of comparison, we begin with no diffusion situation, where the optimal pricing strategies on five different product life-time probability functions are representatively shown in <Graph 2> says first that there are group difference between different types of product life-time functions.

Graph 2 : Optimal Pricing (Sales1-Diffusion2)



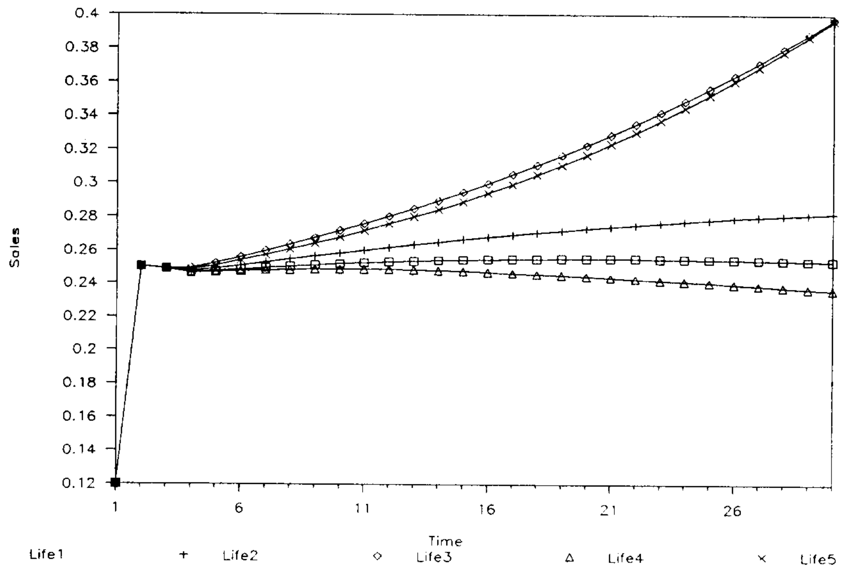
More specifically, the consistent sales trend group (consistent group) shows higher pricing strategies compared to that of the transitional sales trend group (transitional group). In the former, the higher discontinuity probability of the product at the initial state leads to the higher optimal pricing strategy, although it shows eventual convergence to the cost level.

Graph 3 : Discounted Profit(Sales1-Diffusion2)



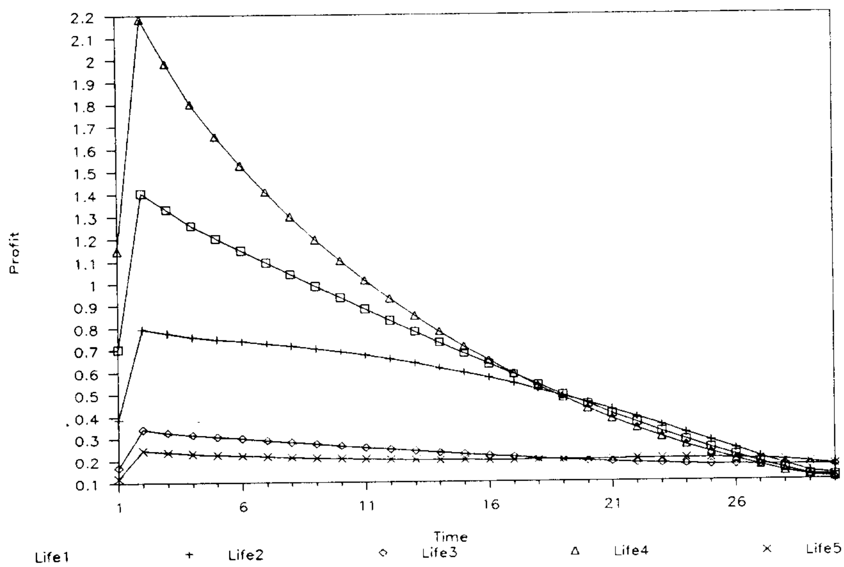
However, while in the incremental sales, the opposite trends effect to the pricing is shown in the later stage in the consistent sales trend group, in the transitional sales trend group, the order is not always kept in reverse way.

Graph 4 : Incremental Sales (Sales1-Diffusion2)



In the transitional group, the incremental sales get distinctively increased compared to the consistent, but the less drastic transitional sales trend situation such as 3) shows rather higher sales amount at least in the initial stage. In the transitional case, we can see that the less transitional life-time situation can lead to the higher pricing strategy with higher incremental sales, which result more gap in profit between them.

Graph 5 : Profit (Sales1-Diffusion2)



Although shows the reversal of profit trend in the later stage, the model constraint that cost decreases with increased cumulative sales (due to learning effect) and price converges to the cost at the later stage restrict distinctive difference.

Pricing strategy directly relates to the corresponding cumulative sales. In general, initial higher price leads to the less sales in the later stage during product life-time. The total discounted profit shows that the first (consistent) group has a distinctive advantage compared to that of the second (transitional). Its gap still remains, although not expanding.

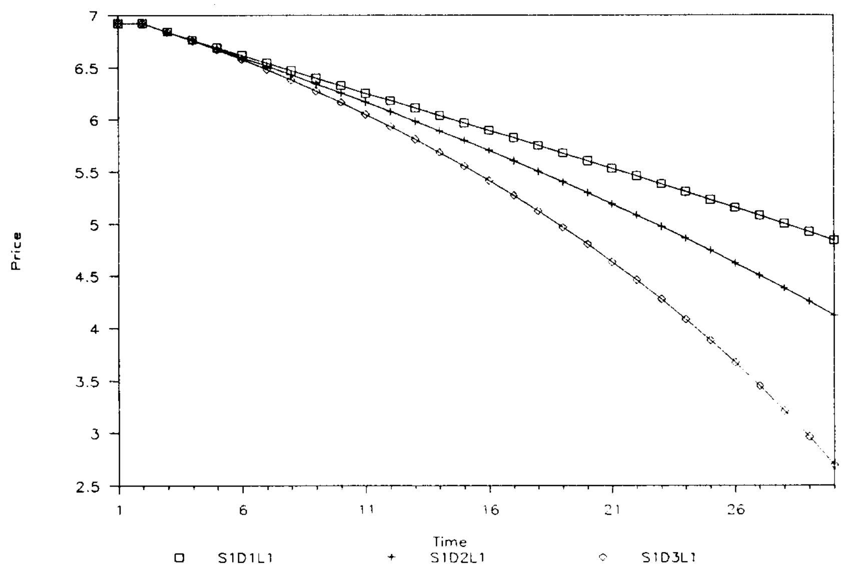
Under negative diffusion situation (in which the more past sales leads to the less current sales), the optimal pricing strategy shows quite different trend that price increasing in the initial stage and then consistent price decreasing, regardless of different life-time functions. In this situation, the higher possibility of product's discontinuity in the initial stage leads to adopt the higher pricing strategy in the initial. But in the transitional group, the case of the more distinctive transitional sales shows the higher initial pricing strategy. Also the initial higher pricing strategy always leads to lower pricing as the time goes. It means the if the initial pricing is relatively higher, it should experience lower pricing at the later stage, due to its negative diffusion effect.

In the incremental sales under negative diffusion, first the consistent group shows the higher sales amount and also the initial higher death probability leads to more sales at the initial stage but its trend gets reversed at the later stage although difference does not get quite distinctive. Here we can temporary conclude that in the profit sense, 1) the consistent sales trend products, 2) the higher probability of product discontinuity at the initial-stage, and also 3) the drastic sales transitional product lead to the higher initial profit and also the eventual larger profit although all the profit relatively converges at later stage including reverse cases. Also in the very beginning stage, the price can be lower than the cost is and in general the logistic product life-time case, such trend can be relatively continued for a while at the initial stage. In the cumulative sales, the consistent group and also the initial-stage higher discontinuity probability product show the higher sales trend.

In the positive diffusion case, the general pricing strategy is almost similar to that of no diffusion case (although it is possible that positive diffusion effect could not be exaggerated enough). But while in the consistent group, there is no reversal trend within life-time T=30 unlike no diffusion case, in the transitional group they have the same result. In general, positive diffusion affects to only the scale. In other words, it has the same but expanded trend. However, in the transitional group, the difference gap gets more distinctive. Optimal pricing in this case shows the consistent decrease with the same trend compared to the negative diffusion but in the transitional group high pricing leads to the higher sales when the transition is less dramatic. In sum, in the consistent group, the effect of product life-time probability is quite distinctive, but in the transitional group, the diffusion offset the life-time effect and reverses the pricing trend.

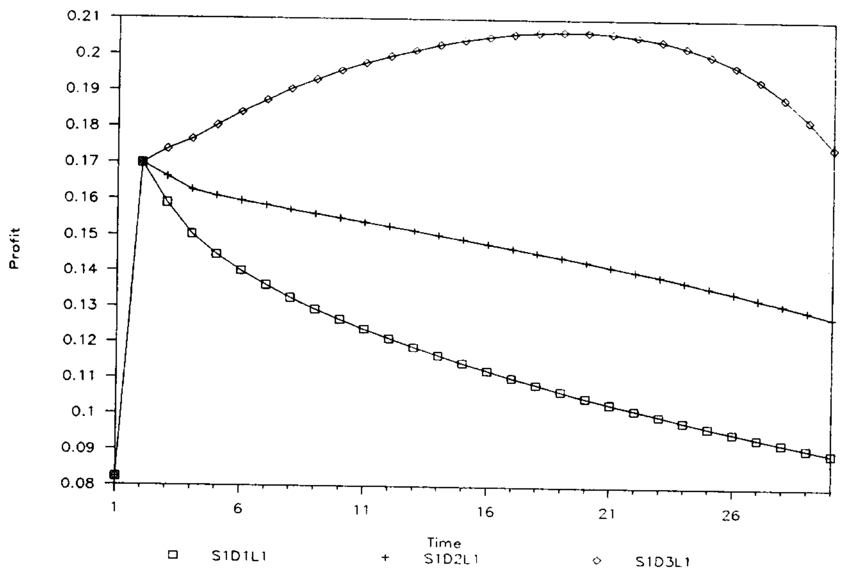
For the second approach in uncertain product life-time situation, we assume the situation in which we have no idea of the product life-time at all. In such case, we can still derive optimal pricing of maximizing discount profit only by using the past price information. Under the constant life-time probability situation, while all the pricing strategies show the initial higher pricing, the negativity of diffusion rather seems to imply the higher initial pricing strategy

Graph 6 : Optimal Pricing (Not-Know T Sales1)



As expected, the incremental sales shows the exact reverse trend compared to the pricing, such that positive diffusion shows almost exponential increase of sales. Even in negative diffusion, the incremental sales shows stable trend. While the cost trend directly reflects such cumulative sales effect, the profit trend reflects such combined effect between unit revenue (price-cost) and incremental sales f(P, x; t). Thus while in no diffusion case, the profit shows quite stable trend, negative diffusion affects the profit into the decreasing trend and the positive one does into the increasing .

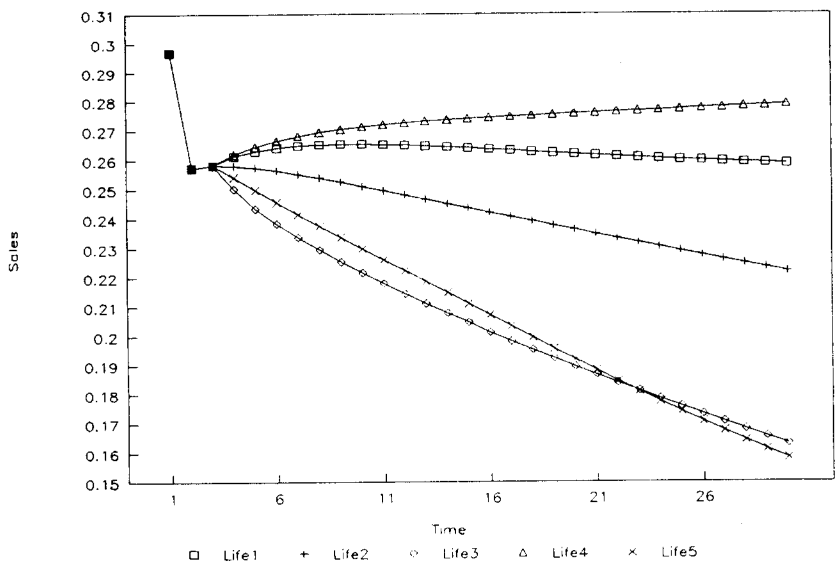
Graph 7 : Discount Profit (Not-Know T Sales1)



Even in the positive case, however, the profit does not explode (it rather has maximum before life-time T).

Now move to the case of combined sales function, for example f(p, x;t) = $\alpha x^{\beta p^{\gamma}}$

Graph 8 : Incremental Sales (Sales2-Diffusion1)



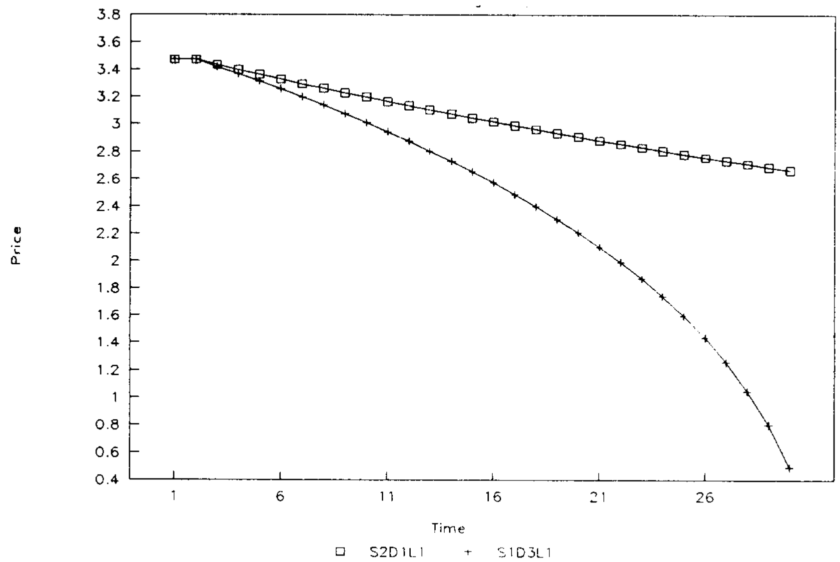
In the negative diffusion situation with the above sales function, the optimal pricing eventually converge to the cost level regardless of diverse product life-time expectation. However, in the consistent group the pricing order remains the same, it is reversed in the transitional group. Also the effect of the combined sales function could change the incremental sales from the convergence to the divergence among the different product life-time probability functions.

In result, as the price effect gets larger but sales difference gets smaller, all the profit trends converge to the zero as the time goes. Again, the pricing position of 5) and 3) in combined sales function is changed to that 3) is larger than 5).

For the positive diffusion case of the combined sales function, pricing strategies under different life-time situations diverge in the same as that of the negative diffusion, whereas the incremental sales show the reverse pattern. In sum, we can conclude that lifetime expectation is still dominant to that of diffusion and the profit trends exactly reflect the pricing due to trivial differences of incremental sales.

Finally, when life-time is unknown under the combined sales assumption, the optimal pricing with different diffusion effects show quite distinctive trends.

Graph 9 : Optimal Pricing (Not-Know T Sales2)



Negative diffusion maintains optimal pricing higher, compared to that of positive diffusion, which leads to opposite direction of the incremental sales. Positive diffusion leads to the exponential sales increase, whereas the negative diffusion makes the incremental sales approach to trivial amount. In result, in the positive diffusion case, the profit increases up to the maximum and fall rapidly to a certain amount but in the negative case, it slowly decreases to the zero. Nevertheless, for the latter, the higher pricing strategy becomes optimal compared to that of the former.