Proceedings of the Korean Nuclear Society Spring Meeting
Cheju, Korea, May 1996

CRX-Hex: A Transport Theory Assembly Code Based on Characteristic Method for Hexagonal Geometry

Nam Zin Cho and Ser Gi Hong

Korea Advanced Institute of Science and Technology
Department of Nuclear Engineering
373-1 Kusong-dong, Yusong-gu
Taejon, Korea 305-701

Abstract - A transport theory code CRX-Hex based on characteristic methods with a general geometric tracking routine is developed for the heterogeneous hexagonal geometry. With the general geomtric tracking routine, the formulation of the characteristic method is not changed. To test the code, it was applied to two benchmark problems which consist of complex meshes and compared with other codes (HELIOS, TWOHEX).

I. Introduction

Recently, the method of characteristics[1,2,3] first proposed by Askew that combines desirable features of the integral transport and S_N method has been considered as an effective method for the cell and assembly calculations. But the method of the characteristics has not yet been applied to hexagonal geometry. In this paper, the method of characteristics is applied to hexagonal geometry. In this method, the outgoing flux is calculated by integrating the differential form of the Boltzmann equation along its characteristic lines with the incoming flux and the source for each discretized direction. Therefore, the method of characteristics has no limitation on geometry. In this aspect, this method resembles the Monte Carlo method and integral transport method. In the geometric tracking routine in CRX-Hex of this paper, the constraints in existing characteristic transport codes are almost relaxed. In most characteristic transport codes, the ray tracing is complicated due to enforcing the return of the rays on the exactly same point on reflective boundaries. But, in CRX-Hex, the reflective boundary is divided into several edges and the angular fluxes on each edge of the reflective boundary are assumed to be same. In treating the hexagonal external boundary, the formulation of the characteristic method is not changed, but the routine for the determination of starting positions of rays and the routine for the returning of rays on the reflective boundaries must be changed.

II. Theory and Methodology

In this section, the formulation of the characteristic method will be briefly reviewed. The derivation of the characteristic method starts with the differential form of the multigroup transport equations. The within group (g) equation for the discretized direction (n, m) is given as follows :

$$\sin\theta_n \frac{d\Psi^g_{m,n}}{dp} + \sigma^g_t \Psi^g_{m,n} = q^g_{m,n} \quad (1)$$

where p is the projected coordinate on $x-y$ plane of the coordinate along the neutron trajectory for the direction (m, n) and θ is the polar angle. In the above equation, m and n represent the azimuthal angle index and the polar angle index, respectively. The equation for computational mesh (i, j) with flat source approximation is simply obtained by intergating Eq.(1) :

$$\Psi^{g,out}_{i,j,m,n,l} = \Psi^{g,\mathrm{in}}_{i,j,m,n,l} e^{-\tau^g_{i,j,m,n,l}/\sin\theta_n} + \frac{q^g_{i,j,m,n}}{\sigma^g_{t,i,j}}(1 - e^{-\tau^g_{i,j,m,n,l}/\sin\theta_n}) \quad (2)$$

where l is the ray index and $\tau^g_{i,j,m,n,l}$ is the optical length.

The average angular flux for the direction (m, n) along the l'th ray is obtained by integrating Eq.(1). The equation is given as follows :

$$\overline{\Psi}^g_{i,j,m,n,l} = \frac{q^g_{i,j,m,n}}{\sigma_{t,i,j}} + \frac{\Psi^{g,\mathrm{in}}_{i,j,m,n,l} - \Psi^{g,out}_{i,j,m,n,l}}{\sigma_{t,i,j} L_{i,j,m,n,l}} \quad . \quad (3)$$

However, to perform the scattering source iteration, the average angular flux over the computational mesh is required for the generation of the source. The equation for the average flux over the computational mesh is obtained by summing the average fluxes (Eq.(3)) over rays that pass through the mesh. The equation is given by the following expression :

$$\overline{\Psi}^g_{i,j,m,n} = \frac{q^g_{i,j,m,n}}{\sigma^g_{t,i,j}} + \frac{\sin\theta_n}{A_{i,j}\sigma_{t,i,j}} \sum_{l \in ij\ cell} \delta_m (\Psi^{g,\mathrm{in}}_{i,j,m,n,l} - \Psi^{g,out}_{i,j,m,n,l}) \quad (4)$$

where $A_{i,j}$ represents the area of the (i,j) mesh and δ_m represents the spacing between adjacent rays for the m'th azimuthal direction.

In treating the hexagonal geomtry by using the method of characteristics, the above formulation is not changed. But new routines for determination of the starting positions (see Fig. 1) and the returning of rays on the reflective boundaries are required. Also, the azimuthal angular quadrature set must satisfy the 1/6 symmetry. At present, in CRX-Hex, each azimuthal angular sector is uniformly divided into discrete angles and uniform azimuthal angular weigt is used.

III. Applications and Results

For verification of the code, two benchmark problems were tested. The first is a heterogeneous hexagonal multicell problem that consists of five fuel cells and one control rod cell. The configuration is given in Fig. 2. For comparision with the HELIOS code[4], thirty-four group macroscopic cross sections were directly extracted from the cross section library of the HELIOS code and the same mesh divisions were used. The HELIOS code is an integral transport code that uses interface current coupling method with angular dependent collision probability. It is also noteworthy that the HELIOS code uses transport corrected macroscopic cross section for consideration of linearly anisotropic scattering. This treatment may in some cases lead to a negative value of the selfscattering cross section. The infinite multiplication factors are given in Table I. The control rod is replaced by a fuel rod in the second case. The second (see Fig. 3) is a homogeneous one group problem that consists of three types of homogeneous cells (water, fuel, control rod). This benchmark problem was selected for comparison with the well known TWOHEX code[5] that uses linear characteristic method. The cross sections are given in Table II and the power relative error(%) distribution is given in Fig. 4. The result of the TWOHEX code (twenty-four triangles per hexagonal cell, S_{12}) is used as reference solution. The results of CRX-Hex are obtained with six triangles per hexagonal cell and twenty-four triangles per hexagonal cell. In Fig. 3, it is noted that fine meshes are required in CRX-Hex to obtain an accurate solution comparable to the reference soution (TWOHEX solution). But this requirement in CRX-Hex can be compensated by its capability of the heterogeneous calculation.

IV. Conclusions

In this paper, a characteristic transport theory code CRX-Hex with a general geometric tracking routine for hexagonal geometry was described and tested. The code was developed to accurately analyze the heterogeneous hexagonal assembly with complicated mesh shapes. In CRX-Hex, for treatment of the reflective boundary condition, we assumed that the angular fluxes for all rays on each edge are the same with the edge average value. With this treatment of the reflective boundary, the constraints of the existing characteristic transport codes are almost relaxed in CRX. To test its accuracy, the code was applied to two benchmark problems. The infinite multiplication factors were compared with the HELIOS code for a heterogeneous problem (with slightly different cross sections). The numerical results (power distribution, multiplication factor) were also compared with the TWOHEX code for a homogeneous problem. From the numerical results, it is concluded that CRX-Hex with fine meshes gives accurate solution in comparison with TWOHEX.

Acknowledgment

The authors are grateful to Young Jin Kim of KAERI for his interest in and support to this work.

References

1. Nam Zin Cho' *et al.*, "Development of a Code System for Fuel Assembly Calculation," prepared by Korea Advanced Institute of Science and Technology for Korea Atomic Energy Research Institute, KAERI/CM-063 /94 (1995). *(in Korean)*
2. M. J. Halsall, "CACTUS - A Characteristic Solution to the Neutron Transport Equation in Complicated Geometries," AEEW-R 1291 (1980).
3. N. Z. Cho, S. G. Hong, "CRX: A Characteristic Transport Theory Code for Cell and Assembly Calculations in Reactor Core Design," Proceedings of the Korean Nuclear Society Autumn Meeting, Vol.1, pp.85-90, October 1995, Seoul.
4. Eduardo A. Villarino *et al.,* "HELIOS: Angularly Dependent Collision Probabilities", Nucl. Sci. Eng., 112 (1992) 16-31.
5. Walters W. F. *et al.,* "Users Guide for TWOHEX: A Code Package for Two-Dimensional, Neutral-Particle, Transport in Equilateral Triangular Meshes," LA-10258-M, Los Alamos National Laboratory(1989).

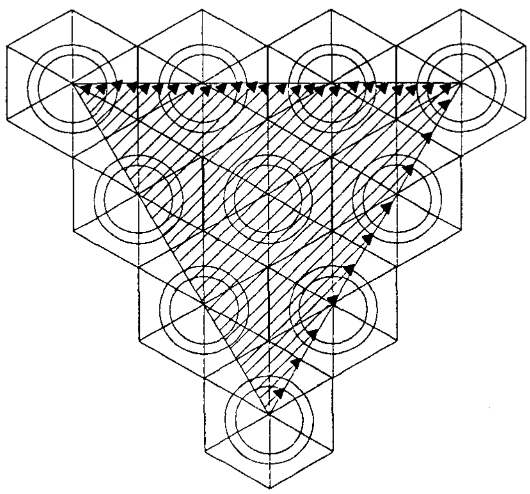


Fig. 1. Ray tracing in the CRX-Hex code

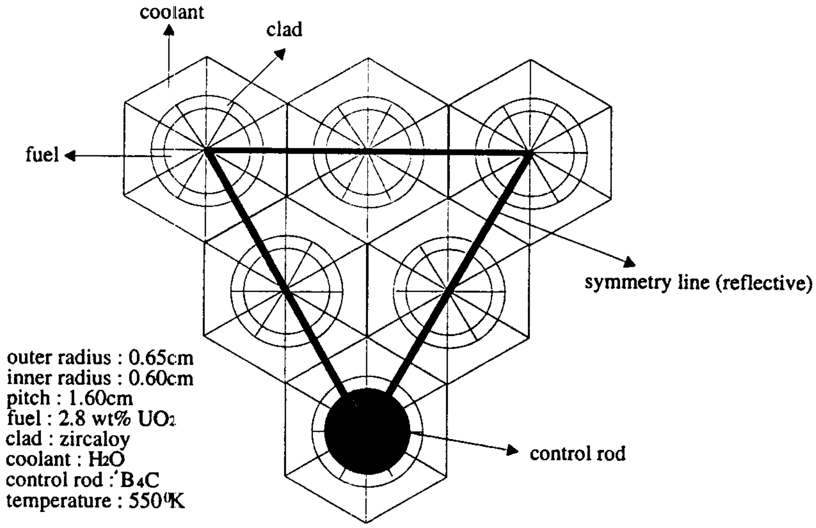


Fig. 2. Configuration for the benchmark problem I

Table I. Infinite multiplication factors for the benchmark problem I

	HELIOS	CRX-Hex[a]
case I	0.679104	0.678090
case II	1.097121	1.107400

[a]number of polar angles: 2
number of azimuthal angles for one sector : 2
number of rays : 220

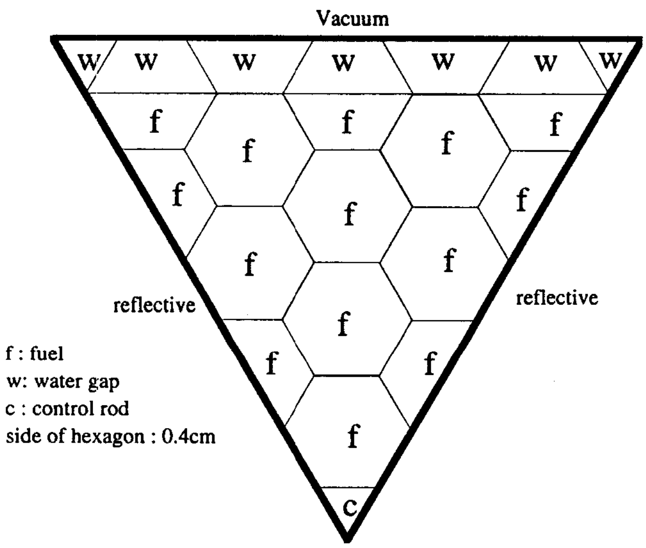
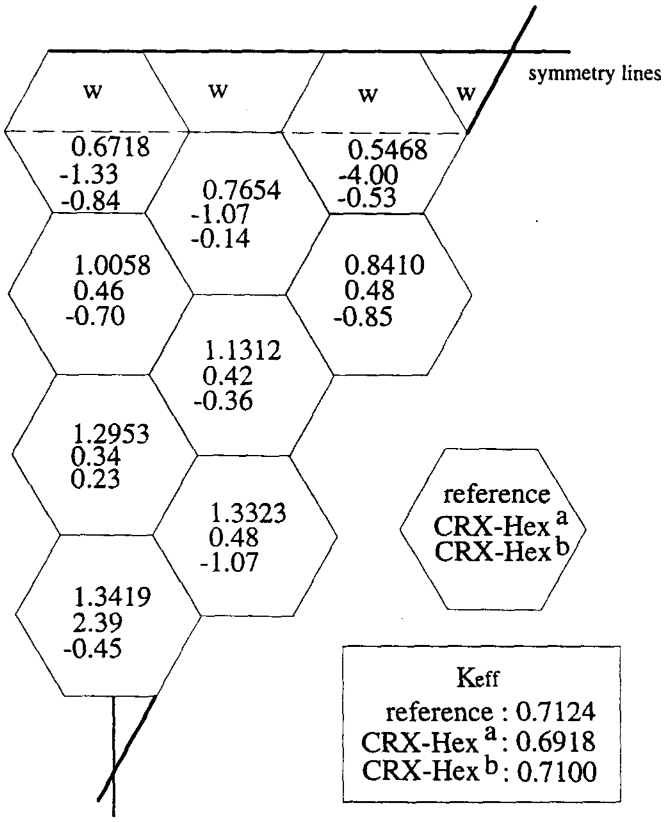


Fig. 3. Configuration for the benchmark problem II

Table II. Cross sections (cm^{-1}) for the benchmark problem II

material	fuel	water	control rod
σ_t	0.93480	1.32956	1.24980
σ_s	0.83220	1.02093	0.46580
$\nu\sigma_f$	0.17970	0.00000	0.00000



CRX-Hex [a] : six triangles/hexagon
CRX-Hex [b] : twenty four triangles/hexagon

Fig. 4. Relative power error distribution for the benchmark problem II