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Development of an Analytic Nodal Expansion Method of Neutron Diffusion Equation in Cylindrical Geometry

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ABSTRACT

An analytic nodal expansion method has been derived for the multigroup neutron diffusion equation in 2-D cylindrical(R-Z) coordinate. In this method we used the second order Legendre polynomials for source, and transverse leakage, and then the diffusion eqaution was solved analytically.

This formalism has been applied to 2-D LWR model. k_{eff}, power distribution, and computing time have been compared with those of ADEP code(finite difference method). The benchmark showed that the analytic nodal expansion method in R-Z coordinate has good accuracy and quite faster than the finite difference method. This is another merit of using R-Z coordinate in that the transverse integration over surfaces is better than the linear integration over length. This makes the discontinuity factor useless.

I. INTRODUCTION

The nodal method is a scheme for analyzing multidimensional core avoiding large storage and execution time, and have proved to be very successful in analyzing light water reactors with Cartesian geometry.[1-3] However, the nodal methods in itself require more computational efforts than the finite difference method in deriving the numerical scheme, and the more numerical complexity is expected as they are applied in curvilinear coordinates as oppose to rectangular geometry. In the result, much of the development of methods have been restricted to the systems having Cartesian geometry.

In spite of these shortcomings, the cylindrical geometry is used often since the light water reactor vessel has a cylindrical type with the symmetrically distributed control rods along axial direction. Therefore, if we can model in the cylinder nodes, we can reduce the of number of dimensions from 3 to 2 for full core analysis. A few researchers[4-5] tried and obtained good results. In this study, an analytic nodal expansion method is developed for the cylindrical R-Z geometry.

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II. Theory

The multigroup diffusion equation in the R-Z geometry for the energy group g and in the node (i,j) is given by

$$-D_{gij}\nabla^2\phi_{gij}(r,z)+\Sigma_{Rgij}\phi_{gij}(r,z)=S_{gij}(r,z), \tag{1}$$

where

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2},$$

and

$$S_{gij}(r,z)=\left[\sum_{g'\neq g}^{G}\Sigma_{sgg'ij}+\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma_{fg'ij}\right]\phi_{g'ij}. \tag{2}$$

The key point in the derivation is to separate the spatial variables of Eq.(1), and convert it into one-dimensional problems. In order to obtain a set of 1-D ordinary differential equations, Eq.(1) is transverse-integrated and the process is repeated over each of the other variables. The resulting set of one-dimensional differential equations is given as

$$-D_g\frac{d^2}{dz^2}\phi_g^r(z)+\Sigma_{Rg}\phi_g^r(z)=S_g^r(z)-L_g^r(z), \tag{3a}$$

$$-D_g\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\phi_g^z(r)+\Sigma_{Rg}\phi_g^z(r)=S_g^z(r)-L_g^z(r), \tag{3b}$$

where the node indices have been dropped and the superscripts, r, z, indicate the integration over that variable, and L indicates transverse leakage..

If the right sides of Eqs.(3a) and (3b) are expressed in terms of second order Legendre polynomials, then Eqs.(3a) and (3b) can be solved analytically. Since these equations are valid only within each node, and the boundary conditions are known only on the reactor boundaries and its center(at r=0), it must be done to connect all the nodes in a given one-dimensional block.

The analytic solutions of Eq.(3a) and Eq.(3b) are

$$\phi_g^r(z)=\alpha_g\cosh(\lambda_g z)+\beta_g\sinh(\lambda_g z)+\sum_{k=0}^{2}a_{gk}^r P_k\left(\frac{z}{h_z}\right), \tag{4a}$$

$$\phi_g^z(r)=\mu_g I_0(\lambda_g r)+\eta_g K_0(\lambda_g r)+\sum_{k=0}^{2}a_{gk}^z P_k\left(\frac{r-r_i}{h_r}\right), \tag{4b}$$

where

$$\lambda_g=\sqrt{\frac{\Sigma_{Rg}}{D_g}}.$$

Derivations for the first node in radial direction must be derived independently, because modified Bessel functions of the second kind, $K_l(\lambda_g r)$, diverge at r=0. The flux distribution in this node is given by

$$\phi_g^z(r)=\mu_g I_0(\lambda_g r)+\sum_{k=0}^{K}a_{gk}^z P_k\left(\frac{r-r_1}{h_r}\right). \tag{5}$$

Because Eq.(4a) and (4b) include the surface net currents on both sides, to obtain complete flux equations, the net currents must be known in advance. This can be done by coupling three neighboring nodes. Connection of the adjacent nodes was done via interface net currents. Applying the following continuity of interface fluxes

$$\phi^r_{gij}(+h_{zj}) = \phi^r_{gij+1}(-h_{zj+1}), \tag{6a}$$

$$\phi^z_{gij}(r_i+h_{ri}) = \phi^z_{gi+1j}(r_{i+1}-h_{ri+1}). \tag{6b}$$

to Eq.(4), relationships between interface net currents relating the three neighboring nodes are obtained. These relationships are

$$R^r_{ij}J^r_{ij-1}(-h_{zj-1})+D^r_{ij}J^r_{ij}(-h_{zj})+U^r_{ij}J^r_{ij+1}(-h_{zj+1})=Q^r_{ij}, \tag{7a}$$

$$R^z_{ij}J^z_{i-1j}(r_{i-1}-h_{ri-1})+D^z_{ij}J^z_{ij}(r_i-h_{ri})+U^z_{ij}J^z_{i+1j}(r_{i+1}-h_{ri+1})=Q^z_{ij}. \tag{7b}$$

Eq.(7a) and (7b) are three point linear equations. And the connection of all 1-D neighboring nodes yields a set of independent tri-diagonal systems for each direction and each energy group.

In order to update the source terms which are expanded in polynomials, fluxes are assumed to be in the form of Legendre polynomials, i.e.,

$$\phi^v_g(u)=\overline{\phi^v_g}+\sum_{k=1}^{2}\phi^v_{gk}P_k\left(\frac{u}{h_l}\right), \tag{8}$$

where $\overline{\phi^v_g}$ is node average flux.

In axial direction, the node average flux is equal to the zeroth order flux moment of Legendre polynomial, but in radial direction this is not the case. In this study flux moment for zeroth order is replaced by the node averaged flux for radial direction.

III. NUMERICAL RESULTS

We have performed numerical analysis by applying the derived formalism to a 2-D LWR model[6] with 2-group constants. The results of k_{eff}, power distributions along radius and axis, and the computing time were compared with those of ADEP[7] code which uses finite difference method.

Fig.1 shows the model of benchmark problem. And the two-group constants for the problem are given in Table 1. Table 2 shows the eigenvalue and CPU time obtained from ADEP and this study(program named CYLANEM). For the same mesh size, e.g., Δr=8cm, Δz=18.75cm, nodal method produces much more accurate value than ADEP(finite difference method) by the order of 2 even without using discontinuity factors. Even for larger mesh sizes the results of nodal method are still accurate while those of ADEP are quite inaccurate. The elapsed CPU times of ADEP are much longer than this study, so we used 10^{-6} for the convergence criterion of ADEP, while 10^{-7} for CYLANEM.

Fig.2 and 3 show that the normalized relative power averaged over height and radius for Δr=8cm, Δz=18.75cm, respectively.

IV. CONCLUSION

An analytic nodal expansion method to solve the multigroup neutron diffusion equation for the cylindrical R-Z geometry has been derived. The results of the two-group test problem applied in this study demonstrate that the nodal method developed in cylindrical coordinate is much more accurate in the calculation of multiplication factor than the finite difference method for the same mesh size. Highly accurate values can also be obtained even for larger mesh sizes. These good accuracies are supposed to be the results of the radial integration, which means the integration over radial surface rather than length. Therefore, discontinuity factors are not necessary to apply for the currents between neighboring nodes in cylindrical geometry. The derived nodal method for cylindrical geometry(R-Z) has been proved its usefulness and the extension to three dimensional R-θ-Z geometry is direct.

ACKNOWLEDGEMENT

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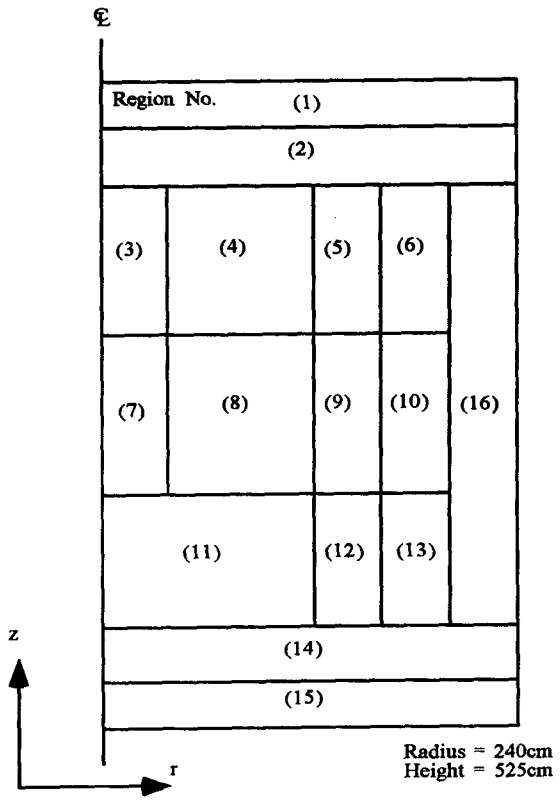

Fig. 1 Two-Dimensional R-Z Reactor Model

Table 1. Two-Group Constants for the LWR Model

Region	Group	D(cm)	Σ_R(cm^{-1})	$\nu\Sigma_f$(cm^{-1})	$\Sigma_{s1\to2}$(cm^{-1})
1,15	1	1.0684	2.8E-2	0	2.6E-2
	2	0.32051	3.3E-3	0	0
2,14	1	1.3495	1.201E-2	0	1.2E-2
	2	0.87032	1.900E-2	0	0
3,4,11	1	1.3052	1.0475E-2	1.1776E-3	8.0351E-3
	2	0.88857	1.3063E-2	1.3268E-2	0
5,12	1	1.3052	1.0475E-2	1.1776E-3	8.0351E-3
	2	0.88857	1.2623E-2	1.3268E-2	0
6,13	1	1.3052	1.0475E-2	1.1776E-3	8.0351E-3
	2	0.88857	1.2183E-2	1.3268E-2	0
7,8	1	1.3052	1.0475E-2	1.1776E-3	8.0351E-3
	2	0.88857	1.3453E-2	1.3268E-2	0
9	1	1.3052	1.0475E-2	1.1776E-3	8.0351E-3
	2	0.88857	1.2973E-2	1.3268E-2	0
10	1	1.3052	1.0475E-2	1.1776E-3	8.0351E-3
	2	0.88857	1.2933E-2	1.3268E-2	0
16	1	1.2997	1.0470E-2	1.2875E-3	7.9061E-3
	2	0.87951	1.3065E-2	1.4246E-2	0

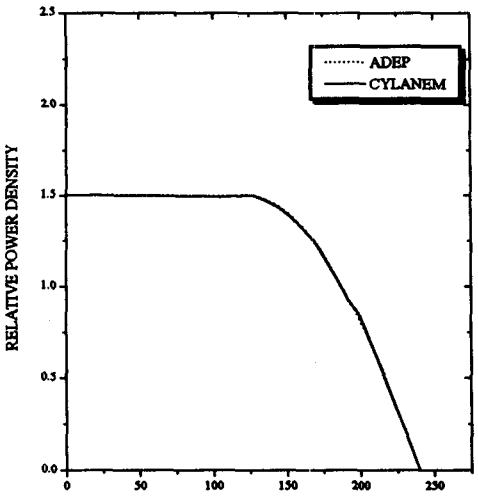

Fig.2 Relative Power Density Averaged over Height vs. Radius for Δr=8cm, Δz=18.75cm

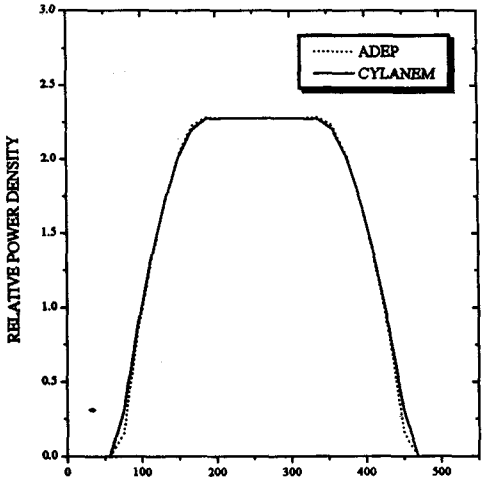

Fig.3 Relative Power Density Averaged over Radius vs. Height for Δr=8cm, Δz=18.75cm

Table 2. Eigenvalues and Elapsed CPU Times Obtained from ADEP and CYLANEM

Mesh Size		ADEP		CYLANEM	
Δr(cm)	Δz(cm)	eigenvalue (%error)	CPU time# (sec)	eigenvalue (%error)	CPU time# (sec)
Irregular Size**		*	*	0.8633767 (0.424)	3.36
20	37.5	*	*	0.8663588 (0.080)	19.53
20	18.75	0.7903087 (8.851)	20.15	0.8663212 (0.084)	34.37
10	37.5	*	*	0.8669894 (0.007)	39.02
10	18.75	0.8699382 (0.333)	42.70	0.8669611 (0.011)	70.95
8	37.5	*	*	0.8670641 (0.0013)	50.54
8	18.75	0.8668486 (0.024)	39.28	0.8670470 (0.00068)	99.10
Reference Eigenvalue		0.867053			

* : Not available.

** : 40,80,40,40,40cm in radial direction.
37.5,37.5,112.5,150.0,112.5,37.5,37.5cm in axial direction.

\# : Under 486 DX2-66.