*Proceedings of the Korean Nuclear Society Spring Meeting*
*Cheju, Korea, May 1996*

# The Robust Controller Design for Nuclear Steam Generator Using $H_\infty$ Control Theory

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## Abstract

$H_\infty$ robust control theory is applied to the nuclear steam generator level control. Nuclear steam generator has the properties such as nonlinearity, non-minimum phase, and so, has some difficulties on level control. In a nuclear plant, it is more important to keep the operating variables under certain safety limits against various uncertainties than to meet the optimal performance. The designed $H_\infty$ controller shows robust level control against modelling error, disturbance in the nonlinear simulation. As the $H_\infty$ controller has both robustness and design transparency, it is adequate to the automation of level control and in licensibility

## 1. Introduction

The level control of nuclear steam generator has some difficulties in automation with the existing PID control technique because of non-minimum phase, nonlinearity, and uncertain flow measurements which result from the thermal shrink and swell phenomena at low power. So, manual operation is required for the level control at low power, which is one of the main causes for the reactor trips.

In our study, $H_\infty$ robust control theory is applied to feedwater control system in order to design robust stable level controller. In a nuclear plant, it is more important to keep the operating variables under certain safety limits against various uncertainties than to meet the optimal performance. The robustness- the capability to treat disturbance destabilizing the system, is a very important requirement in nuclear plant. Accordingly, in the core power control and level control at low power, manual operation that is the more robust than automatic control is mainly taken.

As the intelligent control techniques with high robustness, the fuzzy control[6], and neural network control[7] based upon manual operation have been studied, but those are not transparent in the design of stability and performance.

Contrary, in the $H_\infty$ robust control theory, the uncertainties destabilizing system are modelled, and the effects resulting from those uncertainties are estimated. The controller that has the robust stability against uncertainties is designed in a transparent manner. That is to say, the $H_\infty$-norm that represents the effects of system in the worst case is kept under the prescribed limiting value, and the controller that minimizes that value is to design.[1]

## 2. Steam Generator Dynamic Model for Level Control

It is necessary to model the nuclear steam generator dynamics for a controller design. A nonlinear model that can simulate the shrink and swell is[3] adopted. This model is of decoupled third order for primary and of fifth order for secondary system.[3] The secondary side dynamics can be expressed in a state-space equation with 5 state variables as follows.

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state equation :

$$\underline{A}\frac{d}{dt}\begin{bmatrix} U_0 \\ L_w \\ \langle \alpha_r \rangle \\ \langle \alpha_n \rangle \\ p \end{bmatrix} = \begin{bmatrix} \overline{W}(H_0 - H_r) + q_B \\ \overline{W}(H_r - H_n) \\ \overline{W}(H_n - H_k) - W_s(H_{vs} - H_k) \\ W_{fw}(H_{fw} - H_k) - \overline{W}(H_0 - H_k) \\ W_{fw} - W_s \end{bmatrix}$$

The brief expression of this model can be given by,

$$\underline{A}d\underline{x}/dt = b\ (\underline{x},\ u,\ \underline{w},\ t) \qquad (1)$$

Where, $\underline{x} = [\ U_o\ ,\ L_w\ ,\ \alpha_r\ ,\ \alpha_n\ ,\ P\ ]^T$
$u = g(\ W_{fw}\ )$
$w = h(\ W_s\ ,\ T_{fw}\ ,\ P_{pr}\ ,\ W_{pr}\ ,\ T_h\ )$

In the above notation, $U_o$, $L_w$, $\alpha_r$, $\alpha_n$, P are the state variables meaning internal energy at the tube bundle inlet, the water level, the void fractions at the riser inlet and outlet, and the secondary pressure respectively. The control input is the feedwater, $W_{fw}$. The steam flow, $W_s$, feedwater temperature, $T_{fw}$, primary pressure, $P_{pr}$, primary flow, $W_{pr}$, primary hot-leg temperature, $T_h$, are considered as the uncontrollable disturbances.
All the variables are in SI units. The elements of matrix A in Eq. (1) are shown in detail in reference 3.

## 3. Linearization of Steam Generator Model

In order to design $H_\infty$ controller, nonlinear state equation is linearized at various nominal operating points. Eq. (1) can be rearranged as following equation.

$$d\underline{x}/dt = A^{-1}\ b\ (\underline{x},\ u,\ \underline{w},\ t) = f\ (\underline{x},\ u,\ \underline{w},\ t)$$

The linear model can be given by,

$$\delta\dot{x} = A_l\ \delta x + B\ \delta u + F\ \delta w$$
$$\delta y = C\ \delta x$$

Where, $A_l = \partial f(x^o,\ u^o,\ w^o)/\partial x$,
$B = \partial f(x^o,\ u^o,\ w^o)/\partial u$,
$F = \partial f(x^o,\ u^o,\ w^o)/\partial w$,
$C = [0,\ 1,\ 0,\ 0,\ 0]$,

and $\delta x = x - xo, \quad \delta u = u - uo$
$\delta w = w - wo, \quad \delta y = y - yo$

In our study, the 10% nominal power is adopted as the nominal operating point, and the operating variables at that point are obtained by the steady state simulation. Those variables are given as follows.

$x^o$ =[124995.0, 12.78, .2185E+0, .2185E+0, 7441641.0]$^T$,
$u^o$ =[ 33.2844 ],
$w^o$ =[ 33.2844, 318.2, 15510000.0, 4502.0, 568.0996 ]$^T$

The system matrices of the linear model are given by,

$A_1$ = [1.047e-03, 0.00, -4.154e+02, 1.827e+02, 3.359e-03
9.580e-07, 0.00, -2.088e+00, 3.412e+00, -2.069e-07
6.601e-07, 0.00, -2.166e-01, -7.125e-03, -7.758e-08
-6.286e-07, 0.00, 3.068e-01, -1.738e-01, 6.676e-08
-4.620e-02, 0.00, 1.303e+05, 2.557e+03, 1.586e-02]

B = [-6.769e+01
2.907e-03
8.093e-04
-6.927e-04
-1.710e+02]

F = [-1.276e-01, 8.488e+00, -.904E-07, -.724E-03, -.324E-02
-6.838e-03, -4.900e-04, .345E-12, -.230E-00, .288E-08
3.289e-04, -1.036e-04, .337E-13, -.408E-10, -.938E-09
1.116e-04, 8.611e-05, -.127E-13, -.305E-10, .244E-09
-6.459e+02, 2.574e+01, -.123E-00, .000E-00, -.318E-01]

C = [0, 1, 0, 0, 0]

## 4. System Uncertainties Representation

To generalize the model of nuclear steam generator, it is necessary to develop a representation of the uncertainties that are present in the actual plant. Uncertainties in the nonlinear model that are inherited by the linearization occur as a result of parametric uncertainties. In addition, the inaccuracy of the linearization increases because of the use of the nominal linearized model for operating point, and the local nature of linear approximations of plant dynamics around equilibrium point.[10]

As the feedwater is the input that has overriding effect on the water level of steam generator in the steam generator dynamics, the uncertainties of the transfer function from feedwater to water level are estimated. It is common to represent the plant uncertainties as an output-side unstructured multiplicative dynamic modelling. If the transfer functions from feedwater to water level are $G_i(s)$, the transfer function at the nominal operating point is $G_0(s)$, multiplicative modelling errors is $L_i(s)$, these satisfy the following relation.

$$G_i(s) = (1 + L_i(s))G_0(s)$$

The maximum value of the singular values of $L_i(s)$, $l(w)$ stands for the maximum modelling error over all frequency. In our study, the linearized models are obtained at 20%, 30%, 80%, 100% power respectively, and the modelling errors are analyzed at the nominal operating point, 10% power. The modelling error at each power is shown in Fig. 1., and the operating variables at each power is listed in Table 1.

## 5. $H_\infty$ Control Design

### A. $H_\infty$ Robust Control Theory

The level control loop can be transformed as the standard compensator configuration. Generally, the major control problems (disturbance rejection, robust stability, tracking, etc.) can be transformed into the standard compensator configuration.

In the standard compensator configuration, the input 'w' is an exogeneous input representing the disturbance acting on the system, and the output 'z' is all signal to be controlled such as reference signal, tracking error. The output 'y' is the measurement to be made on the system, and finally 'u' is the all control input of the generalized plant. The design of the controller in $H_\infty$ space is to compose K(s) such that minimize the closed-loop $H_\infty$ norm, $\|T_{zw}(s)\|_\infty$ with internal stability maintained. The transfer matrix G(s) is given by,

$$G(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \qquad \begin{aligned} \dot{x} &= Ax + B_1w + B_2u \\ z &= C_1x + D_{12}u \\ y &= C_2x + D_{21}w \end{aligned}$$

To assure well-posedness of the problem, $D_{12}^T D_{12}$ and $D_{21} D_{21}^T$ should have inverse matrices, and the above elements should satisfy the following conditions.

(i) $(A, B_1)$ stabilizable, $(C_1, A)$ detectable

(ii) $(A, B_2)$ stabilizable, $(C_2, A)$ detectable

(iii) $D_{12}^T [C_1 \quad D_{12}] = [0 \quad I]$

(iv) $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$

For the prescribed $\gamma$, the $H_\infty$ controller, K(s) satisfying $\|T_{zw}\|_\infty < \gamma$ is obtained as follows.[5,8,9]

$$K(s) = \left[ \begin{array}{cc} A - K_f C_2 - B_2 K_c + Y_\infty C_1^T (C_1 - D_{12} K_c)/\gamma & K_f \\ K_f & 0 \end{array} \right]$$

B. $H_\infty$ ROBUST CONTROLLER DESIGN FOR FEEDWATER SYSTEM

As Fig 2., the generalized plant reflecting the uncertainties is constructed, and the weighting functions are tuned for the trade-off of desired performance and robust stability. The weighting functions and maximum modelling error are showed in Fig. 3. To avoid the condition in which the plant has a pole on the imaginary axis, the bilinear transformation is used. The computation of controller is processed by the $H_\infty$ control algorithms implemented in MATLAB.

Finally, the system matrices ($A_c$, $B_c$, $C_c$, $D_c$) of the computed controller and locations of the closed-loop poles are listed in Appendix A.

## 6. Simulation

The designed controller is connected to the nonlinear simulation code, and is tested in some conditions. The time responses of the controller under step changes of steam flow, feedwater flow, feedwater temperature, primary hot-leg temperature, are shown in Fig 4-7. In the case of power increase from 10% to 15%, the result is shown in Fig 8.

Also, PI controller is employed comparatively in the same condition as $H_\infty$ controller. To the each case, $H_\infty$ controller shows more less oscillation than PI controller, and robust control action.

## 7. Conclusion

An $H_\infty$ controller is presented for the nuclear steam generator level control problem which exhibits nonlinearities, nonminimum phase, etc. The designed controller achieves automatic control with robustness to modelling error and disturbance. Also, it provides better licensibility in actual implementation because of the design transparency that comes from the mathematical foundation of $H_\infty$ control theory. But it will not necessarily provide good performance for large deviation of the system dynamics from those of the nominal model because of the system nonlinearities. To extend the applicability of this design, further refinements are required such as the inclusion of smooth nonlinear plants with a wide operating range and the gain scheduling techniques.[10]

## References


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## Appendix A.

$A_c$ = [ -7.397e-01, 5.147e-02, -1.489e-02, -5.395e-03, -1.224e-03, 1.940e-04, -5.496e-03
1.958e-01, -7.524e-02, 1.656e-02, 1.417e-02, 5.877e-04, 3.436e-03, 1.948e-03
-2.195e-01, -6.896e-02, 9.477e-03, 1.110e-02, 8.351e-03, 2.772e-04, 3.241e-02
2.136e+00, 6.924e-01, -1.446e-01, -1.651e-01, -2.050e-02, -4.183e-02, -6.424e-02
2.404e-02, 7.856e-03, -5.389e-03, 6.354e-03, -7.740e-04, 1.019e-02, -4.640e-04
1.896e+00, 6.136e-01, -1.268e-01, -1.439e-01, -8.661e-04, -3.632e-02, -6.708e-02
4.499e-01, 1.455e-01, -3.154e-02, -3.336e-02, -1.798e-03, -5.357e-03, -1.393e-02 ]

$B_c$ = [ -3.821e+01
-4.975e+01
5.576e+01
-5.426e+02
-6.106e+00
-4.817e+02
-1.142e+02 ]

$C_c$ = [ 2.659e-01, -2.187e-02, 6.328e-03, 2.297e-03, 5.273e-04, -8.384e-05, 2.371e-03 ]

$D_c$= [ 1.6481e+01 ]

Locations of closed-loop poles,

-8.9038e-01, -1.5974e-01+5.1470e-02i, -1.5974e-01-5.1470e-02i, -1.3952e-01,
-7.5944e-02, -5.9760e-03+4.3662e-03i, -5.9760e-03-4.3662e-03i, -5.6911e-04,
-2.1785e-03+1.3904e-02i, -2.1785e-03-1.3904e-02i, -6.6987e-04+1.3825e-02i,
-6.6987e-04-1.3825e-02i

Table 1. The Data of State Variables, Control input and Disturbances

| Power | 20% | 30% | 80% | 100% |
|---|---|---|---|---|
| $U_o$ | 124331.00 | 123852.00 | 120422.00 | 119464.00 |
| $L_w$ | 12.7800 | 12.7800 | 12.7800 | 12.7800 |
| $\alpha_r$ | .330428 | .420640 | .648834 | .699916 |
| $\alpha_n$ | .330428 | .420640 | .648834 | .699916 |
| P | 7353955.00 | 7278491.00 | 6980441.00 | 6890000.00 |
| $W_{fw}$ | 76.1606 | 125.492000 | 370.2658 | 475.7121 |
| $W_s$ | 76.1606 | 125.492000 | 370.2658 | 475.7121 |
| $T_{fw}$ | 395.6 | 443.0 | 488.5 | 499.7 |
| $P_{pr}$ | 15510000.0 | 15510000.0 | 15510000.0 | 15510000.0 |
| $W_{pr}$ | 4502.0 | 4502.0 | 4502.0 | 4502.0 |
| $T_h$ | 571.5992 | 575.0500 | 591.8159 | 598.2292 |

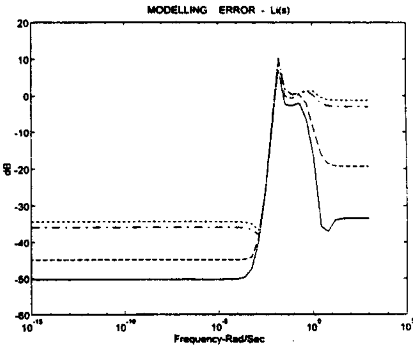

Figure 1. Modelling Errors of Linearized Model

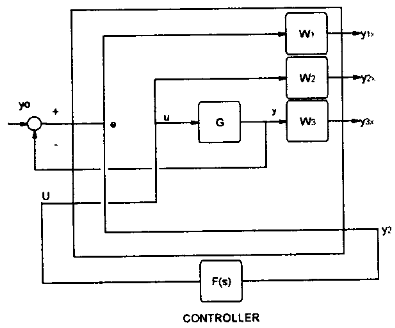

Figure 2. Generalized Plant

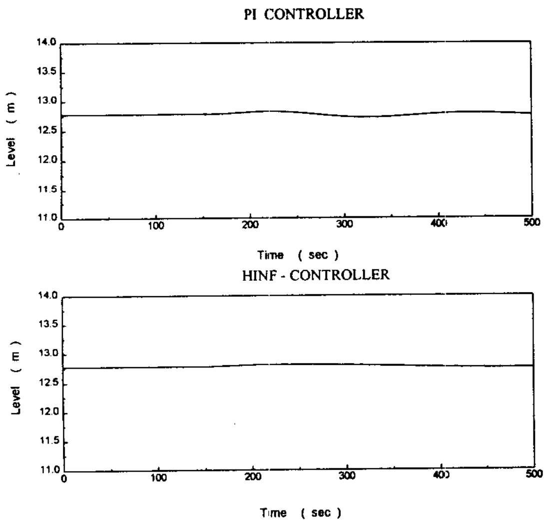

Figure 4. Steam Flow Step Change

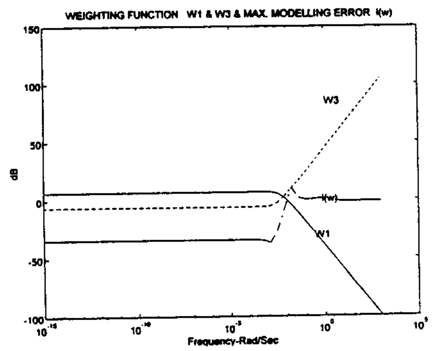

Figure 3. Weighting Functions and Maximum Modelling Error

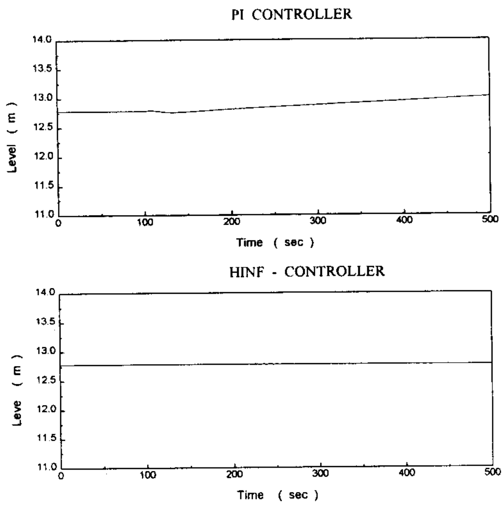


Figure 5. Feedwater Flow Step Change

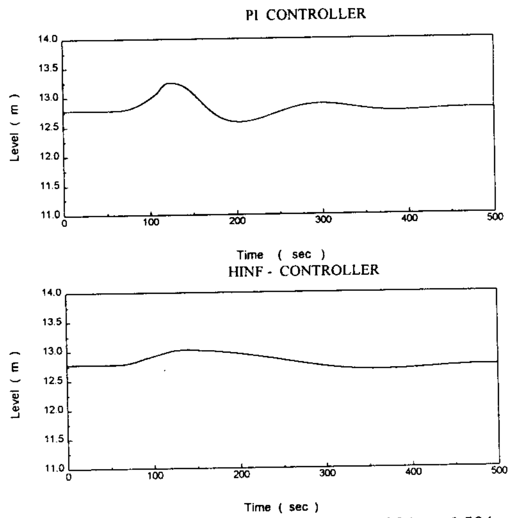


Figure 8. Power Increase from 10% to 15%

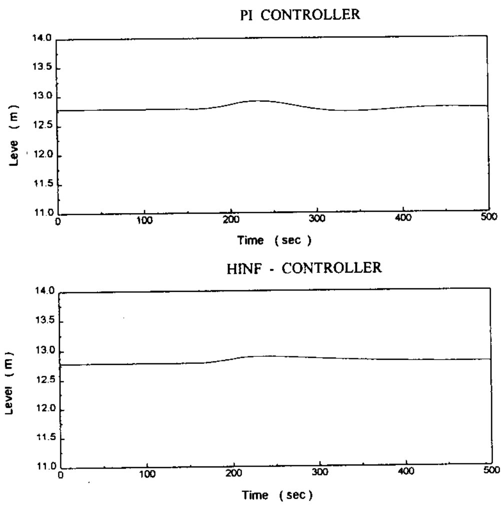


Figure 6. Feedwater Temperature Step Change

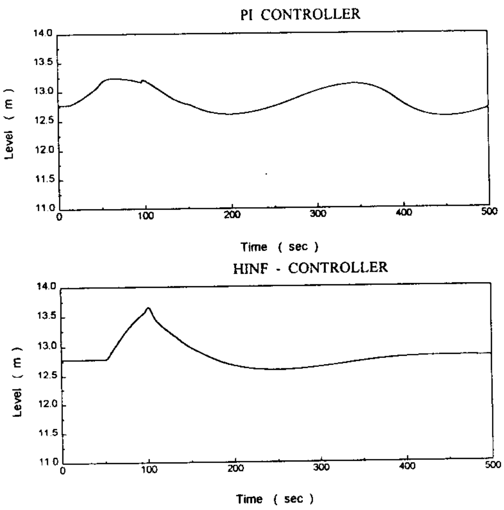


Figure 7. Primary Hot-Leg Temperature Step Change