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A New Transition Criterion for Stratified and Nonstratified Flows in Pipes

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Abstract

A two-step approach has been used to obtain a new transition criterion for the stratified and nonstratified flow in horizontal pipe: (1) In the first step, a more general expression than the existing models for the flow transition criterion has been derived from the analysis of singular points and neutral stability conditions, or the parallel lines conditions of the transient one-dimensional two-phase flow equations of two-fluid model. (2) In the second step, introducing simplifications and incorporating a parameter into the general expression obtained in the first step to satisfy a number of physical conditions a priori specified, a new simple flow transition criterion for horizontal pipes has been derived. Comparison between results predicted by the present theory with the experimental data and theories in the pipe flow conditions, show good agreement.

I. Introduction

The main purpose of this paper is to present the two-step approach used to extend the method of characteristics and stability analyses of one-dimensional two-phase flow equations presented by Lyczkowski et al.[1] and Bilicki et al. to the derivation of a new criterion for the onset of slug formation. In the first step, a more general form for the onset of slug flow criterion is derived from the analyses of singular points and neutral stability conditions of the transient one-dimensional two-phase flow equations of the two-fluid model in a manner similar to the procedure used in the derivation of a flooding correlation by Lee and No [2]. In the second step, the final form of a new criterion for the onset of slug flow has been obtained by simplification of the general expression derived in the first step and incorporation of a parameter to satisfy a number of physical conditions a priori specified.

II. Two-Phase Flow Equations

Consider a one-dimensional transient stratified two-phase flow shown in Fig.1. The two-phases are assumed to be weakly coupled cocurrent gas-liquid in a pipe of diameter D, (or a channel of height H and infinite width) with an inclination θ to the gravity vector.

The one-dimensional, two-fluid transient model is formulated by considering each phase separately in terms of two sets of conservation equations governing the balance of mass and momentum of each phase as follows:

$$\frac{\partial}{\partial t}(\alpha_f \rho_f) + \frac{\partial}{\partial z}(\alpha_f \rho_f u_f) = \Gamma_f \quad (1a)$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \frac{\partial}{\partial z}(\alpha_g \rho_g u_g) = \Gamma_g \quad (1b)$$

$$\frac{\partial}{\partial t}(\alpha_f \rho_f u_f) + \frac{\partial}{\partial z}(\alpha_f \rho_f u_f^2) = \alpha_f \rho_f\, g \sin\theta - \frac{\partial(\alpha_f P_f)}{\partial z}$$
$$+ P_{if}\frac{\partial \alpha_f}{\partial z} - \tau_i \frac{\partial \alpha_f}{\partial z} + \frac{\partial}{\partial z}(\alpha_f \tau_f) + \Gamma_f(u_{if} - u_f) + M_{if} \quad (2a)$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g u_g) + \frac{\partial}{\partial z}(\alpha_g \rho_g u_g^2) = \alpha_g \rho_g\, g \sin\theta - \frac{\partial(\alpha_g P_g)}{\partial z}$$
$$+ P_{ig}\frac{\partial \alpha_g}{\partial z} - \tau_i \frac{\partial \alpha_g}{\partial z} + \frac{\partial}{\partial z}(\alpha_g \tau_g) + \Gamma_g(u_{ig} - u_g) + M_{ig} \quad (2b)$$

Here α, ρ, u, P, θ and τ_i are the volume fraction, the density, the average velocity of the two fluids, the pressure, the inclination to the horizontal, and the interfacial shear stress. Γ_k and M_{ik} are the rate of mass generation of phase k at the interface and the rate of momentum generation of phase k at the interface (which results from surface tension, and depends on the geometric state of the interface). The subscripts f and g denote liquid-phase and gas-phase and i stands for the value at the interface. In Eqs.(1) and (2), the spatial coordinate is denoted by z and the time is denoted by t. From the interfacial momentum and mass conservation $M_{if} + M_{ig} = 0$ and $\Gamma_f + \Gamma_g = 0$.

In the present derivation of the flow transition criterion based on characteristics and stability analyses of the transient one-dimensional two-fluid formulations of two-phase flow, the energy conservation equations are not included since the effect of thermal energy transfer on the rapid flow regime transition phenomenon such as the onset of slugging can be considered to be a secondary factor in comparison with the effect of mass and momentum transfer on the flow regime transition.

Noting that $\Gamma_g = -\Gamma_f = \Gamma$, the mass conservation equations for liquid and gas, Eqs.(1a) and (1b), can be rewritten as

$$\frac{\partial}{\partial t}(\alpha_f) + \frac{\partial}{\partial z}(\alpha_f\, u_f) = -\Gamma \;/\; \rho_f \quad (3)$$

$$\frac{\partial}{\partial t}(\alpha_g) + \frac{\partial}{\partial z}(\alpha_g\, u_g) = \Gamma \;/\; \rho_g \quad (4)$$

Substituting the geometric and special relations, into Eqs.(2a) and (2b), and multiplying Eqs.(2a) and (2b) by α_g and α_f, respectively, and subtracting the latter from the former, one can obtain the combined momentum equation for two-phase flow as follows :

$$\alpha_f \alpha_g \left[\rho_f \frac{\partial u_f}{\partial t} - \rho_g \frac{\partial u_g}{\partial t} \right] + \alpha_f \alpha_g \left[\rho_f u_f \frac{\partial u_f}{\partial z} - \rho_g u_g \frac{\partial u_g}{\partial z} \right] - F \frac{\partial \alpha_g}{\partial z} = G \quad (5)$$

where F and G are defined as

$$F \equiv \alpha_f \alpha_g [(\frac{\partial \Delta P_i}{\partial \alpha_g}) + \frac{\Delta\rho g D \cos\theta}{\sin\gamma} \frac{\pi}{4}], \quad (6)$$

$$G \equiv \alpha_f \alpha_g \Delta\rho\, g \sin\theta - \tau_f \frac{S_f}{A}\alpha_g + \tau_g \frac{S_g}{A}\alpha_f + \tau_i \frac{S_i}{A}(\alpha_g + \alpha_f)$$
$$-\alpha_g \Gamma(u_{if} - u_f) - \alpha_f \Gamma(u_{ig} - u_g) \quad (7)$$

and the constitutive relations for shear stresses, $\Delta\rho$ and Γ are given by the following :

$$\tau_f = \frac{f_f}{2}\rho_f |u_f| u_f \tag{8a}$$

$$\tau_g = \frac{f_g}{2}\rho_g |u_g| u_g \tag{8b}$$

$$\tau_i = \frac{f_i}{2}\rho_g |u_r| u_r \tag{8c}$$

$$\Delta\rho = \rho_f - \rho_g \tag{9}$$

$$\Gamma = \frac{Q_w}{i_{fg}} \tag{10}$$

In Eq.(8c), u_r denotes the relative velocity between the two-phase (i.e., $u_r = u_g \pm u_f$), where the positive sign is applicable for cocurrent flow whereas the negative sign is applicable for countercurrent.

III. Irregular Singular Points and Hyperbolicity Breaking for Transient One-Dimensional Two-Phase Flow Equations

The mathematical models of the above transient one-dimensional two-phase flow equations, i.e., Eqs.(3), (4), and (5) can be brought to the form of a system of first-order, quasi-linear, partial differential vector equation as follows:

$$A_{ij}(X_j)\frac{\partial X_j}{\partial t} + B_{ij}(X_j)\frac{\partial X_j}{\partial z} = C_j \tag{11}$$

It may be noted here that Eq.(11) is similar in form to Lyczkowski et al.'s Eq.(A.1). The 3×3 square matrices A_{ij} and B_{ij} and the column vector C_j depend only on (X_j, t, z), whereas X_j is a column vector of 3 that depends on the two-phase flow dynamic quantities (i.e., u_g, u_f, and α_g).

Introducing the new coordinate $\xi = \lambda t + z$, Equation (11) can be transformed into the following:

$$(A_{ij}\lambda + B_{ij})\frac{\partial X_j}{\partial \xi} = C_j \tag{12}$$

For this hyperbolic equation, the characteristics λ must be real and non-zero. To find the condition under which Eq.(11) becomes singular points, Eq.(12) is rewritten by application of Cramer's rule as follows:

$$\frac{\partial X_j}{\partial \xi} = (A_{ij}\lambda + B_{ij})^{-1} C_j = \frac{N_j}{\Delta(X_j)} \tag{13}$$

where $\Delta(X_j)$ is determinant given by

$$\Delta(X_j) = |A_{ij}\lambda + B_{ij}| \tag{14}$$

and $N_j(X_j, \xi)$ are also determinants, each obtained from $(A_{ij}\lambda + B_{ij})$ by replacing the j-th column by C_j. The phase space of Eq.(13) is constructed of the $n+1$ dimensions which consists of the n components X_j (i.e., u_g, u_f, and α_g) and coordinate ξ. In this phase space, there are three classes of points: (1) regular points (if $\Delta(X_j) \neq 0$), (2) turning points (if $\Delta(X_j) = 0$ and $N_j \neq 0$), and (3) singular points (if $\Delta(X_j) = 0$ and $N_j = 0$) as illustrated in Fig. 2.

If $\Delta(X_j) \neq 0$, a point in the phase space is a 'regular point', whereas all points in the space which satisfy the condition $\Delta(X_j) = 0$ are either 'turning points' or 'singular points'. However, these conditions depend on the values of characteristics λ in Eq.(22). Lyczkowski showed that most two-phase flow models proposed in the literature yield complex-valued characteristics in the practical regions of interest for the two-phase steam-water system.

There are three different types of solutions to the characteristic equation corresponding to Eq.(14): (1) two different real roots (hyperbolic equation domain), (2) only one real root (parabolic equation domain), and (3) no real roots and two complex roots (elliptic equation domain). Physically, the hyperbolic equation domain represents a regular flow, the elliptic equation domain represents a different type of regular flow (instability), and the parabolic equation domain represents the neutral stable state between regular flow and unstable flow .

If the characteristics at singular points are inflected nodes ($\lambda_1 = \lambda_2$ real), the hyperbolicity of the two-phase flow equation is broken and a regular flow pattern changes to another flow pattern as shown in Fig. 3.

For the purpose of the present work, the above mathematical background can be briefly summarized as follows: The 'irregular singular points' occur when both conditions of (1) $\Delta(X_j) = 0$ and the hyperbolicity breaking are satisfied and (2) $N_j = 0$. The conditions of hyperbolicity breaking, on the other hand, are (1) inflected nodes ($\lambda_1 = \lambda_2$ real) and (2) parallel lines ($\lambda_1 \neq 0$, $\lambda_2 = 0$ real).

IV. 'Onset of Slugging Criterion' from Analyses of Singular Point and Neutral Stability Conditions

(1) Singular Point and Neutral Stability Conditions

Since the parabolic domain that corresponds to the single real root of Eq.(14) represents the bifurcation of the instability, this 'neutral stability condition' in addition to the condition of $\Delta(X_j) = 0$ (i.e., 'turning points' or 'singular points') is used to derive the 'onset of slugging criterion' as follows:

From Eq.(13), the gradient of the void fraction along the gradient of X_j is given as

$$\frac{\partial \alpha_g}{\partial \xi} = \frac{N_{\alpha_g}}{\Delta(X_j)} \tag{15}$$

where

$$\Delta(X_j) = \alpha_f \rho_g (\lambda + u_g)^2 + \alpha_g \rho_f (\lambda + u_f)^2 - \alpha_f \alpha_g [(\frac{\partial \Delta P_i}{\partial \alpha_g}) + \frac{\pi}{4} \frac{\Delta \rho g D \cos\theta}{\sin\gamma}] = 0 \tag{16}$$

and

$$N_{\alpha_g} = \alpha_f \alpha_g (\rho_f - \rho_g) g \sin\theta - \tau_f \frac{S_f}{A} \alpha_g + \tau_g \frac{S_g}{A} \alpha_f + \tau_i \frac{S_i}{A} (\alpha_g + \alpha_f)$$
$$+ \Gamma \left[\alpha_f (\lambda + u_g) + \alpha_g (\lambda + u_f) \right] = 0 \tag{17}$$

Now, the condition of hyperbolicity breaking at singular (and/or turning) points can be found by first finding the characteristics from the solutions of the characteristic equation $\Delta(X_j) = 0$ and then by checking whether the characteristics obtained from this equation become inflected nodes (discriminant in Eq.(21) is zero) or parallel lines (In Eq.(21), $q = 0$):

Equation (17) can be rewritten as follows:

$$\lambda^2 + 2r\lambda + q = 0 \tag{18}$$

where

$$r = \frac{\alpha_f \rho_g u_g + \alpha_g \rho_f u_f}{\alpha_f \rho_g + \alpha_g \rho_f} \tag{19}$$

$$q = \frac{\alpha_f \rho_g u_g^2 + \alpha_g \rho_f u_f^2 - F}{\alpha_f \rho_g + \alpha_g \rho_f} \tag{20}$$

Solving Eq.(26) for λ, one can obtain as:

$$\lambda = -r \pm \sqrt{r^2 - q} \tag{21}$$

Now, the neutral stability condition can be obtained by setting the discriminant of Eq.(21) to zero. Using Eqs.(19) and (20) in Eq.(21), the following relations can be obtained:

$$(u_g - u_f)^2 = u_r^2 = [(\frac{\partial \Delta P_i}{\partial \alpha_g}) + \frac{\pi}{4}\frac{\Delta\rho g D\cos\theta}{\sin\gamma})](\frac{\alpha_f \rho_g + \alpha_g \rho_f}{\rho_f \rho_g}) \tag{22}$$

Equation (22) is a primitive form of the onset of slugging criterion that results from the analysis of a singular point and neutral stability conditions for the transient one-dimensional two-fluid governing equations of two-phase flow. This equation may be considered to be a more general expression than the existing models for the flow transition criterion in the sense that Eq.(22) includes the effects of both surface tension and pipe inclination. Equation (22) in its present form, however, is not useful for comparisons with existing models and experimental data in examining its applicability and accuracy. To obtain a simpler and more useful criterion for the onset of slugging, therefore, simplifications and modifications of Eq.(22) are made as shown in the following.

(2) Final Form of the Flow Transition Criterion

The first term on the right hand side (R.H.S.) of Eq.(22) represents the effect of surface tension force whereas the second term includes the effect of gravitational force. In those waves which form slugs, it can be shown that the first term on the R.H.S. of Eq.(22) is negligibly smaller than the second term. When the first term on the R.H.S. of Eq.(22) is neglected, Eq.(22) reduces to the following:

$$u_{r,crit} = [(\frac{\pi}{4}\frac{\Delta\rho\alpha_g g D\cos\theta}{\rho_g \sin\gamma})(1 + \frac{\alpha_f \rho_g}{\alpha_g \rho_f})]^{0.5} \tag{23}$$

When Eq.(23) is compared with the existing models, it can be noticed that the numerical coefficient of the existing models obtained by Wallis and Dobson, Mishima and Ishii, and Chun et al. varies from 0.47 to 0.5, whereas Eq.(24) has an extra coefficient of $(1 + \alpha_f \rho_g / \alpha_g \rho_f)^{0.5}$.

For flow in round pipes and for disturbances of finite amplitude, Taitel and Dukler speculated that the coefficient, $1 - H_f / D$, used in their model can be estimated. A close examination of Eq.(23) shows that this equation does not explicitly include the effect of the liquid depth (H_f) on the slug flow formation for a given pipe diameter. Therefore, a parameter to be used in Eq.(23) is sought to satisfy the following physical conditions:

(1) When the equilibrium liquid level approaches the top of the pipe (i.e.,$H_f \cong D$), slug flow occurs at zero critical relative velocity ($u_{r,crit} = 0$).

(2) Conversely, when the equilibrium liquid level approaches zero (i.e., $H_f \cong 0$), the slug flow occurs at the maximum critical relative velocity for given conditions. In fact, slug flows cannot occur in this case.

(3) The wave amplitude (or the critical relative velocity) at which slug flows occur decreases linearly as the equilibrium liquid level (H_f) is increased. For example, when H_f is increased from $H_f = D/2$ to $H_f = 3D/4$, the wave amplitude (or $u_{r,crit}$) needed to form a slug flow decreases by one-half.

It can be recognized that the coefficient used by Taitel and Dukler, $1 - H_f / D$, satisfies all the physical conditions specified above. To incorporate the above physical conditions into the present model, Eq.(23) is modified as follows:

$$u_{r,crit} = (1 - \frac{H_f}{D})[(\frac{\pi}{4}\frac{\Delta\rho\alpha_g g D\cos\theta}{\rho_g \sin\gamma})(1 + \frac{\alpha_f \rho_g}{\alpha_g \rho_f})]^{0.5} \qquad (24)$$

Equation (24) is the final form of the onset of slug flow criterion. Note that Eq.(24) gives the critical relative velocity at which the transition occurs from a stratified to a slug flow in horizontal or inclined pipes. It may be noted here that for all practical conditions of air-water (or steam-water) two-phase flow $\alpha_f \rho_g / \alpha_g \rho_f << 1$. Therefore, Eq.(24) can be further reduced to a simpler form as follows:

$$u_{r,crit} = (1 - \frac{H_f}{D})[\frac{\pi}{4}\frac{\Delta\rho\alpha_g g D\cos\theta}{\rho_g \sin\gamma}]^{0.5}, \text{ or}$$

$$u_{r,crit} = (\frac{1+\cos\gamma}{2})[\frac{\pi}{4}\frac{\Delta\rho\alpha_g g D\cos\theta}{\rho_g \sin\gamma}]^{0.5} \qquad (24a)$$

(3) Parallel Lines Conditions for Eq.(16)

Since the parallel lines in hyperbolic equations domain that correspond to one zero root and a single real root of Eq.(18) represent the bifurcation of the instability, 'critical flow conditions' in addition to the condition of $\Delta(X_j) = 0$ (i.e., 'turning points' or 'singular points') are used to derived this 'criticality conditions' or 'critical flow conditions' in stratified two-phase flow as follows:

Now, the criticality condition can be obtained by setting q to zero. Using Eq.(20) in Eq.(18), following relations can be obtained:

$$\frac{\rho_g u_g^2}{\alpha_g} + \frac{\rho_f u_f^2}{\alpha_f} = [(\frac{\partial \Delta P_i}{\partial \alpha_g}) + \frac{\Delta\rho g D\cos\theta}{\sin\gamma}\frac{\pi}{4})] \qquad (25)$$

The present critical flow condition, Eq.(25), can be simplified to compare with previous critical flow conditions of the stratified two-phase flow in horizontal conditions. It should be noted that the effect of surface tension is negligible for normal stratified flow. Let us introduce the dimensionless volumetric fluxes, j_g^* and j_f^*, and consider the geometric value , $D\sin\gamma = S_i$. Then Eq.(25) can be rewritten as follows:

$$\frac{j_g^{*2}}{\alpha_g^3} + \frac{j_f^{*2}}{\alpha_f^3} = \frac{\pi D\cos\theta}{4S_i} \qquad (26)$$

The present results for the critical flow condition in horizontal condition ($\theta = 0^o$), Eq.(26) , can be reduced to the same expression obtained by Gardner [4]. In his theoretical work with large lossless waves and with two fluids in horizontal closed channels of arbitrary cross-section, Gardner [4] derived from the critical flow condition (i.e. the condition for a stationary, infinitesimal interfacial wave) for stratified two-phase flow as follows:

$$\frac{j_g^{*2}}{\alpha_g^3} + \frac{j_f^{*2}}{\alpha_f^3} = \frac{\pi D}{4w} \tag{27}$$

where w is the width of water surface ($w = S_i$). Eq.(27) represents the conditions for the transition to super-critical flow where any small interfacial disturbance cannot propagate the flow. It should be noted that this equation for the critical flow condition of infinitesimal interfacial wave for stratified two-phase flow conditions in a horizontal or an inclined condition, Eq.(25) or Eq.(26), is more general than the existing critical flow condition criteria derived by previous workers.

V. Results and Discussion

In Fig.4, j_g^* versus α_g curves obtained by various models expressed in dimensionless forms are compared with existing data. It can be seen from Fig. 4 that for the onset of slugging in horizontal pipes Eq. (24) agrees very well with Taitel and Dukler's model , in particular, and also with existing data for a broad range of α_g values.

In Fig.5, on the other hand, j_f^* versus j_g^* curves obtained by various models for an upward inclined pipe ($\theta = -2^\circ$) are shown along with experimental data reported by Crowely et al.. This figure shows that prediction of the present model agrees more closely with the experimental data than those of existing models.

Reference

1. R.W. Lyczkowski, et al., Nucl. Sci. Eng., 66, 378, (1978).
2. J. Y. Lee. and H. C. No, 146, 225, (1992).
3. Y. Taitel and A. E. Dukler, AIChE J. 22, 47, (1976)
4. G. C. Gardner., Int. J. Multiphase Flow, 5, 201, (1979).
5. M. H. Chun et al., Int. Comm. Heat Mass Transfer, 23, 11, (1996).
6. H. Nakamura et al., ANS Proceedings 1991 National Heat Transfer Conference, July 1991, Minneapolis, Minnesota, 5, 269 (1991).
7. Crowely et al., Int. J. Multiphase Flow, 18, 249,(1992).

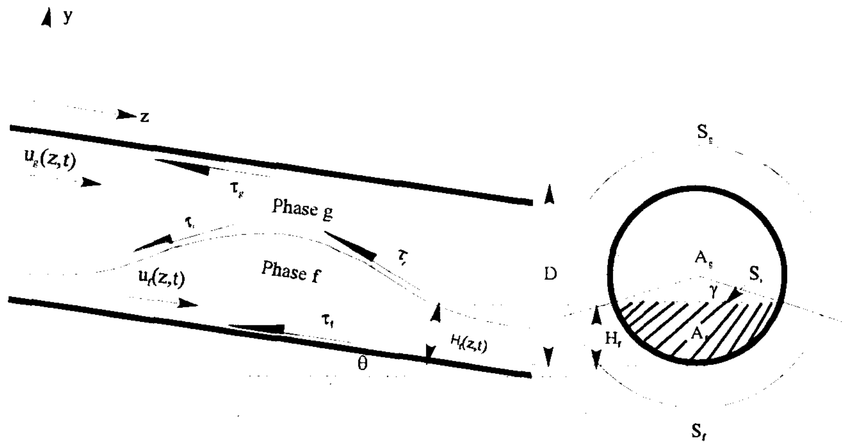

Fig. 1. Physical Model for Analysis of the Flow Transition Criterion.

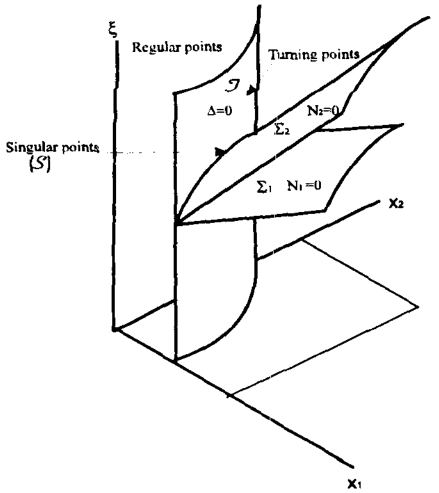

Fig.2 Definition of Manifold $\mathcal{S}$ as the Intersection between $\Delta(X_j) = 0$ and All $N_j = 0$ and Illustration of Three Classes of Points in the Phase Space Ω.

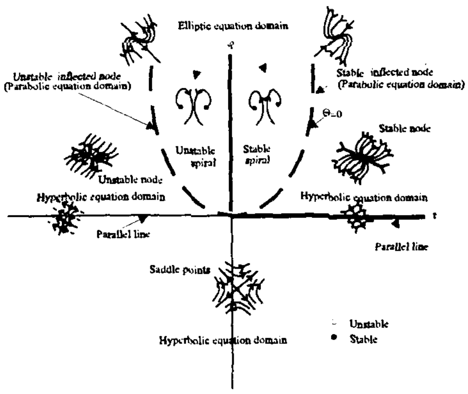

Fig.3 General Classification of Topological Patterns for the Linear System

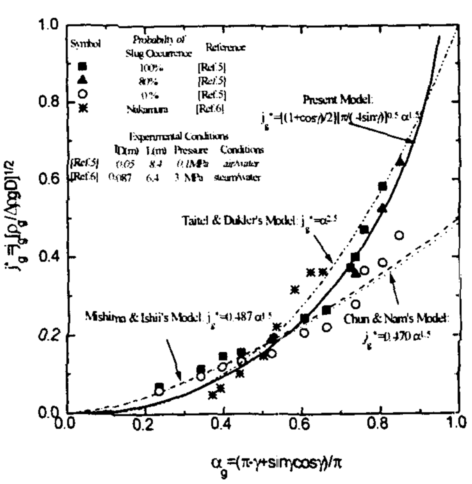

Fig.4 Comparison of Present and Existing Model Predictions with Experimental Data for the Onset of Slugging on Horizontal Pipes.

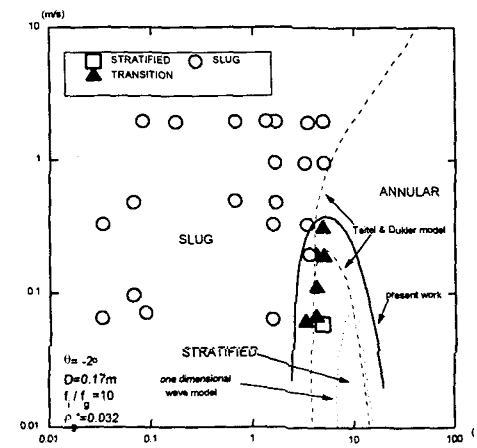

Fig.5 Stratified Transition for an Upward Inclined (−2°) 0.17m Dia. Pipe (Experimental Data: Crowley & Sam 1986)