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Derivation of Mechanistic Critical Heat Flux Model and Correlation for Water Based on Flow Excursion

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Abstract

In this study, the mechanistic critical heat flux (CHF) model and correlation for water are derived based on flow excursion (or Ledinegg instability) criterion and the simplified two-phase homogeneous model. The relationship between CHF for the water and the principal parameters such as mass flux, heat of vaporization, heated length-to-diameter ratio, vapor-liquid density ratio and inlet subcooling is derived on the developed correlation. The developed CHF correlation predicts very well at the applicable ranges, $1 < P < 40$ bar, 1,300 < G 27,00 kg/m^{2}s and inlet quality is less than -0.1. The overall mean ratio of predicted to experimental CHF value is 0.988 with standard deviation of 0.046.

1. Introduction

The CHF is a major parameter which determines the cooling performance and therefore the prediction of CHF with accuracy is of importance for the design and safety analysis of nuclear power plant (NPP). The occurrence of CHF limits the high cooling capability and can ultimately leads to the burnout of a heated surface and its destruction. Therefore, the accurate prediction of the CHF is of importance for design and safety analysis of NPP. Until now, most of the existing CHF correlations have been developed under stable flow conditions [1, 2]. However, the flow trend of NPP can be changed from the stable state to the unstable state in transient and accident conditions, such as a loss-of-coolant-accident (LOCA). It is generally known that boiling channels are subject to various flow instabilities such as flow excursion, pressure drop oscillations and density-wave oscillations [3]. In particular, the temporary flow reduction due to flow excursion may be the cause of premature burnout as a consequence. Under certain conditions, the CHF mechanism is influenced by the coupling and the existence of hydrodynamic instabilities such as excursive flow instability (Ledinegg instability) and oscillatory instabilities [4]. Ishii *et al.* [5] investigated the flow excursion CHF condition of liquid metal under low flow from the relevant experimental data. Chang *et al.* [6], especially, studied on the CHF for liquid metal in consideration of flow excursion under low heat flux-low flow conditions. But, the study and investigation on the accurately predictable correlation and the mechanistic understanding of the CHF due to flow instability is still not sufficient in condition of water.

This study aims to derive the predictable correlation and to increase the understanding of the CHF due to flow excursion. To derive the flow excursion CHF correlation, a homogeneous two-phase frictional pressure drop correlation is used. By using the homogeneous two-phase frictional pressure drop correlation and the Ledinegg instability criterion, the relationship between CHF for water and the parameters has been derived.

2. Flow Excursion

2.1 General Considerations

This is the most interesting form of static instability which is one where a small change in one of the independent variables leads to a large change in one of the dependent variables. The flow excursion was first analyzed by Ledinegg as one of the flow instabilities which are occurred in two-phase flow systems. The criterion for the instability is as follows

$$\left(\frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}}\right) \le \left(\frac{\partial \Delta P_{ext}}{\partial u_{fi}}\right)$$

where

ΔP_{ext} : the variations of pressure drop induced by the external circuit when inlet flow rate is varied.

ΔP_{int} : the variations of pressure drop that would be induced by two-phase flow (with constant power and P_{out})

This means that the flow excursion occurs when the slope of the internal pressure-drop vs flow-rate curve becomes smaller (more negative) than the external pressure-drop vs flow-rate curve (or pump characteristic pressure head). This is represented in the shape of "S" curve as shown in Fig. 1. If the flow is changed from the onset of boiling point B to the high quality point C because of the large liquid to vapor density ratio, a two-phase system may encounter with the CHF due to flow excursion. They often cause a sudden reduction or/and oscillation of flow rate with a small increase in the heat flux. The temporary reduction of the flow rate due to instability causes total liquid starvation which may lead to a premature burnout as a consequence.

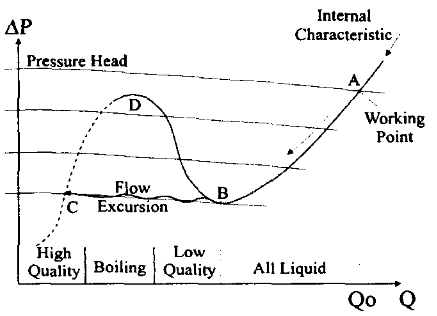

Fig. 1. Flow Excursion and ΔP Characteristic Curve

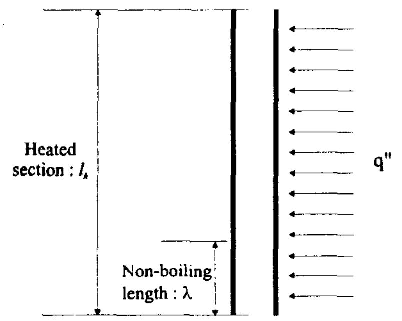

Fig. 2. Schematic of Test Section Geometry

2.2 CHF due to Flow Excursion

The flow excursion CHF for water has been studied by Mishima *et al.* [7, 8] by analyzing their experimental CHF data for round tube. But, the study is limited on the their experimental loop. The objective of this study is to obtain a general CHF correlation due to flow excursion for water. So, the basic geometry under consideration is given as Fig. 2 to simplify the derivation of the CHF correlation. The flow excursion stability criterion is as follows; $\frac{\partial \Delta P_{ext}}{\partial u_{fi}} \le \frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}}$.

According to the Ledinegg criterion, the above criterion have to be satisfied to prevent the premature burnout at the two-phase systems.

3. Derivation of Correlation from the Mechanistic Model

Based on homogeneous frictional pressure drop models the internal pressure drop can be expressed approximately as

$$\Delta P_{\text{int}} = \rho_{fi} u_{fi}^2 \frac{f_{fo}}{2D}[\lambda + \phi_{Lo}^2 (l_h - \lambda)] + g\rho_{fi} l_h - g\Delta\rho_i (l_h - \lambda/2) - g\Delta\rho\bar{\alpha}(l_h - \lambda) \tag{1}$$

where

$$\lambda = \frac{\Delta h_i \rho_{fi} u_{fi} D}{4q} \tag{2}$$

$$f_{fo} = \frac{0.316}{\text{Re}_f^{0.25}} \text{ and } \text{Re}_f = \frac{\rho_{fi} u_{fi} D}{\mu_f} \tag{3}$$

ϕ_{Lo}^2 : Homogeneous two-phase multiplier (4)

$$\bar{\alpha} = \frac{1}{1 + \frac{1-x}{x}\frac{\rho_g}{\rho_f}} = \frac{1}{1 + \frac{1 - x_e/2}{x_e/2}\frac{\rho_g}{\rho_f}} \tag{5}$$

$$x_e = \left(\frac{4ql_h}{\rho_{fi} u_{fi} D} - \Delta h_i\right)\frac{1}{h_{fg}} \tag{6}$$

The CHF (q''_{CHF}) can be derived by solving for the flow excursion condition $\frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}} = 0$ and seeking the solution for the equation.

The partial derivative of internal pressure drop can be expressed as follows;

$$\frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}} = \rho_{fi}(2u_{fi})\frac{f_{fo}}{2D}\left[\lambda + \phi_{Lo}^2(l_h - \lambda)\right] + \rho_{fi} u_{fi}^2 \frac{1}{2D}\frac{\partial \lambda}{\partial u_{fi}}\left[\lambda + \phi_{Lo}^2 (l_h - \lambda)\right]$$
$$+ \rho_{fi} u_{fi}^2 \frac{f_{fo}}{2D}\left[\frac{\partial \lambda}{\partial u_{fi}} + \frac{\partial \phi_{Lo}^2}{\partial u_{fi}}(l_h - \lambda) - \phi_{Lo}^2 \frac{\partial \lambda}{\partial u_{fi}}\right] + g\Delta\rho_f \frac{1}{2}\frac{\partial \lambda}{\partial u_{fi}} - g\Delta\rho \frac{\partial \bar{\alpha}}{\partial u_{fi}}(l_h - \lambda) + g\Delta\rho\bar{\alpha}\frac{\partial \lambda}{\partial u_{fi}} \tag{7}$$

where,

$$\frac{\partial \lambda}{\partial u_{fi}} = \frac{\Delta h_i \rho_{fi} D}{4q} = \frac{\lambda}{u_{fi}}, \quad \frac{\partial f}{\partial u_{fi}} = -\frac{0.079}{u_{fi}}\left(\frac{\rho_{fi} u_{fi} D}{\mu_{fi}}\right)^{-0.25} = -\frac{f}{4u_{fi}}$$

$$\frac{\partial \bar{\alpha}}{\partial u_{fi}} = \left(1 + \frac{1 - 0.5x_e}{0.5x_e}\frac{\rho_g}{\rho_f}\right)^{-2} \frac{\rho_g}{\rho_f}(2x_e{}^{-2})\frac{\partial x_e}{\partial u_{fi}}, \quad \frac{\partial x_e}{\partial u_{fi}} = -\frac{4ql_h}{\rho_{fi} h_{fg} D}\frac{1}{u_{fi}^2}$$

Therefore the above equation reduces to

$$\frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}} = \frac{7}{4}\rho_{fi} u_{fi} \frac{f_{fo}}{2D}[\lambda + \phi_{Lo}^2(l_h - \lambda)] + \rho_{fi} u_{fi}^2 \frac{f_{fo}}{2D}\frac{\partial \lambda}{\partial u_{fi}}\left[\frac{\lambda}{u_{fi}}(1 - \phi_{Lo}^2) + \frac{\partial \phi_{Lo}^2}{\partial u_{fi}}(l_h - \lambda)\right]$$
$$+ \frac{\lambda}{u_{fi}}\left(\frac{1}{2}\Delta\rho_f + \Delta\rho\bar{\alpha}\right) - g\Delta\rho\frac{\partial \bar{\alpha}}{\partial u_{fi}}(l_h - \lambda) \tag{8}$$

In Eq. (8), the 3rd and 4th term (~10^3) are negligible in comparison to the 1st and 2nd term (~10^5) Therefore Eq. (8) is rearranged as

$$\frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}} = \rho_{fi} u_{fi} \frac{f_{fo}}{2D}\left[\frac{7}{4}\phi_{Lo}^2(l_h - \lambda) + \frac{\partial \phi_{Lo}^2}{\partial u_{fi}} u_{fi}(l_h - \lambda) - (\phi_{Lo}^2 - \frac{11}{4})\lambda\right] \tag{9}$$

To satisfy the condition of $\frac{\partial \Delta P_{\text{int}}}{\partial u_{fi}} = 0$ to Eq. (9), it can be rewritten as

$$\frac{7}{4}\phi_{Lo}^2(l_h - \lambda) + \frac{\partial \phi_{Lo}^2}{\partial u_{fi}} u_{fi}(l_h - \lambda) - (\phi_{Lo}^2 - \frac{11}{4})\lambda = 0 \tag{10}$$

The two-phase multiplier is the function of pressure and exit quality, so the partial derivative of

$$\frac{\partial \phi_{Lo}^2}{\partial u_{fi}} = \frac{\partial \phi_{Lo}^2}{\partial x_e} \cdot \frac{\partial x_e}{\partial u_{fi}} \tag{11}$$

$$\frac{7}{4}\phi_{Lo}^2(l_h-\lambda)+\frac{\partial\phi_{Lo}^2}{\partial x_e}\cdot\frac{\partial x_e}{\partial u_{fi}}u_{fi}(l_h-\lambda)-(\phi_{Lo}^2-\frac{11}{4})\lambda=0 \tag{12}$$

3.1 Two-phase Multiplier (ϕ_{Lo}^2) for the Linear Equation of x_e

$$\phi_{Lo}^2=A_0+A_1x_e \tag{13}$$

$$\frac{\partial\phi_{Lo}^2}{\partial x_e}=A_1 \tag{14}$$

By substituting Eq. (13) and (14) into Eq. (12), the equation can be rewritten as,

$$\frac{7}{4}(A_1x_e^2+A_0x_e)-A_1x_e\frac{4l_hq}{GDh_{fg}}-\left[(A_1x_e+A_0)-\frac{11}{4}\right]\frac{\Delta h_i}{h_{fg}}=0 \tag{15}$$

The exit quality is expressed as

$$x_e=\frac{4l_hq}{GDh_{fg}}-\frac{\Delta h_i}{h_{fg}}=\frac{4}{C}q-t\,,\;\frac{4l_hq}{GDh_{fg}}=\frac{4}{C}q=(\frac{4}{C}q-t)+t \tag{16}$$

Since, $C=\frac{Gh_{fg}}{l_h/D},\quad t=\frac{\Delta h_i}{h_{fg}}$ (17)

$$\frac{3}{4}A_1(\frac{4}{C}q-t)^2+(\frac{7}{4}A_0-2A_1)(\frac{4}{C}q-t)+(A_0-\frac{11}{4})t=0 \tag{18}$$

Finally, the above equation can be rearranged as follows

$$(\frac{4}{C}q-t)^2+\frac{1}{3A_1}(7A_0-8A_1)(\frac{4}{C}q-t)-\frac{1}{3A_1}(4A_0-11)t=0 \tag{19}$$

The solution of the Eq. (19) may be given in the following form,

$$\frac{4}{C}q-t=\frac{-a\pm\sqrt{a^2-4b}}{2} \tag{20}$$

where,

$$a=\frac{1}{3A_1}(7A_0-8A_1)\,,\;b=-\frac{1}{3A_1}(4A_0-11)t \tag{21}$$

$$\therefore\; q=\frac{C}{4}\left\{t+\frac{-a\pm\sqrt{a^2-4b}}{2}\right\} \tag{22}$$

3.2 Two-phase Multiplier (ϕ_{Lo}^2) for Homogeneous Model

$$\phi_{Lo}^2=\left[1+x(\frac{\upsilon_g}{\upsilon_{fg}})\right]\left[1+x(\frac{\mu_{fg}}{\mu_g})\right]^{-0.25} \tag{23}$$

The two-phase multiplier is simplified as follows

$$\phi_{Lo}^2=\left[1+x(\frac{\upsilon_g}{\upsilon_{fg}})\right] \tag{24}$$

$$\phi_{Lo}^2=\left[1+\frac{x_e}{2}(\frac{\upsilon_g}{\upsilon_{fg}})\right] \tag{25}$$

by Eq. (25)

$$A_0=1,\;A_1=\frac{\upsilon_g}{2\upsilon_{fg}} \tag{26}$$

By substituting Eq. (26) into Eq. (21), Eq. (22) can be rewritten as

$$q=\frac{C}{8}\left\{2t-a-\sqrt{a^2-4b}\right\} \tag{27}$$

Finally, from the two-phase multiplier for the homogeneous model and Ledinegg instability criterion the relationship between CHF and the principal parameters is derived and can be expressed as

$$q = 0.125\frac{Gh_{fg}}{l_h / D}\left\{2t - a - \sqrt{a^2 - 4b}\right\} \quad (23)$$

where

$$t = \frac{\Delta h_i}{h_{fg}},\quad a = \frac{14}{3}\frac{\upsilon_f}{\upsilon_{fg}} - \frac{8}{3},\quad b = \frac{14}{3}\frac{\upsilon_f}{\upsilon_{fg}}t$$

Table 1. Comparison of the Two-Phase Frictional Multiplier for the Homogeneous Model Steam-Water System

Steam quality % by wt.	Pressure, bar (psia)					
	1.01 (14.7)		20.59 (100)		34.48 (500)	
	Eq. (28)	Eq. (29)	Eq. (28)	Eq. (29)	Eq. (28)	Eq. (29)
0.0	1.0	1.0	1.0	1.0	1.0	1.0
0.5	9.0	8.8	1.4	1.4	1.2	1.2
1.0	17.0	16.2	1.8	1.8	1.7	1.4
1.5	25.0	23.3	2.2	2.2	1.9	1.6
2.0	33.0	30.2	2.6	2.5	3.4	1.8

Table 2. Parameter Ranges of CHF Experimental Data for Water

Parameter	Range
Pressure (MPa)	0.1 - 4.0
Mass flux (kg/m²s)	1,300 - 27,000
Inlet quality (-)	< -0.1
Heated length (m)	0.025 - 0.86
Tube diameter (m)	0.001 - 0.0108
Number of data	21

4. Results and Discussion

The parameter ranges of the selected data are $1 < P < 40$ bar, $1{,}300 < G < 27{,}000$ kg/m^{2}s and $x_i(\approx \frac{\Delta h_i}{h_{fg}}) < -0.1$ as shown in Table 2. To compare the prediction accuracy of the present model with the other CHF correlations, Bowring [2] and Katto [1] correlations of water are selected. In Table 3 these correlations are presented and the prediction accuracy is compared. The overall mean accuracy ratio of Bowring and Katto's correlations are 1.10 and 1.614 with the standard deviation of 0.314 and 0.935, respectively.

Also, the various comparisons of each prediction on mass flux, pressure, inlet subcooling and exit quality are presented in Fig. 3 through Fig. 7. From the results, the present mechanistic CHF correlation for water shows that the predictions agree well with the CHF data of water.

5. Conclusions

(1) From the simplified two-phase homogeneous frictional pressure drop and the Ledinegg instability(or Flow Excursion) criterion the relationship between CHF for the water and the principal parameters such as mass flux, heat of vaporization, heated length-to-diameter ratio, vapor-liquid density ratio and inlet subcooling is derived.

(2) The present mechanistic model to predict the CHF of water agree well with the extensive KAIST CHF data. The overall mean ratio of predicted to experimental CHF value is 0.988 with standard deviation of 0.046.

(3) Finally, the CHF correlation of water based on Ledinegg instability has been derived as follows; $q = 0.125\frac{Gh_{fg}}{l_h / D}\left\{2t - a - \sqrt{a^2 - 4b}\right\}$, $t = \frac{\Delta h_i}{h_{fg}}$, $a = \frac{14}{3}\frac{\upsilon_f}{\upsilon_{fg}} - \frac{8}{3}$, $b = \frac{14}{3}\frac{\upsilon_f}{\upsilon_{fg}}t$

Table 3. Comparison of the Accuracy for Water CHF Correlations

Correlations	Mean	Standard deviation
Present : $q = 0.125\frac{Gh_{fg}}{l_h / D}\left\{2t - a - \sqrt{a^2 - 4b}\right\}$	0.988	0.046
Bowring : $q = \frac{A' + 0.25DG\Delta h_i}{C' + l_h}$	1.110	0.314
Katto : $q = q_{co}(1 + K\frac{\Delta h_i}{h_{fg}})$	1.614	0.935

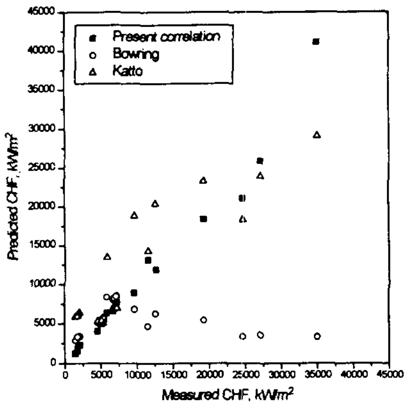

Fig. 3. Comparison of Predicted and Measured CHF Values

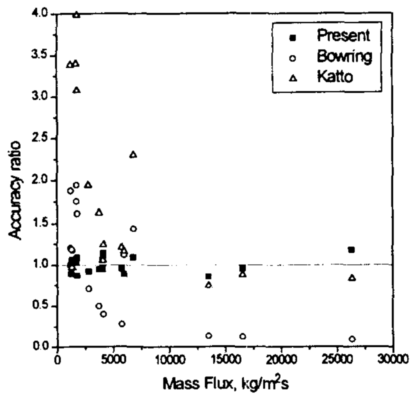

Fig. 4. Prediction Accuracy on Mass Flux

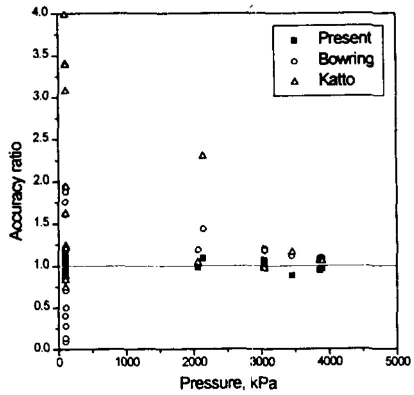

Fig. 5. Prediction Accuracy on Pressure

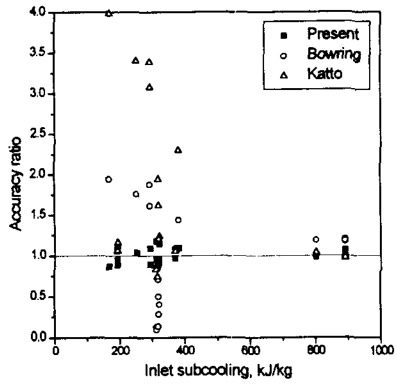

Fig. 6. Prediction Accuracy on Inlet Subcooling

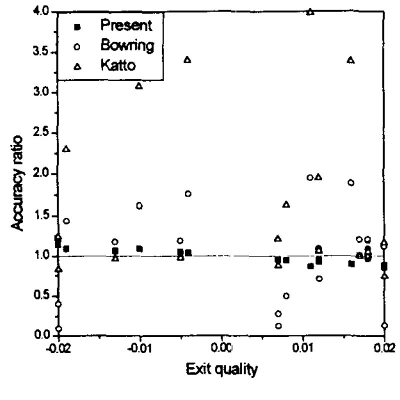

Fig. 7. Prediction Accuracy on Exit Quality

Nomenclature

A	cross-sectional area (m^2)
D	hydraulic diameter (m)
f	friction factor of liquid
g	gravitational acceleration (m/s^2)
G	mass flux (kg/m^2s)
h_{fg}	heat of vaporization (J/kg)
l_h	heated length (m)
P	pressure (Pa)
q	heat flux (W/m^2)
u_{fi}	inlet liquid velocity (m/s)
υ	specific volume (m^3/kg)
z	axial coordinate
Δh_i	inlet subcooling (J/kg)
$\Delta\rho$	density difference(ρ_f-ρ_g), (kg/m^3)
$\Delta\rho_f$	density change($\beta\rho_{fi}\Delta T$), (1/°C)
ϕ^2_{Lo}	two-phase multiplier
μ	viscosity ($N\cdot s/m^2$)
x_i	inlet quality
x_e	exit quality

Subscripts

ext	external
f	fluid
fi	inlet fluid
g	vapor
int	internal
meas	measured(or experimental)
pred	predicted

References

1. Y. Katto and H. Ohno, "An improved version of the generalized correlation of critical heat flux for the forced convective boiling in uniformly heated vertical tubes," Int. J. Heat Mass Transfer 27, pp1641-1648, 1984.
2. R.W. Bowring, "A simple but accurate round tube, uniform heat flux, dryout correlation over the pressure range 0.7 - 17 MN/m^2 (100-2500 psia)," AEEW-R 789, 1972.
3. J.A. Boure, A.E. Bergles and L.S. Tong, "Review of two-phase flow instability," Nucl. Eng. & Design., 25, pp. 165- , 1973.
4. M. Ledinegg, "Instability of flow during natural and forced circulation," Die Warme, 61, p.8, 1938.
5. M. Ishii and H.K. Fauske, "Boiling and dryout behavior in a liquid-metal fast breeder reactor subassembly bundle under low heat flux and low flow conditions," Nucl. Sci. Eng., 84, pp. 131-146, 1983.
6. S.H. Chang and Y.B. Lee, "A new critical heat flux model for liquid metals under low heat flux-low flow conditions," Nucl. Eng. & Design., 148, pp. 487-498, 1994.
7. K.Mishima, H. Nishihara and I. Michiyoshi, "Boiling burnout and flow instabilities for water flowing in a round tube under atmospheric pressure," Int. J. Heat Mass Transfer 28[6], pp.1115-1129, 1985.
8. K. Mishima, "Boiling burnout at low flow rate and low pressure conditions," Ph.D. Thesis, Kyoto Univ., Japan, 1984.