Control of voltage collapse in an electrical power system using center manifold theory

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Abstract

In this paper center manifold theory is reviewed and its application to the control of bifurcations is explored. When applying the theory to a sample power system, we study the stabilization of bifurcation points using controls depending only on the rotor angular velocity of a generator. Under such a control it is shown that the system is not locally stabilizable when control is applied through mechanical power, and the system is locally stabilizable when the control is applied to the capacitor compensator.

1 Introduction

Recently a number of utilities in the world have experienced voltage related problems. In some cases blackouts occurred as a result of voltage collapse. In response to the growing concern an international workshop on voltage stability and security was recently convened under the sponsorship of EPRI and NSF [1]. The concensus was that today's highly stressed and heavily loaded power system networks are part of the reason for these types of problems and concluded further work is needed to fully understand this phenomena.

It has been proved the coexistence of oscillatory type instabilities and voltage collapse in a sample power system model[2,3]. It is generally admitted that a saddle node bifurcation of the nominal equilibrium and an oscillatory transient from a subcritical Hopf bifurcation lead to voltage collapse phenomena[2,4]. Therefore the control of the bifurcation becomes crucial. Colonius and Kliemann studied the one-dimensional bifurcation control and Hopf bifurcation control from the control set point of view under local accessibility assumptions[5,6]. In [7] the cubic feedback for the control of Hopf bifurcation and linear feedback for the control of saddle node bifurcation in a power system was first proposed. In this paper, we investigate the stabilization of the saddle node bifurcation using center manifold theory, and apply this to the analysis of a sample power system. Our approach differs from the ref.[4,7] in the following ways. First we apply only output feedback. Secondly we study stabilization when the control is applied to either the mechanical power or the compensated capacitor. It is shown in a specific system that the linear feedback proposed in [4] does not control the bifurcation modes, so a nonlinear feedback is required.

2 Bifurcation control using center manifold theory

Consider the lift of the control system on n dimensional Euclidean space,

$$\begin{aligned} \dot{x} &= f(x, p, u) \\ \dot{p} &= 0 \end{aligned} \tag{1}$$

with states $x \in \mathcal{R}^n$, system parameters $p \in \mathcal{R}^p$, and control $u \in \mathcal{R}^1$. Suppose $f(0,0,0)=0$, $D_u f(0,0,0) \neq 0$ and f is C^∞ on neighborhood of the origin, and dynamical system $\dot{x}=f(x,p,0)$ exhibits a bifurcation at $(x,p) = (0,0)$. Now the problem is to design a feedback $u(x) \in C^1$ with $u(0)=0$ to stabilize the bifurcation point. Here we will apply center manifold theory to this stabilization problem. We start with a review of the linear theory.

Linearization of the system (1) gives

$$\begin{aligned} \dot{x} &= Ax + \Lambda p + Bu + \tilde{f}(x,p,u) \\ \dot{p} &= 0 \end{aligned}$$

where $A = D_x f(0,0,0)$, $\Lambda = D_p f(0,0,0)$, $B = D_u f(0,0,0)$, and $\tilde{f}(0,0,0)=0$, $D\tilde{f}(0,0,0)=0$. If the eigenvalues of A not on the left half plane are controllable by linear feedback $u=Fx$, then the system is stabilizable using linear system theory. If the eigenvalues in the right half plane are not controllable by linear feedback, then the system definitely cannot be stabilized by any feedback control under the conditions above. The remaining case, in which some uncontrollable eigenvalues are placed on the imaginary axis, becomes interesting because nonlinear theory comes into effect. We will assume this in the sequel.

Let $u = Fx + \alpha(x)$, *where* $\alpha(0) = 0$ and $D\alpha(0) = 0$ so that A+BF has all the controllable eigenvalues in left half plane. Then there is linear transformation $y = T^u x$ under which (1) can be written as :

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} f_1^u(y_1, y_2, p) \\ f_2^u(y_1, y_2, p) \\ 0 \end{pmatrix} \qquad (2)$$

where the u denotes the dependence on feedback functions chosen, A_{11} has eigenvalues in the left half plane, A_{22} has eigenvalues on the imaginary axis, and $f_i^u(0,0,0) = 0$, $Df_i^u(0,0,0) = 0$. for $i = 1,2$.

The following theorem insures the existence of a center manifold and has the reduction principle as a consequence [8].

Theorem 2.1 *There exists a center manifold for (2),* $y_1 = h(y_2, p)$, $|(y_2, p)| < \delta$, *where h is* C^2, *and the flow on the center manifold is governed by the system*

$$\frac{d}{dt}\begin{pmatrix} y_2 \\ p \end{pmatrix} = \begin{pmatrix} A_{22} & A_{23} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y_2 \\ p \end{pmatrix} + \begin{pmatrix} f_2^u(y_1, y_2, p) \\ 0 \end{pmatrix} \qquad (3)$$

Furthermore if the equilibrium point (0,0) *of (3) is stable (asymptotically stable) (unstable), then the equilibrium point* (0,0,0) *of (2) is stable (asymptotically stable) (unstable).* Denote

$$z = \begin{pmatrix} y_2 \\ p \end{pmatrix}, \quad B = \begin{pmatrix} A_{22} & A_{23} \\ 0 & 0 \end{pmatrix}, \quad g(\cdot,\cdot) = \begin{pmatrix} f_2^u(\cdot,\cdot) \\ 0 \end{pmatrix}.$$

We have a center manifold $y_1 = h(z)$ and rewrite (3) as :

$$\dot{z} = Bz + g(h(z), z) \qquad (4)$$

To find $h(z)$, one needs to solve the system of partial differential equations :

$$Dh(z)\cdot[Bz + g(h(z), z)] - A_{11}h(z) - f_1^u(h(z), z) = 0.$$

However, solving these equations could be harder than solving our original problem. Carr[8] showed the following useful approximation theorem, hence simplified the computation.

For a function $\phi: \mathcal{R}^{c+p} \to \mathcal{R}^s$ which is C_1 in a neighborhood of origin, define

$$(M\phi)(z) = D\phi(z)\cdot[Bz + g(\phi(z), z)] - A_{11}\phi(z) - f_1^u(\phi(z), z).$$

Then $(Mh)(z) = 0$, and furthermore(page 25 in [8]).

Theorem 2.2 *Let* ϕ *be a* C^1 *mapping of a neighborhood of origin in* $\mathcal{R}^{c+p}$ *into* $\mathcal{R}^s$ *with* $\phi(0) = 0$ *and* $D\phi(0) = 0$. *Suppose that as* $z \to 0$, $|(M\phi)(z)| = O(|z|^q)$ *for some* $q > 2$. *Then as* $z \to 0$, $|h(z) - \phi(z)| = O(|z|^q)$.

This theorem provides a way to approximate the center manifold to any arbitrary order of accuracy. For examples to approximate center manifold, see Chapter 5 of [8]. Since any power system consists of highly nonlinear dynamics and large of equations, applying the above approximation theorem directly will result in a heavy computational load. Therefore we need the following consequence before we analyze a sample power system.

Corollary 2.3 *Given any integer* $p > 2$, *let* $\tilde{f}_1^u$ *and* $\tilde{g}$ *be the approximate functions of* f_1^u *and* g *respectively so that* $|\tilde{f}_1^u(x) - f_1^u(x)| = O(|x|^p)$, $|\tilde{g}(x) - g(x)| = O(|x|^p)$. *Define*

$$(M^p\phi)(z) = D\phi(z)\cdot[Bz + \tilde{g}(\phi(z), z)] - A_{11}\phi(z) - \tilde{f}_1^u(\phi(z), z).$$

If $\phi(0) = 0$, $D\phi(0) = 0$ *and* $|(M^p\phi)(z)| = O(|z|^p)$, *then* $|h(z) - \phi(z)| = O(|z|^p)$.

Note that if $\phi(z)$ is such an approximation of $h(z)$, then the dynamics restricted to the center manifold can be approximated by $\dot{z} = Bz + \tilde{g}(\phi(z), z)$.

3 Voltage stabilization using output feedback

Consider a sample power system that consists of a load which is supplied by two generators [2]. The load is represented by an induction motor in parallel with a constant PQ load. The load reactive power Q_1 is chosen as the bifurcation parameter. In the example described in [2], assume that control can be applied to either the mechanical power P_m or the capacitor C *such as* $P_m = 1.0 + u$, $C = 12$ or $P_m = 1.0$, $C = 12 + u$.

Without controls, a codimension one saddle node bifurcation appears at $Q_1 = 11.4115$, and the dynamics restricted on the center manifold can be computed to be $\dot{z} = -79.4359\, z^2 + h.o.t.$ see also [2]. Therefore we need to apply control around this bifurcation point. If we use state feedback, we have shown that the linearized system is complete controllable for both mechanical power control and capacitor control. However, in practice not all the states can be tracked and observed, hence only some output feedback can be applied. Here we use the output function $y = \omega = D \cdot x$ where, $D = [0\,1\,0\,0]$ and $x = [\delta\ \omega\ \delta_L\ V]^T$. When linearizing the system around the bifurcation point, we see that the zero eigenvalue is not controllable by the linear output feedback, for both excitation control and capacitor control. Therefore we will apply a nonlinear output control $u = F(y) = F(D \cdot x)$.

Case 1 : control applied through mechanical power, i.e. $P_m = 1.0 + u,\ C = 12.$

In this case, $f(x, p, u)$ in (1) is linear in u. And the bifurcation point is at $Q_1 = 11.4115$, $\delta = 0.3476$, $\omega = 0$, $\delta_L = 0.1380$, and $V = 0.9250$. Let $u = u_1(Dx) + u_2(Dx)^2 + u_3(Dx)^3 + \cdots$. In order to make the equilibrium of (3) asymptotically stable, the second order term $K_2 y_2^2$ in the Taylor expansion of $f_2''(h(y_2, p), y_2, p)$ has to be zero. This means, after some computations, $u_1 = 9861.2581235$. When applying $u = u_1(Dx) + \cdots$, the linearized system around the bifurcation point is $\dot{x} = \bar{A}x + h.o.t.$, where the eigenvalues of $\bar{A}$ are 0, -1, -89.32871. Therefore the linearized system is unstable. Hence we conclude that the bifurcation point cannot be stabilized by excitation control depending only on rotor angle velocity of the generator.

Case 2 : control applied through the capacitor, i.e. $P_m = 1.0,\ C = 12 + u$

In this case, $f(x, p, u)$ in (1) is nonlinear in u. And one of the codimension one saddle node bifurcation points is at $Q_1 = 11.3226$, $\delta = 0.3532$, $\delta_L = 0.1421$, and $V = 0.9184$. Again using output feedback, we apply $u = u_1(Dx) + u_2(Dx)^2 + \cdots$. Following the dame procedure as in case 1, we have $u_1 = 7570.821611$. The eigenvalues of the controlled linear system are 0, -1.8, $-44.8 \pm 1558i$. Therefore we have to apply center manifold theory and the approximation theorems above on order to see of the system is stable. It turns out that we can apply a control with u_1 given above and $u_2 = 0$, $u_3 = 0$. Then the approximate center manifold is computed as $\phi_1(z) = -0.159z^2 + \cdots$; $\phi_2(z) = -1.7915z^2 + \cdots$, $\phi_3(z) = 0.4541z^2 + \cdots$; and $\phi(z) = (\phi_1(z), \phi_2(z), \phi_3(z))$. the approximate dynamics restricted to the center manifold is $\dot{z} = -0.0364255z^3$. Therefore we conclude that under such a control, the bifurcation point is stabilized.

4 Discussions

In this paper, by employing center manifold theory, we investigate the feedback stabilization of codimension one saddle node bifurcation point which arises from a sample power system. Applying the control depending on rotor anglu;ar velocity to the capacitor, we have shown that the bifurcation point can be stabilized. However, such a control would have two drawbacks. First, the approximate dynamics restricted to the center manifold is structurally unstable. Second, such a control should be regarded as temporary and secondary because we did not globally design a control which moves bifurcation point to stable non-bifurcation equilibrium. Therefore additional fault clearing of the power system is needed as a primary tool.

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