Chaotic 비선형 동역학 시스템의 Chaotic 현상 분석 시뮬레이터의 개발과 궤환제어에 관한 연구

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A Study on Feedback Control and Developmemt of chaotic Analysis Simulator for Chaotic Nonlinear Dynamic Systems

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In this Paper, we propose the feedback method having neural network to control the chaotic signals to periodic signals. This controller has very simple structure, it is immune to small parameter variations, the precise access to system parameters is not required and it is possible to follow ones of its inherent periodic orbits or the desired orbits without error, The controller consist of linear feedback gain and neural network. The learning of neural network is achieved by error-backpropagation algorithm. To prove and analyze the proposed method, we construct a software tool using c-language.

1. Introduction

The chaotic behavior of dynamic systems has been extensively studied in recent years and many approaches have been followed.

The characteristics of chaotic behavior are as follows.

1) It is happened that a small difference in the initial conditions produce very great ones in the final phenomina.

2) A chaotic spectrum is not composed solely of discrete frequency, but has a continuous broad-band nature. This noise-like spectrum is characteristic of chaotic systems.

3) The steady-state Poincare' orbits of chaotic systems do not lie on a simple geometrical objects as is the case with periodic and quasi-periodic behaviors. It appears to be layers within layers, much like fine pastry. This structure is typical of chaotic systems-called "Fractal structure".

In this paper, we will study how to control the chaotic trajectory of a continuous-time nonlinear systems to converge to its ones or more periodic orbits such as limit cycles.

Many research papers have proposed the method to control and suppress chaos. ,there are representatively parameter variation technique, shock absorber concept and the entrainment method [1]. But this methods have problems in that the goal behavior has to be chosen by trial-and-error, there is no feedback and any solution of original system can not be a goal of the control. And, in the more developed method there is the linear feedback method proposed by Chen and Dong [3]. This method has merits in that access to system parameters is not required, it is immune to small parameter variations and any solution of original system can be a goal of the control. To achieve our goal, however, the feedback control gains of sufficiently large value are required. Although we use the large control gain, the control purpose is not $x(t)=x_m(t)$ but $|x(t)-x_m(t)| < \varepsilon$. And it might be difficult to apply the large control gain in real systems.

In this paper, it is our achieve $x=x_m$ and $y=y_m$, although we used a relatively small value for the feedback gain. For this purpose, we use neural network and linear feedback controller.

A small feedback gain attracts chaotic signal to periodic signal having $|x_m(t)-x(t)| \le \varepsilon$. Then, to follow the desired periodic orbits without error can be achieved afterwards by a learning of neural network.

To prove the proposed method, we construct a software tool and use a 4-order runge-kutta method for solving the solution of differential equations.

2. The Analysis for chaotic dynamic systems

Duffing equation describes the hardening spring effect observed in many mechanical problems. Since then, this has become one of the most popular methods, like the well-known Van-der pole equation, in the studies of nonlinear oscillation, bifurcation and chaos [3][6].

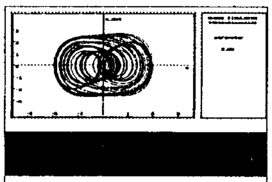
In this paper control and analysis Duffing equation.

$$\ddot{x}+p\cdot\dot{x}+p_1\cdot x+x^3=q\cdot\cos(wt)$$

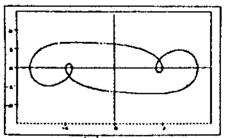
where $p>0$, $p_1<0$ (1)

This solution of trajectory of Duffing equation display complex phenomena, including various periodic orbits and some chaotic orbits, when certain parameters of the equation are varied.

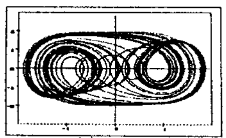
In figure 1,we have seen such changes when the parameter q,the magnitude of the external periodic forcing term, is varied within certain range.



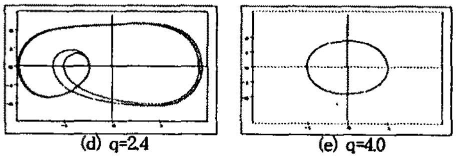
(a) The phase and time diagram of a constructed graphic simulator



(b) q=1.5



(c) q=2.3



(d) q=2.4 (e) q=4.0

Fig. 1 Typical periodic and chatic orbits of the Duffing equation.(x axis: x, y axis: $\dot{x}$)

1) Bifurcation diagram

The dynamics may also be viewed more globally over a range of parameter values, thereby allowing simultaneous comparison of periodic and chaotic behavior.

The bifurcation diagram provides a summary of the essential dynamics and is therefore a useful method of acquiring the overview. In dynamics a change in the number of solution to a differential equations as a parameter is viewed is called a bifurcation.

For the Duffing equation, bifurcation can be easily detected by examing a graph of x (at a fixed phase in the drive cycle) versus the drive amplitude q.

This diagram is allowed to come to a steady-state by omitting the some drive cycles. Suppose first that the Duffing equation is driven to a small value.

The phase trajectory is limit cycle that is symmetric about the origin (show Fig. 1.(b)) : the corresponding Poincare' section shows a fixed point. The x or $\dot{x}$ takes only a single value in the bifurcation diagram.

If the driving force q is slightly increased, the diagram has two different shapes or many different shapes. This two and many valuedness means two limit cycles and chaotic behavior. In this diagram, we can know that the chaotic behavior is non-periodic but is bounded. This means that is stable in sense of Lyapunov.

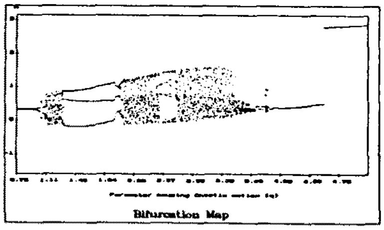


Fig. 2 Bifurcation diagram Duffing equation with q

3. The control of chaotic systems

1) Linear feedback method

Chen and Dong have studied in the problem of controlling trajectory of the Duffing equation to one of its inherent orbits [3]. The block diagram of control system is shown in Fig 4.

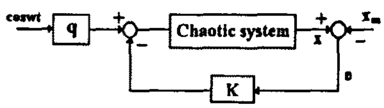


Fig. 4 Block diagram of the linear feedback controller (Chen-Dong approach)

Let $(x_m, y_m) = (x_m(t), y_m(t))$ be one of its period-1 orbits that we are targeting.

The approximate steadily oscillating period-1 solution is defined

$$x_m(t) = c + a\cdot\cos wt + b\cdot\sin wt \qquad (2)$$

To determine unknown coefficients a, b and c, $x(t)$ of (1) is substitued for (2).

And the equilibrium point (a, b, c) of system, which corresponds to steady oscilations, can be solved from a little algebra.

For some T>0 we want to have

$$x(t) = x_m(t)\ ,\ \dot{x}(t) = \dot{x}_m(t), \qquad t \geq T \qquad (3)$$

But, since it is impossible to perfectly follow a desired periodic orbits, we modify the control purpose such that for any given $\varepsilon > 0$

$$|x_m(t) - x(t)| \leq \varepsilon \text{ and } |\dot{x}_m(t) - \dot{x}(t)| \leq \varepsilon \quad \text{for all } t > T, v \qquad (4)$$

For this purpose, Chen and Dong consider the conventional feedback controller of the form

$$\begin{bmatrix} u \\ v \end{bmatrix} = -K\begin{bmatrix} x - x_m \\ \dot{x} - \dot{x}_m \end{bmatrix} = -\begin{bmatrix} k_{11}, k_{12} \\ k_{21}, k_{22} \end{bmatrix}\begin{bmatrix} x - x_m \\ \dot{x} - \dot{x}_m \end{bmatrix}$$

where K is 2x2 matrix (5)

which yeild, after being added to the original system, the following " controlled Duffing equation" :

$$\dot{x} = f_c(x, y)$$
$$\dot{y} = g_c(x, y) \qquad (6)$$

To determine a suitable controller designed by K, the jacobian matrix is considered :

$$J_c = J_c(x_m, y_m) = \begin{vmatrix} \dfrac{\partial f_c}{\partial x}, & \dfrac{\partial f_c}{\partial y} \\ \dfrac{\partial g_c}{\partial x}, & \dfrac{\partial g_c}{\partial y} \end{vmatrix}_{(x_m, y_m)} \qquad (7)$$

Let us determine K so that the root of its characteristic equation $det[sI - J_c] = 0$ could be located in the open left-half s-plane. Obviously, a sufficient condition is

$$p + k_{11} + k_{12} > 0$$
$$k_{11}(p + k_{22}) + (1 - k_{12})(k_{21} + p_1 + 3x_m^2) > 0 \qquad (8)$$

With the choice $k_{11} = k_{12} = k_{22} = 0$, we have

$$k_{21} > -p_1 - 3x_m^2 \qquad (9)$$

and the controlled Duffing equation becomes

$$\dot{x} = y$$
$$\dot{y} = -p_1 x - x^3 - py + q\cos wt - k_{21}(x - x_m) \qquad (10)$$

in which x_m is given by (). Furthermore, it is clear that if we use a relatively very large value of k_{21}, then this controlled equation reduces to

$$\dot{x} = y$$
$$0 \fallingdotseq -k_{21}(x - x_m)$$

But, in many cases it might be difficult to apply the control gain k_{21} of large value in real systems. For some value of k_{21}, the oscillatory term $q\cos wt$ existing originally in the Duffing equation may dominate the designated feedback control input, so that the controlled trajectory appears to be oscillating in some way.

Therfore, as above mentioned, the linear feedback method have demerit in that

$$|x_m(t) - x(t)| \leq \varepsilon \text{ and } |\dot{x}_m(t) - \dot{x}(t)| \leq \varepsilon$$
$$\text{for all } t > T,$$

,not perfect model following.

2) The improved feedback control using neural network

Since it might be difficult to apply the high gain in real systems, the linear feedback method defined the control goal as follow :

$|x_m(t) - x(t)| \le \varepsilon$ and $|\dot{x}_m(t) - \dot{x}(t)| \le \varepsilon$ for all $t > T_\varepsilon$

In this paper, although we used a relatively small value as the feedback gain k_{21}, our goal is to to achieve $x = x_m$ and $y = y_m$.

The block diagram of control system is shown in Fig 5. A small k_{21} attract chaotic signal to periodic signal having $|x_m(t) - x(t)| \le \varepsilon$ and the perfect model following control without error can be achieved afterwords by a learning of neural network.

The modified control inputs are defined

$$u_a = -k_{21}(x - x_m) + N(t;, x, x_m) \qquad (11)$$

where $N(\cdot)$ is the ouput neural network.

Therefore, the controlled Duffing equation has the form

$$\ddot{x} + p\dot{x} + p_1 x + x^3 = q\cos wt + u_a \qquad (12)$$

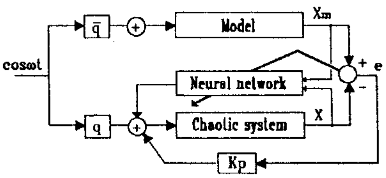


Fig. 5 The block diagram of the proposed feedback control using neural netwotk.

First, suppose that the desired periodic solution x_m satisfies Duffing equation :

$$\ddot{x}_m + p\dot{x}_m + p_1 x_m + x_m^3 = \bar{q}\cos wt \qquad (13)$$

However, since there are some modelling and parameter errors in real applications, the periodic solution x_m satisfies the following form

$$\ddot{x}_m + \bar{p}\dot{x}_m + \bar{p}_1 x_m + x_m^3 = \bar{q}\cos wt \qquad (14)$$

,so that subtracting it from the controlled Duffing equation with neural network gives

$$\ddot{e} + p\dot{e} + (p_1 + k_{21})e + e^3 = -3x x_m e + h(t; x_m, \dot{x}_m) + N(t; x, x_m) \qquad (15)$$

where, $e = x - x_m$ and $h(\cdot)$ is a undesirable term resulting from the modelling and parameter error, and bounded in small region.

Neural network is learned toward $e \to 0$ after some time, 'that is', be approximated to the form :

$$N(t; x, x_m) \fallingdotseq e^3 + 3x x_m e - h(t; x_m, \dot{x}_m) \qquad (16)$$

Therefore,

$$\ddot{e} + p\dot{e} + (p_1 + k_{21})e \fallingdotseq 0 \qquad (17)$$

Since back-propagation neural networks have local-minima, it is impossible for controlled Duffing equation to perfectly follow a desired periodic orbit. Obviously, however, it has a small errors.

In this paper, neural network is learned by back-propagation algorithm, and the proposed method, since neural network can learn all-nonlinear characteristics, have the good robustness [9][10].

The weight adjustment of neural network is computed [9][10]

$$\Delta w_{ij}^k(n+1) = \eta\, \delta_j^{k+1} out_i^k + \alpha\, \Delta w_{ij}^k(n)$$

$$w_{ij}^k(n+1) = w_{ij}^k(n) + \Delta w_{ij}^k(n+1) \qquad (18)$$

where $w_{ij}^k(n)$ = the value of a weight from neuron to q neuron at step n (before adjustment) and k indicate its destination layer

out_i^k = the value of output for neuron i

α, η =the momentum and training-rate coefficient

δ_i^k term of(18) have the following

output layer :

$$\delta_i^k = (target_i - out_i^{k+1}) \cdot f(net_i)$$

hidden layer :

$$\delta_j^k = f(net_i^k) \cdot \sum_j \delta_j^{k+1} \cdot w_{ij}^k \qquad (19)$$

where $f(\cdot)$, called a activation function, use

$$1/(1 + e^{-net_i}) - 0.5 \; [8]$$

A complete proof of the proposed method is already given in Chen and Dong, neural network only eliminate a small oscillating term after some time with learning.

4. Results

To prove and simlulate the proposed method, we construct a software tool using C-language, and the solution of differential equations are solved by 4-order runge-kutta method.

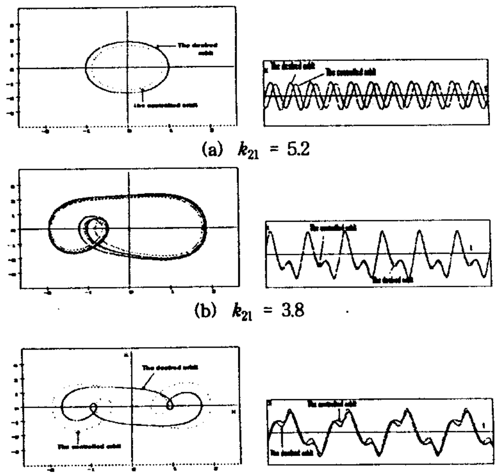
The controlled Duffing equation is given by

$$\ddot{x} + p\dot{x} + p_1 x + x^3 = q\cos wt + k_{21}e + N(t\, x, x_m, \dot{x}, \dot{x}_m) \qquad (20)$$

Where $p = 0.7$, $p_1 = -1.3$, $q = 2.3$, $w = 1.8$($rad/\sec$)

We use a conventional backpropagation network with input, hidden 1, hidden 2 and output layer. Each layers have 4, 4, 4 and 1 neurons, and input neurons consist of x, $\dot{x}$, x_m and $\dot{x}_m$. The momentum and learning coefficient is $\eta = 0.9$ and $\alpha = 0.8$, and the initial values of all weights are randomly chosen between -0.5 and +0.5.

Fig 6. show the result of the linear feedback method proposed Chen and Dong. From figure, we can know that , although a chaotic signal of Duffing equation is controlled to periodic orbit, the controlled signal differs from the desired orbit.



(a) k_{21} = 5.2

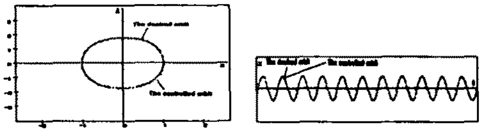
(b) k_{21} = 3.8

(c) k_{21} = 5.8

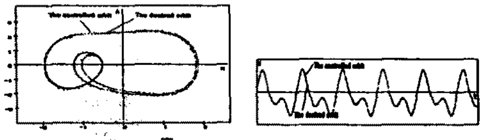
Fig.6 The result of the linear feedback method (Chen-Dong approach)

Fig 7. show the result of the proposed method after 100th periodic learning. we can know that almost perfectly follow the desired multi-period trajectory as well as the desired period 1 trajectory, in spite of small k_{21} . For simulation, the desired model equation use the following form

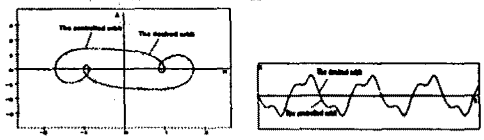
$$\ddot{x}_m + p\dot{x}_m + p_1 x_m + x_m^3 = \bar{q}\cos wt$$



(a) $\bar{q}=4.0$, $k_{21}=0.8$



(b) $\bar{q}=2.4$, $k_{21}=0.8$



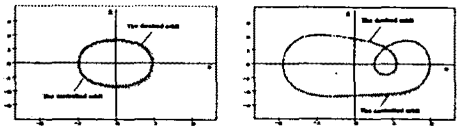
(c) $\bar{q}=1.5$, $k_{21}=0.8$

Fig. 7 The result of the proposed method after 100th periodic learning

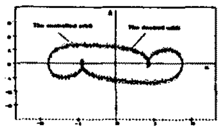
Fig 8. show that the proposed method have the robustness. To illustrate the robustness, the parameter of Duffing equation satisfying $x_m(t)$ differ somewhat from the actual controlled Duffing equation. That is, suppose that $\bar{p}$, $\bar{p}_1$ differ from p, p_1 .

$$\ddot{x}_m + \bar{p}\dot{x}_m + \bar{p}_1 x_m + x_m^3 = \bar{q}\cos wt \qquad (21)$$

where $\bar{p}=0.75$, $\bar{p}_1=-1.4$, $w=1.8$(rad/sec)



(a) $\bar{q}=4.0$, $k_{21}=0.8$ (b) $\bar{q}=2.5$, $k_{21}=0.8$



(c) $\bar{q}=1.5$, $k_{21}=0.8$

Fig. 8 The Robustness of the proposed method

Although the desired periodic trajectory are not the ones of inherent orbits in actual chaotic systems, it has no problem. This means that the precise access of the chaotic systems is not required and it is possible to apply in real systems having interactions of many system variables.

5. Conclusion

In this study, we analyzed the chaotic systems and proved the effectiveness of the feedback method having neural networtk.

The proposed method perfectly controlled the chaotic systems to limit cycles in spite of a small feedback gain.

In this method, Access to system parameter is not require , any particular solution of the autonomous system can be the goal of the control and robustness is gauranteed.

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