Meridional Circulations in a Sliced Cylinder

기울어진 회전 원판에 의한 원통형 용기내의 자오면 유동의 크기에 관한 연구

Jae Won KIM[1], Hong Sick LIM[2]

김 재 원, 임 홍 식

Abstract

Mixing is most important for developing an electric washer which transforms angular momentum from rotating solid wall to laundry clothes inside it. For magnification of this mixing effect, some inventions are introduced to washing machine system, i. e., washing plate, washing rod, and even for washing cap in a model of a Korean manufacture. However, the previous efforts show dissatisfaction up till now. In this paper, a triumph to enhance mixing effects to increase washing performance is presented and verified by numerical investigation. The present model to simulate a washing tub is the simple circular cylinder with two endwall disks which is completely filled with a viscous liquid. The present improvement is to change mounting position of a bottom disk of the model cylinder. Therefore, the aim of this work just proposes a new idea, which is numerically inspected, to a producer of washing machine. In detail, this invention is alternating the mounting position of a rotating bottom disk. Actually skewed pulsator is placed in steady of a flat disk, so the two endwall disks at top and bottom are not in parallel. The angle between an inclined bottom disk and the horizontal plane is fixed as 5 degree and physical domain to consider poses a sliced cylinder. Flow fields in both a right circular cylinder and the present improved model are fully depicted by numerical integration on a body fitted nonorthogonal regular uniform grid system. Numerical data to explain flow structure are plotted for understanding of the effects of the inclined disk. Also enhanced mixing effects by the inclined rotating disk are gauged by accurate numerical data used in this work.

1. Introduction

A numerical investigation has been accomplished for fluid flows in a circular cylinder which has an inclined rotating bottom disk at constant rotational speed Ω with respective to the longitudinal axis of the cylinder. The cylinder is completely filled with a viscous fluid of kinematic viscosity ν. Fluid depth changes from lh in maximum to rh in minimum along periphery path of the oblique disk. Impulsively rotating motion of the inclined disk generates fluid motion in a cylinder. The bottom end wall disk is oblique with angle α against the horizontal plane of the flow configuration. Cylinder aspect ratio, defined as a ratio of maximum height of fluid to radius of the cylinder, is O(1). Figure 1 shows the schematic diagram of the flow system. Inclined angle of the bottom disk α is equal to $\tan^{-1}((lh-rh)/R)$, where lh denotes the longest length of the cylinder, rh the shortest length and R is the radius of the container. The resulting rotational Reynolds number is defined as $Re=\Omega Rh/\nu$, where Ω denotes angular speed of the end wall disk and h the average depth of fluid.

1) 선문대학교 기계자동제어공학과 (336-170, 충남 아산시 탕정면 100, Tel: 0418-530-2337)

2) 영남대학교 대학원 기계공학과 (712-749, 경북 경산시 대동 214-1, Tel: 053-815-1887)

Through this calculation, Reynolds number is equal to 2000.

The fluid flows in the container carry angular momentum generated by a rotating solid wall. The passage for the momentum transfer is in the velocity boundaries adjacent to the end wall disks and side wall. Main focus is on the strength of the meridional circulation according to the variation of the oblique angle, α, of the end wall disk. In fact, an attempt was done for increased mass transfer in a cylindrical container with a rotating flat disk by adjustments of rotational time of the disk [Larousse, 1987]. Fluid mixing methodology, in the previous work, using spin-up and spin-down process only has operating time period as an adjustable parameter. However, in this work, fluid mixing using control of attitude of a rotating solid wall is proposed. The main controllable parameter is oblique angle α, otherwise Reynolds number and aspect ratio are fixed. Two angles, α=0 ° and 5 °, are selected for measuring the effects of the oblique disk.

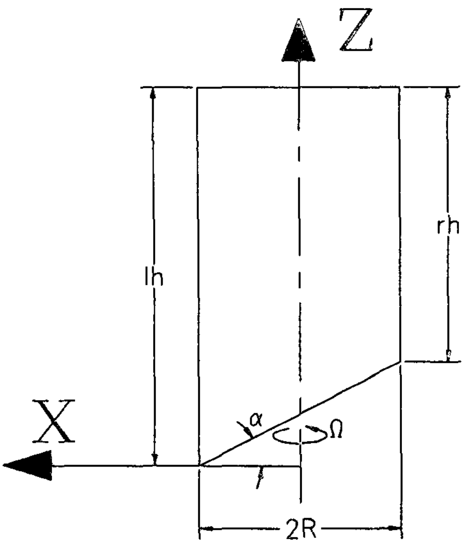

Fig. 1 The present flow system

Numerical integration is performed for continuity equation and momentum equations on a generalized nonorthogonal grid system. The present grid system is three dimensional nonorthogonal body fitted system and divides the physical domain into 42 by 42 by 42 grid points in (x,y,z) direction, respectively. This nonorthogonal grid net is generated by solving a Poisson equations (13.36a) and (13.36b) of recent text of Fletcher's [2] in the computational domain (ξ, η). The generation method is well explained in detail in his book. Figure 2 shows the typical grid on a (x,y) plane at an arbitrary vertical height. Actually the shape of Fig. 2 is gradually skewed against the horizontal plane as the vertical coordinate decreases, i.e., the grid net on the top disk is a right circle otherwise the net on the bottom is extremely inclined at 5 degree to the horizontal (x-y) plane. Dependent variables for numerical calculations are covariant velocity components for convergence and small skewness of discrete equations [3,4].

Fluid flows in a sliced cylinder have been studied for analysis of detached shear layer in rotating flow system. Noticeable work was presented by numerical calculations in work of Page's [5]. Page showed a relationship between parameters to explain flow stability conditions. Also an experimental research was done by Hide and Hocking [6]. The above two results was in good agreements. However, these two previous works had a sliced cylinder model dealt with behaviour of shear layer caused by partially rotating top disk.

It may be first triumph to handle the sliced cylinder problem with a view point of mixing magnifier. Increased magnitude of meridional velocities is observed in case of the present cut cylinder. The amount of intense mixing is quantitatively measured by numerical experiments.

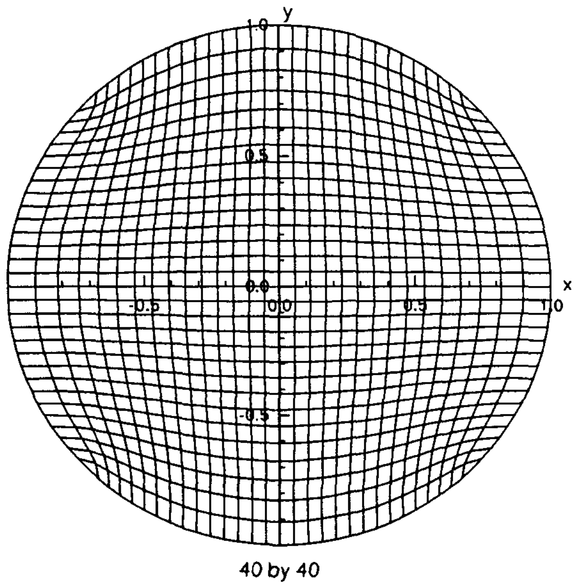

Fig. 2 Exemplar H-type grid net on (x,y) plane.

What we want to obtain is a geometric condition for a rotating disk for enhanced mass transfer in a circular cylinder. For this purpose, the disk oblique angle is selected for a major parameter. For the various angle α, magnitude of the meridional circulation is procured. As we predicted, for moderate angle α, greater than 0, the axisymmetric behaviour is invalid and non-axisymmetric circulations is observed on both (x,y) plane and the meridional plane. The plots of velocity vector on each plane will be shown in the present results. Also flowing fluid particle path will be drawn in the flow field of the container. These results will summarized for development of rotating machinery in a company.

2. The model

Numerical integration for both physical domains which are two kinds of containers ; one is a right circular cylinder with a flat disk and the other with an oblique disk (hereinafter referred to as sliced cylinder) is performed on a generalized body fitted coordinate system. In the case of α=0 (right circular cylinder), the flow system poses axisymmetrical three dimension and for α=5 degree, the flow configuration is completely three dimensional. However the same grid system which has 42 by 42 by 42 points in (x,y,z) direction, respectively is used for the both cases.

The governing equations are momentum equations and continuity equation. The equations in the (x,y,z) coordinate system are transformed into (ξ, η, ζ) coordinate system in a computational domain. Equation (1) shows continuity equation and eqns. (2) denote momentum equations, respectively.

$$\frac{1}{J}\left[\frac{\partial}{\partial \xi}(U)+\frac{\partial}{\partial \eta}(V)+\frac{\partial}{\partial \zeta}(W)\right]=0. \tag{1}$$

$$\begin{aligned}\frac{1}{J}\left[\frac{\partial}{\partial \xi}(U\phi)\right]+\frac{\partial}{\partial \eta}(V\phi)+\frac{\partial}{\partial \zeta}(W\phi)]\\ =\frac{1}{J}\left\{\frac{\partial}{\partial \xi}\left[\Gamma\left(G^{11}\frac{\partial \phi}{\partial \xi}+G^{12}\frac{\partial \phi}{\partial \eta}+G^{13}\frac{\partial \phi}{\partial \zeta}\right)\right]\right\}\\ +\frac{1}{J}\left\{\frac{\partial}{\partial \eta}\left[\Gamma\left(G^{21}\frac{\partial \phi}{\partial \xi}+G^{22}\frac{\partial \phi}{\partial \eta}+G^{23}\partial\frac{\phi}{\partial \zeta}\right)\right]\right\}\\ +\frac{1}{J}\left\{\frac{\partial}{\partial \zeta}\left[\Gamma\left(G^{31}\frac{\partial \phi}{\partial \xi}+G^{32}\frac{\partial \phi}{\partial \eta}+G^{33}\frac{\partial \phi}{\partial \zeta}\right)\right]\right\}+S_{\phi}(\xi,\eta,\zeta).\end{aligned} \tag{2}$$

Velocity components (U,V,W) correspond to (ξ, η, ζ), respectively. The above equations include source term, S_ϕ, and transformation functions which are defined as followings.

$$J=x_\xi y_\eta z_\zeta+x_\zeta y_\xi z_\eta+x_\eta y_\zeta z_\xi-x_\xi y_\zeta z_\eta-x_\zeta y_\eta z_\xi-x_\eta y_\xi z_\zeta, \tag{3}$$

$$\xi_x=\frac{y_\eta z_\zeta-y_\zeta z_\eta}{J},\ \xi_y=\frac{x_\zeta z_\eta-x_\eta z_\zeta}{J},\ \xi_z=\frac{x_\eta y_\zeta-x_\zeta y_\eta}{J}, \tag{4a}$$

$$\eta_x=\frac{y_\zeta z_\xi-y_\xi z_\zeta}{J},\ \eta_y=\frac{x_\xi z_\zeta-x_\zeta z_\xi}{J},\ \eta_z=\frac{x_\zeta y_\xi-x_\xi y_\zeta}{J}, \tag{4b}$$

$$\zeta_x=\frac{y_\xi z_\eta-y_\zeta z_\xi}{J},\ \zeta_y=\frac{x_\eta z_\xi-x_\xi z_\eta}{J},\ \zeta_z=\frac{x_\xi y_\eta-x_\eta y_\xi}{J}. \tag{4c}$$

$$G^{11}=J(\xi_x^2+\xi_y^2+\xi_z^2), \tag{5a}$$

$$G^{22}=J(\eta_x^2+\eta_y^2+\eta_z^2), \tag{5b}$$

$$G^{33}=J(\zeta_x^2+\zeta_y^2+\zeta_z^2), \tag{5c}$$

$$G^{12}=G^{21}=J(\xi_x\eta_x+\xi_y\eta_y+\xi_z\eta_z), \tag{5d}$$

$$G^{23}=G^{32}=J(\eta_x\zeta_x+\eta_y\zeta_y+\eta_z\zeta_z), \tag{5e}$$

$$G^{31}=G^{13}=J(\zeta_x\xi_x+\zeta_y\xi_y+\zeta_z\xi_z). \tag{5f}$$

The present numerical scheme follows QUICK [7] algorithm and pressure is managed by pressure substitute method developed by Hobson [8]. Detail procedure for the numerical calculation is based on the work by Kang and Bae [9].

3. Results and discussion

The present main improvement for enhanced mixing effects is represented by an oblique disk in steady of a flat disk in a cylinder which is simple and characteristic model of a washing tub. Quantitative gauge for the difference between ordinary and enhanced mixing is accomplished by just comparing a magnitude of meridional velocities in a container with a flat bottom disk or an inclined disk.

Figure 3 shows vertical variation of the axial velocity in both cylinders with a flat disk and an inclined disk at 5 degree against the horizontal plane. Along a radial position at r=R/4, the vertical variations of the axial velocities depict in Fig. 3(a) and Fig. 3(b) for the line of R/2.

Each velocity profile is measured at node points along with the axial direction. As predicted, axial velocities in the sliced cylinder is no longer axisymmetric and changes its magnitude over the whole fluid region.

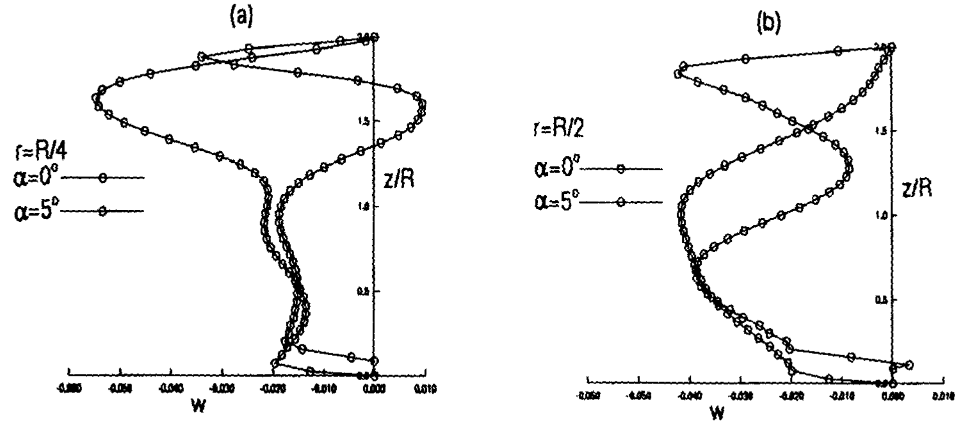

Fig. 3 Axial variation of W for an ordinary and a sliced cylinder along the prefixed radial positions ; (a) r=R/4, (b) r=R/2.

Typical plots of meridional circulations in (x-z) plane are displayed in Figure 4. For validation of the present numerical work, stream lines contours are shown in Fig. 4(a) which is in good agreements with the results of Escudier's [10]. Vortex breakdown shaped a bubble is observed in Fig. 4(a). However, in case of a sliced cylinder, the meridional contours is asymmetric and the center of main cells is shifted from the rotational axis. These phenomena explains that there are radial flows across the central axis of the cylinder. Maybe these flows are what washer manufactures want to find.

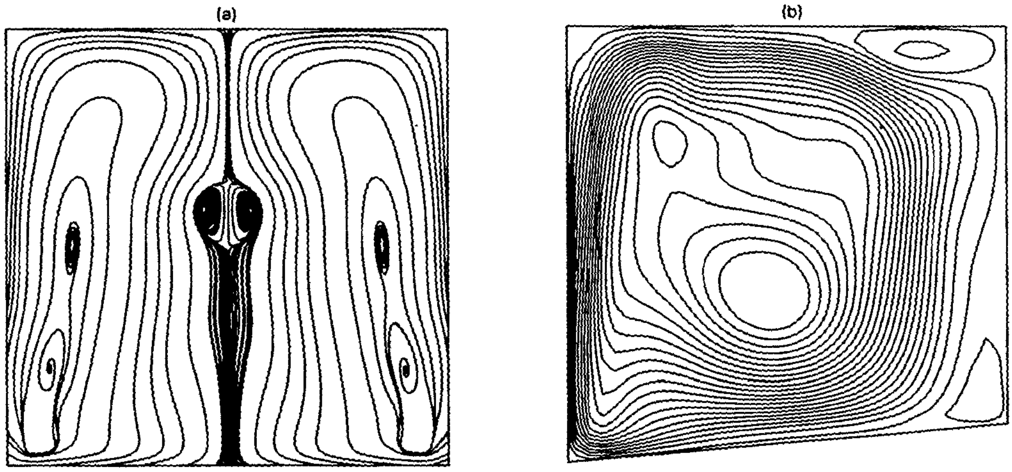

Fig. 4 Meridional flows for a right circular cylinder (a) and a present model (b).

4. Conclusions

A novel idea for pronounced mixing of a container with a rotating bottom disk is proposed and numerically verified in this investigation. The suggestion is to make the bottom disk incline against the top disk of the cylindrical container. This model can practically be applied to a washer with a rotating pulsator. It is obviously confirmed that there are stronger meridional flows in the cut-cylinder than those in the right circular cylinder. The center point of the circulation axially moves according to vertical position of container. Also the magnitude of meridional velocities is increase relative to the case of ordinary container with a flat disk. These pronounced enhancements for mixing of internal flows is just accomplished by introducing an oblique disk to the previous circular tub. The present results means that it is easily possible to vitalize angular momentum transfer in a container by tilting a rotating bottom disk. Consequently, the present results supply new method for home appliance companies. The angle pulling a rotating disk to one side is fixed as 5 degree in this work. However, a continuous investigation will be done for various angles to give optimal value for washing process.

References

[1] Larrousse, M. F., "Transport phenomena during spin-up/spin-down in the Bridgman-Stokbarger technique," Ph. D dissertation, Clarkson university, New York (1987).

[2] Fletcher, C. A. J., *Computational techniques for fluid dynamics 2*, Springer-Verlag, Sydney (1991), p. 101.

[3] Issa, R., I., "Solutions of the implicit discretized fluid flow equations by operator splitting," *Mechanical Eng. Rept.*, 25 (1982).

[4]Lee, D. and Chiu, J. J., "Covariant velocity based calculation procedure with nonstaggered for computation of pulsative flows," *Numerical heat transfer*, 21 (1992), p. 269.

[5] Page, M. A., "A numerical study of detached shear layer in a rotating sliced cylinder," *Geophys. Astrophys. Fluid Dynamics*, 22 (1982), p. 51.

[6] Hide, R. and Hocking, L. M., "On detached shear layers and western boundary currents in a homogeneous liquid," *Geophys. Astrophys. Fluid Dynamics*, 14 (1979), p. 19.

[7] Leonard, B. P., " A stable and accurate convective modeling procedure based on quadratic upstream interpolation," *Comput. Methods Appl. Mech. Eng.*, 19 (1979), p. 59.

[8] Hobson, G. V., and Lakshiminarayana, B., "Prediction of cascade performance using an incompressible Navier-stokes technique," *J. Turbomachinery*, 113 (1991), p. 561.

[9] Kang, D. J., and Bae S. S., "Calculation of the incompressible Navier-stokes equations in a generalized nonorthogonal body fitted coordinate system," *KSME J.*, 20-3 (1996), p. 1015.

[10] Escudier, M. P., " Observations of the flow produced in a cylindrical container by a rotating endwall," *Exp. Fluids*, 2 (1984), p. 189.