고유치 솔버 Davidson Method 의 병렬화

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A Parallel Algorithm of Davidson Method for Eigenproblems

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Abstract - The analysis of eigenvalue and eigenvector is a crucial procedure for many electromagnetic computation problems. However, eigenpair computation is timing-consuming task. Thus, its parallelization is required for designing large-scale and precision three-dimensional electromagnetic machines. In this paper, the Davidson method is parallelized on a cluster of workstations. Performance of the parallelization scheme is reported. This scheme is applied to a ridged waveguide design problem.

1. Introduction

1.1 Waveguide Design Problem: Many applications require the solution of the eigenvalue problem

$$A\,x = \lambda\,x, \qquad (1)$$

where x is an eigenvalue of the matrix A, and x is its corresponding eigenvector. In electromagnetics, we can find many eigenpair computation problems resulting from the dispersion analysis of waveguides or resonance analysis of cavities [4, 12]. One of our target applications is the ridged waveguide design. Ridged waveguides have many applications in microwave and antenna systems because of their unique characteristics of low cut-off frequency, wide bandwidth and low impedance compatible with coaxial cables [12]. The original ridged waveguide design problem produces a generalized eigenvalue problem of the form

$$B\,x = \lambda\,C\,x. \qquad (2)$$

However, (2) can be transformed into (1). The original matrix B is sparse, but A becomes dense. In addition, A is real and symmetric in this paper.

1.2 Eigensystem Solver: There are several iterative methods dealing with the eigenpair computation problems: the Jacobi's diagonalization method [6, 7], the power method [7], the method of Lanczos [6, 8], the Arnoldi's method [9, 10], and the Davidson method [11, 10, 1, 2]. The latter method has been reported to be quite successful [1, 2]. Davidson method can be regarded as a preconditioned version of the Lanczos method. It seems to be a promising method for eigenvalue problem in many applications including electromagnetic fields computation. The Davidson's algorithm that computes the largest or the smallest eigenvalue of the matrix A can be prescribed as follows [1, 2]:

1.3 Hardware Configuration: In our experiment, three HP workstations, the C160s, connected by 10Mbps standard Ethernet, are employed to parallelize the Davidson method. However, our parallelization scheme can be applied to any bus-based multidrop configuration.

1.4 PVM: The program is developed upon PVM, Parallel Virtual Machine [5]. The PVM is a software that permits a network of heterogeneous Unix computers

to be used as a single large parallel computer. The PVM is freely available, well designed, and not restricted to any specified type of machine so it is now widely used and becomes almost a standard for message-passing system developing tool like MPI (Message Passing Interface).

1.5 Parallelization Strategy: Two typical programming models in distributed memory system are SPMD (Single Program, Multiple Data) and Master/Slave [5]. Even though the Davidson method is an eigensystem solver, it computes eigenpairs indirectly from a very small $k \times k$ matrixrather than directly from the large-scale matrix A. The size of the matrix increases as the iteration number k increases. Thus, performance of the Davidson method may depend on the number of iterations. In general, the eigensolver itself is difficult to parallelize. In addition, the matrixis not worth to be parallelized as long as k is small. Thus, it is better to employ the Master/Slave model in which the master solves this small eigenvalue problem while the salves take over the tasks of computing the most time consuming step: the matrix-vector multiplication and orthogonalization [2]. Our main idea is that every slave holds a part of the matrix A to execute the matrix-vector multiplication and the orthogonalization in parallel which are the bottleneck of the Davidson's algorithm. The details of our algorithm is omitted due to the length of the text.

2. Experimental Results

In our experiments, we first apply our algorithm to a real world eigenvalue problem derived from a ridged waveguides design (we denote it *stif* below) [12].

In addition, in order to observe the advantages and constraints of our parallelization scheme, we also select the artificial sparse test matrix set as in [1] given below:

$$a_{ij} = \begin{cases} \text{if } i=j \text{ normally distribued} : N(0, 5^2) \\ \text{if } i \neq j \begin{cases} \text{with probability } \alpha : N(0,1), \\ \text{with probability } (1-\alpha) : 0. \end{cases} \end{cases} \quad (3)$$

Computation stops when

$$| \mu_k - \mu_{k-1} | < 10^{-11},$$

where μ_k is the convergent eigenvalue at t he kth iteration. The parameter is approxi mately the density of the matrix which me ans the ratio $\alpha = (number\ of\ non\text{-}zero\ eleme nts)/(n \times n)$.

Table I:Description of the Testing Matrices

Name	Matrix Resource	Dimesion	# of Iteration	Density	Maximum Eigenvalue
stif	See[12]	677	39	1.0000	7.6411e+7
test1	Eq.(3)	8000	27	0.0002	19.64068
test2	Eq.(3)	8000	66	0.0010	23.15136
test3	Eq.(3)	8000	77	0.0200	28.64236
test4	Eq.(3)	12000	31	0.0002	19.82698
test5	Eq.(3)	20000	23	0.0002	23.91715

Table II: Timing Data of the Testing Matrices Above

# of Workstations	stif	test1	test2	test3	test4	test5
1	18.689	41.394	117.57	227.41	26.330	40.997
2	9.4964	26.850	61.400	114.92	114.92	14.754
3	6.4958	20.718	68.339	106.03	106.03	13.763

Table III: Speed-up Performance of Our Parallelization Scheme

# of Workstations	stif	test1	test2	test3	test4	test5
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.9680	1.5416	1.9149	1.9788	1.7843	1.7513
3	2.8771	1.9980	1.7205	2.1447	1.9128	1.2903

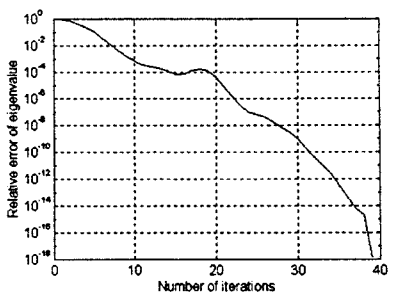

Figure 1. The convergence profile of matrix *stif*

3. Conclusion

From all the above results of the experiments we can see our algorithm is successful on the real-world matrix, *stif*. Its convergence profile is shown in Figure 1. The convergence speed is very fast, supporting the claim that Davidson method provides a second order convergence near the solution [2]. This is one important reason that we recommend using the Davidson method.

When A is sparse, the speed-up is more than 1.5 with two computers. However, the speed-up may not be satisfactory for some sparse problems when three computers are engaged. From the speed-up results of *test1*, *test2* and *test3* in Table III we find that the speed-up increases radically as the number of non-zero elements increases. Results of *test1* and *test4* show that when the dimension of the problem increases, speed-up increases obviously. However, *test5* shows that speed-up with three computers can be lower than with two machines. So it is recommended that when using our algorithm in the computing environment described above with more than three computers may not produce satisfactory result especially with sparse matrices. However, dense matrices produce attractive speed-up. Obviously, the boundary element methods produce dense matrices.

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