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Model-Based Sliding Mode Tracking Control of 6-6 Stewart Platform Manipulator

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Abstract A high speed tracking control for 6-6 Stewart platform manipulator is performed by employing the joint-axis sliding mode control based on dynamics. Because of the complex dynamics and kinematics of Stewart platform manipulator, two computer systems, consisting of a PC and a DSP, are adopted, so that real time tasks are run in synchronous and asynchronous modes. It is experientially proven that the proposed control system leads to an easy to implement and effective control task, and it can achieve the high performance tracking control under the high speed and severe payload condition.

Keywords Stewart platform manipulator, Sliding mode control, Model based control, Forward kinematics, Dual processors

1. INTRODUCTION

In recent years, there has been considerable interest in the area of parallel manipulator, which provides better accuracy, rigidity, load-to-weight ratio, and load distribution than serial manipulator. Such advantages of the fully parallel manipulator[1], which is known as Stewart platform manipulator(SPM), originate from the fact that the actuators act in parallel sharing the common payload. Although the real time calculation of the dynamics and forward kinematics may be difficult task owing to the time consuming nature[2], the high speed motion control of a manipulating system especially needs accurate knowledge of the kinematics and dynamics of the physical plant at every time. This problem has blocked the development of a practical control algorithm capable of real-time trajectory generation, a necessity for application of the SPM.

In this study, the time consuming dynamic problem is resolved introducing the dual-processor based computing architecture to share the complex control tasks synchronously and asynchronously, so that the feedback loop time is reduced and high speed motion control can be realized. Perturbations, which include external disturbances, unpredictable parameter variations and unmodeled plant dynamics, may also be induced in modeling of manipulator dynamics, simplification of actuator model and asynchronous calculation of dynamic properties. To reject the perturbations for high performance tracking control of the SPM, we employ a model based joint-axis sliding mode control(SMC)

2. MODELING OF STEWART PLATFORM MANIPULATOR

The inverse kinematic solution for the 6-6 SPM can be easily derived in a closed form unlike the forward kinematic solution. The conventional forward kinematics approach to get the displacements of upper centroid of the motion base from actuator lengths uses the numerical analysis such as the iterative Newton-Raphson(NR) method[3]. Although the approach is relatively simple and easy to realize, it may be generally hard to complete the task in a given sampling time because the iterative numerical method needs too much time to get the accurate solution, and the calculation time varies depending on its condition. The iterative NR method, however, uses the last calculated data for the displacement of upper centroid so that the increments in moving distance of each actuators may be very small when the time interval of measurement is sufficiently small. In addition, a reasonable allowance of numerical errors may be permitted to reduce the iterations. Then, the iterative NR method becomes very powerful, guaranteeing the convergence of order of two near solutions.

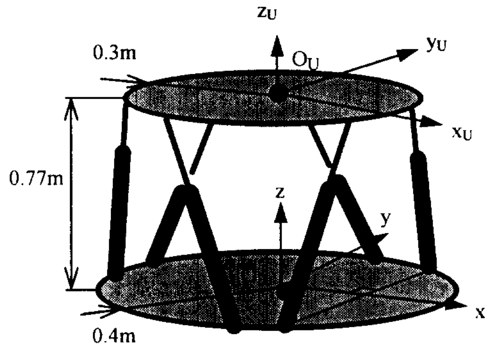

Fig. 1 Typical Stewart platform manipulator and the coordinates

The iterative NR formula can be defined as

$$\mathbf{X}_{n+1} = \mathbf{X}_n - \left[\Psi'(\mathbf{X}_n)\right]^{-1}\Psi(\mathbf{X}_n) \tag{1}$$

where $\Psi(\mathbf{X}_n) = \boldsymbol{l}^2(\mathbf{X}_n) - \boldsymbol{l}_a^2$, and $\Psi'(\mathbf{X}_n) = \dfrac{\partial\Psi(\mathbf{X}_n)}{\partial\mathbf{X}_n}$;

$\mathbf{X} = [x \ \ y \ \ z \ \ \alpha \ \ \beta \ \ \gamma]^T$ is the coordinates vector of the upper centroid, O_u, as shown in Fig. 1; α, β and γ are the rotational angles about the x, y and z axes, respectively; $\boldsymbol{l} = [l_1 \ \ l_2 \ \cdots \ l_6]^T$ and $\boldsymbol{l}_a = [l_{a,1} \ \ l_{a,2} \ \cdots \ l_{a,6}]^T$ are the calculated and measured actuator length vectors, respectively; the subscript n is the iteration number. The $\Psi'(\mathbf{X})$ is obtained as

$$\Psi'(\mathbf{X}) = 2\mathrm{diag}(\boldsymbol{l})\mathbf{J} \tag{2}$$

where $\mathbf{J}$ and diag($\bullet$) are the Jacobian matrix of the SPM and a diagonal matrix, respectively. The $[\Psi'(\mathbf{X})]^{-1}$ should not be near singular or singular for safe convergence to the solution in the operation space. Since the actuator length can not be zero, singularity of $[\Psi'(\mathbf{X})]^{-1}$ means the singularity of the $[\mathbf{J}]^{-1}$. Thus the formulation singularity of this method depends on the architecture singularity of the SPM.

The dynamic equation of the SPM considering all inertia effect is known to be very difficult to derive, if not impossible. Zhang and Song[4] modified the SPM to get a virtual multiple open loop mechanism, then derived the dynamic equation using the Newton-

Euler method and virtual work principle, and obtained the forces, u_p, of the actuated joints as

$$u_{p,j}(t) = H_j\big(X(t), \dot{X}(t), \ddot{X}(t)\big), \quad j = 1,2,\cdots,6 \tag{3}$$

Although this formula is computationally efficient, it can not be directly used for derivation of control law. After some algebraic operation, equation (3) can be re-expressed as

$$u_{p,j}(t) = \sum_{k=1}^{6}\left\{m_{p,jk}\big(X(t)\big)\ddot{l}_k(t)\right\} + V_{p,j}\big(X(t), \dot{X}(t)\big) + G_j\big(X(t)\big), \quad j = 1,2,\cdots,6 \tag{4}$$

which can be used for design of the joint-axis controller. Here $\left[m_{p,jk}\right] \in R^{6\times6}$ is the inertia mass matrix associated with the acceleration of actuator length coordinates, $\left[V_{p,j}\right] \in R^{6\times1}$ corresponds to the centrifugal and Coriolis forces vector, and $\left[G_j\right] \in R^{6\times1}$ is the gravity force vector.

A linear actuator system consisting of an AC servo motor and a linear ball screw system is employed in this study. The AC servo motor(Max. power: 200W) equipped with a 3000 pulse encoder generates torque and rotational motion, and the linear ball screw system(lead: 25mm) converts them into the linear force(Max. rated force:240N; Peak force:720N) and linear motion. The motor core, ball nut and coupling, which are the spinning units relative to the fixed actuator body, may result in a complex nonlinear dynamic equation. Assuming that such a complex nonlinear part acts as a perturbation to the modeled dynamics, we can obtain the simple linear dynamics as[6]

$$u_{a,j} = m_{a,j}\,\ddot{l}_j + c_j\,\dot{l}_j, \quad j = 1,2,\cdots,6 \tag{5}$$

where $m_{a,j}$ is the summation of equivalent inertia masses of all the rotating parts in the actuator system, c_j is the friction damping caused by the relative motion of those inertia masses.

The electromechanical interaction between the motor and drive is neglected in this study since the control bandwidth of the system is far wider than the operating range.

The complete nominal dynamic equation of the SPM system possessing manipulator and actuator dynamics becomes

$$u_j(t) = \sum_{k=1}^{6}\left\{m_{jk}\big(X(t)\big)\ddot{l}_k\right\} + V_j\big(X(t), \dot{X}(t), \dot{l}_j(t)\big) + G_j\big(X(t)\big), \quad j = 1,2,\cdots,6 \tag{6}$$

where $\left[u_j\right] = \left[u_{p,j}\right] + \left[u_{a,j}\right] \in R^{6\times1}$ is the control force vector, $\left[m_{jk}\right] = \left[m_{p,jk}\right] + \left[m_{a,j}\right] \in R^{6\times6}$ represents the inertia mass matrix of the SPM system, which is symmetric and nonsingular, $\left[V_j\right] = \left[V_{p,j}\right] + \left[c_{a,j}\dot{l}_j\right] \in R^{6\times1}$ correspond to the centrifugal and Coriolis force vector of the SPM system, and the friction force of the actuator system.

3. CONTROL SYSTEM

3.1 Control hardware and software

A model based control strategy generally needs calculation of system dynamics at every given sampling time. However, the SPM dynamics including numerical forward kinematics is too complex for general computing system to calculate within a sampling time interval, which should be small in value for a high speed motion control.

Thus we employed two processors for digital servo control of SPM, consisting of a personal computer(PC) and a digital signal processor(DSP) interfaced to the PC bus. As shown in Fig. 2, two computer systems communicate each other through a dual port RAM installed in the DSP board so that a high speed digital data transfer is achieved. The DSP system is equipped with ADCs, DACs and counters for communication with real world signal. All the hardware used in this control system is listed in Table 1.

TABLE 1 Specification of the control system

Parts	Characteristics	Parts	Characteristics
PC	Intel Pentium 133MHz	DSP	TMS320C40 50MHz
ADCs	16 ch., 12bits, 48kSPS	DACs	8ch., 12bits
Counter	6ch., 24bits, Programmable	Timer	18bits

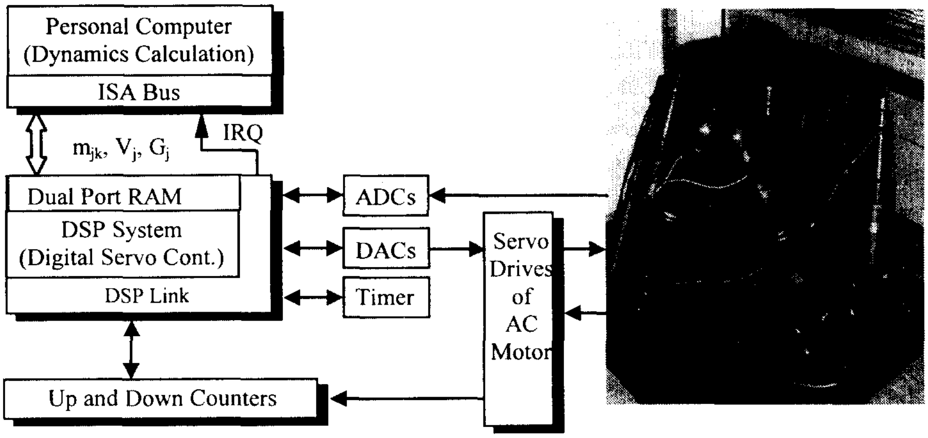

Fig. 2 Control block diagram of SPM system

The tasks required for the dynamic based servo control of the SPM are grouped into three. The first real-time task, the digital servo control except the dynamic calculation, is executed in a sampling time interval by the DSP. The second real-time task, the command generation and data transfer between two processors, is executed by the PC which takes the command signal from user or PC interfaced peripherals. The forward kinematics and dynamic equation are calculated on the PC asynchronously to the digital servo control.

3.2 Design of the joint-axis SMC

The dynamic system (6) can be re-written, in the presence of perturbation, as

$$u_j(t) = \hat{m}_{jj}\big(X(t)\big)\left\{\ddot{l}_j(t) + p_{E,j}(t)\right\} + n_j(t) + \hat{V}_j\big(X(t), \dot{X}(t)\big) + \hat{G}_j\big(X(t)\big), \quad j = 1,2,\cdots,6 \tag{7}$$

where

$$P_{E,j} = \frac{1}{m_{jj}} \left\{ \sum_{k=1}^{6} \left(\Delta m_{jk} \ddot{l}_k\right) + \Delta V_j + \Delta G_j + d_j \right\}$$

$$n_j = \sum_{\substack{k=1 \\ j \neq k}}^{6} \left(\hat{m}_{jk} \ddot{l}_k\right)$$

Here '^' represents the nominal value, Δm_{jk}, ΔV_j and ΔG_j are the uncertainties of $\hat{m}_{jk}$, $\hat{V}_j$ and $\hat{G}_j$, respectively, and d_j denotes the external disturbance.

The sliding function s is defined by[5]

$$s_j = \dot{e}_j + \lambda_j e_j \quad j = 1, 2, \cdots, 6 \tag{8}$$

where $e_j = l_j - l_{d,j}$, the positive constant λ_j is the desired control bandwidth, and, l_j and $l_{d,j}$ are the measured and desired actuator lengths. Let the time derivative of the Lyapunov function candidate be given by $\frac{1}{2}\frac{d(s_j^2)}{dt} < 0$ to satisfy the boundary layer attraction condition, which gives the conventional SMC law defined as

$$u_j = \hat{m}_{jj}\left[\ddot{l}_{d,j} - \lambda_j \dot{e}_j - k_j \mathrm{sat}(s_j, s_{o,j})\right] + \hat{n}_j + \hat{V}_j + \hat{G}_j \tag{9}$$

where

$$k_j = max\left| \sum_{\substack{k=1 \\ j \neq k}}^{6} \left\{ \frac{\hat{m}_{jk}}{\hat{m}_{jj}} \left(\ddot{l}_k - \ddot{l}_{d,k}\right) \right\} + P_{E,j} \right|$$

$$\hat{n}_j = \sum_{\substack{k=1 \\ j \neq k}}^{6} \left(\hat{m}_{jk} \ddot{l}_{d,k}\right), \quad j = 1, 2, \cdots, 6$$

$$sat(s, s_o) = sgn(s) \quad \text{for} \quad |s| \geq s_o$$
$$= s/s_o \quad \text{for} \quad |s| < s_o$$

Here the positive constant s_o is the thickness of boundary layer[5].

4. EXPERIMENT

A simple tracking control is performed in this study to check the control system and tracking performance. The command trajectories chosen for tracking are given by

$$\begin{cases} x(t) = 0.03\cos(0.54\pi t), \quad m \\ y(t) = 0.03\sin(0.56\pi t), \quad m \\ z(t) = \dfrac{0.02}{1+2t}\sin\left(2\pi t\left(\dfrac{0.1+11t}{12.5}\right)\right), \quad m & ;2 \leq t \leq 12 \\ \alpha(t) = 0 \\ \beta(t) = 4\sin(0.86\pi t), \quad deg \\ \gamma(t) = 5\sin(0.74\pi t), \quad deg \end{cases} \tag{10}$$

$$x(t) = y(t) = z(t) = \alpha(t) = \beta(t) = \gamma(t) = 0 \quad ;\text{otherwise}$$

The tuned control parameter values throughout the test are: $\lambda_j = 60$ rad/sec, $s_{o,j} = 0.05$ m/sec and $k_j = 1.5$ N/kg for j=1,2, ..., 6. The payload of the SPM is 74 kg.

Because most of time consuming routines are executed by the PC asynchronously, all the routines executed by the DSP can be carried out within the sampling time interval of 1m*sec*. The asynchronous task executed by the PC required about 2-3m*sec* for finishing its all routines. Figure 3 shows the percentage uncertainties induced by asynchronous calculation of dynamic properties in PC, which was very small in value compared to the nominal values. Figures 4-6 show the tracking errors in the operating coordinates, a typical sliding function associated with tracking error in an actuator direction coordinate and a typical actuator control force, respectively. Note that the SMC did not induce chattering in the control input, and kept the sliding functions inside the boundary layer throughout the control experiment.

5. SUMMARY AND CONCLUSIONS

The forward displacement analysis necessitated use of the iterative NR method because it is very fast in convergence and robust to the system environments without introducing formulation singularity. Since the dynamic equation of the SPM, including the inertia effects, requires a time consuming calculation, dual processing computer system to share the time consuming tasks was proposed for dynamic based real-time tracking control of the general SPM. The uncertainties resulted from asynchronous execution of tasks by the two processors and the model uncertainties were treated as perturbations to the nominal dynamics. It is shown experimentally that the robust joint-axis SMC to the perturbations worked well for the SPM under high frequency motion and large payload conditions.

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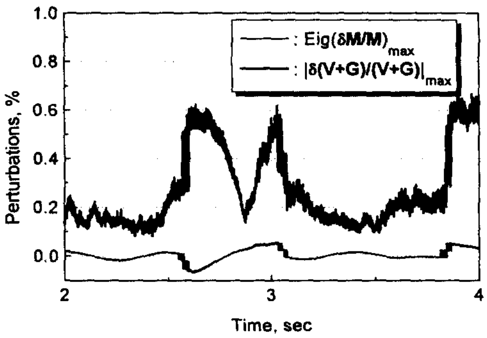

Fig. 3 The perturbations caused by time delayed data

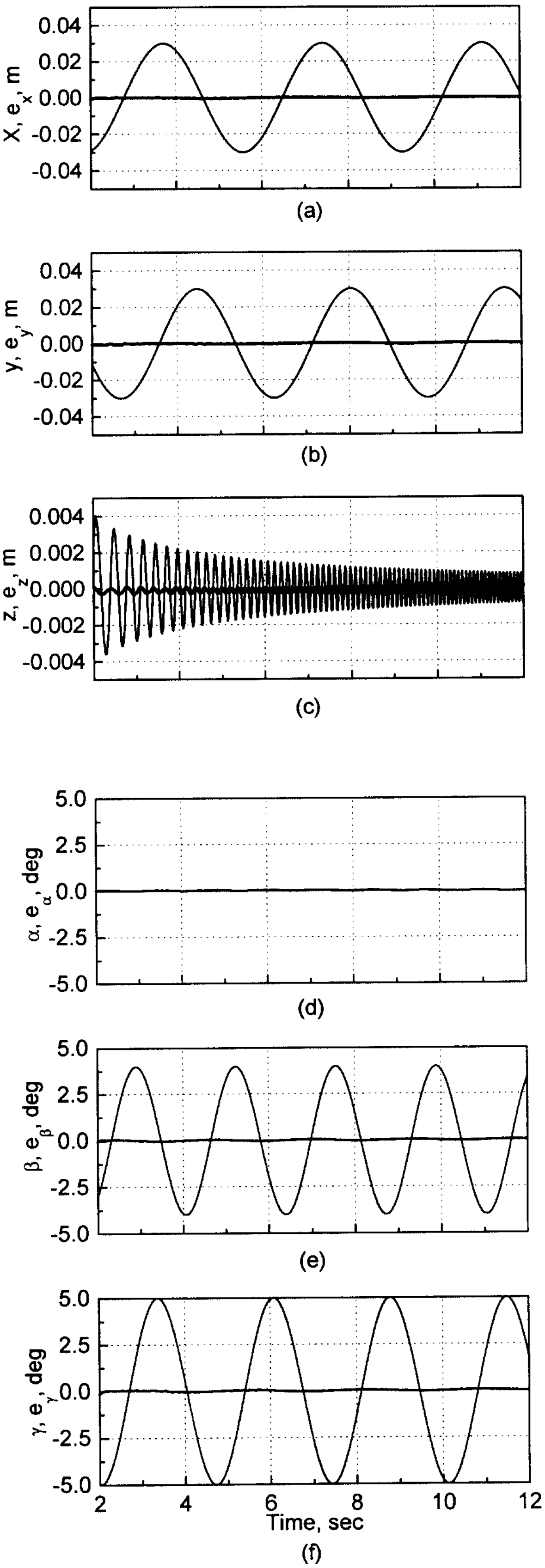

Fig. 4 Command displacements and tracking errors :
—— Command, ▬▬ Error

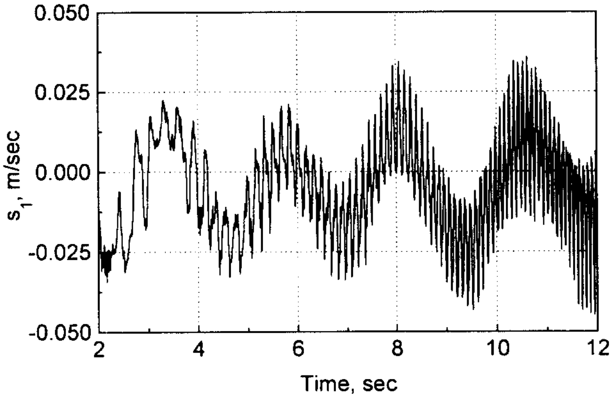

Fig. 5 Typical sliding function

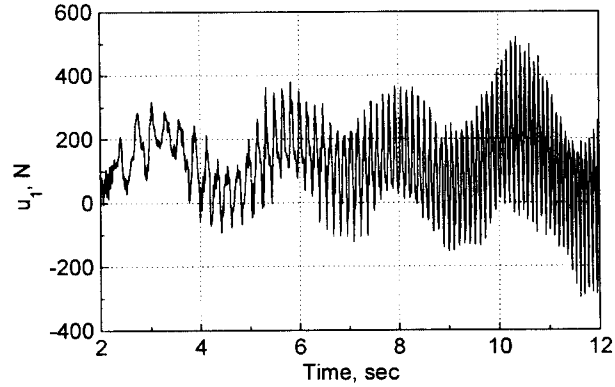

Fig. 6 Typical control force