

3차원 균열의 응력확대계수에 대한 해석의 자동화

Automation of Analysis for Stress Intensity Factor of 3-D Cracks

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ABSTRACT

This paper describes an automated system for analyzing the stress intensity factors(SIFs) of three-dimensional (3D) cracks. A geometry model, i.e. a solid containing one or several 3D cracks is defined. Several distributions of local node density are chosen, and then automatically superposed on one another over the geometry model by using the fuzzy knowledge processing. Nodes are generated by the bucketing method, and ten-noded quadratic tetrahedral solid elements are generated by the Delaunay triangulation techniques. The singular elements such that the mid-point nodes near crack front are shifted at the quarter-points are automatically placed along the 3D crack front. The complete finite element (FE) model generated, i.e. the mesh with material properties and boundary conditions is given to one of the commercial FE codes, and a stress analysis is performed. The SIFs are calculated using the displacement extrapolation method. To demonstrate practical performance of the present system, a semi-elliptical surface crack in a plate subjected to uniform tension is solved.

Key Words : Stress Intensity Factor(응력확대계수), Surface Crack(표면균열), Automatic Mesh Generation(자동요소분할), Fuzzy Theory(퍼지이론), Bucketing Method(버킷법), Delaunay Triangulation(데로우니삼각형), Singular Element(특이요소)

1. Introduction

3D cracks such as surface or embedded cracks are more common flaws being found in practical structures. Analyses of the 3D cracks are desirable in structural integrity studies of practical structures. The SIFs for an elliptical or a semi-elliptical crack have been obtained by the finite element method (FEM)[1-5]. However, there are still some problems to be solved. The main concern for the FEM is a relatively higher computation cost, especially when dealing with 3D crack problems. It should be also noted here that the data preparation for 3D crack analyses require special element arrangement near the crack front, and that much efforts are necessary to generate such special meshes. Dramatic progress in computer technology now shortens computation time. However in reality, labour intensive tasks to prepare a FE model of a structural component with 3D cracks are still a bottle neck. The author has proposed an automatic FE mesh generation method for 3D structures consisting free-

form surfaces[6]. In the present study, by integrating this mesh generator, one of commercial FE analysis codes and some additional techniques to calculate the SIF, a new automated system for analyzing the SIFs of 3D cracks was developed.

To examine accuracy and efficiency of the present system, the SIF for a semi-elliptical surface crack in a plate subjected to uniform tension is calculated and compared with Raju-Newman's solutions[5,7].

2. Outline of the System

2.1 Definition of Geometry Model

A whole analysis domain is defined using one of commercial geometry modelers, Designbase[8] which has abundant libraries which enable us to easily operate, modify and apply to a solid model. Any information related to a geometry model can be easily retrieved by the libraries of Designbase

2.2 Attachment of Material Properties and Boundary Conditions to Geometry Model

Material properties and boundary conditions are directly attached onto the geometry model by clicking the loops or edges that are parts of the geometric model using a mouse, and then by inputting values. The present system deals with displacement as well as force boundary conditions.

2.3 Designation of Node Density Distributions

A node density distribution over a whole geometry model is constructed as follows. The present system stores several local node patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite and whole domain uniformly. A user selects some of those local node patterns and designates where to locate them. Then a global distribution of node density over the whole analysis domain is automatically calculated through their superposition using the fuzzy knowledge processing [9].

2.4 Node and Element Generation

Node generation is one of time consuming processes in automatic mesh generation. In the present study, the bucketing method[10] is adopted to generate nodes which satisfy the distribution of node density over a whole analysis domain.

The Delaunay triangulation method[11] is used to generate tetrahedral elements from numerous nodes produced within a geometry. When the Delaunay triangulation method is utilized to generate elements in a geometry with cracks, mis-match elements across surface crack front tend to occur as shown in Fig. 1(a). To avoid the mis-match elements, node densities on the crack front are automatically controlled to be slightly higher than those near the crack as shown in Fig. 1(b).

2.5 Automatic Attachment of Material Properties and Boundary Conditions to FE Mesh

Through the interactive operations mentioned in section 2.2, a user designates material properties and

boundary conditions onto the geometry model. Then these are automatically attached on nodes, edges, faces and volume of elements. Such automatic conversion can be performed owing to the special data structure of FEs such that each part of element knows which geometry part it belongs to. Finally, a complete FE model consisting of mesh, material properties and boundary conditions is obtained. The current version of the system produces FEs compatible to quadratic tetrahedral elements implemented in one of the commercial FE codes, MARC.

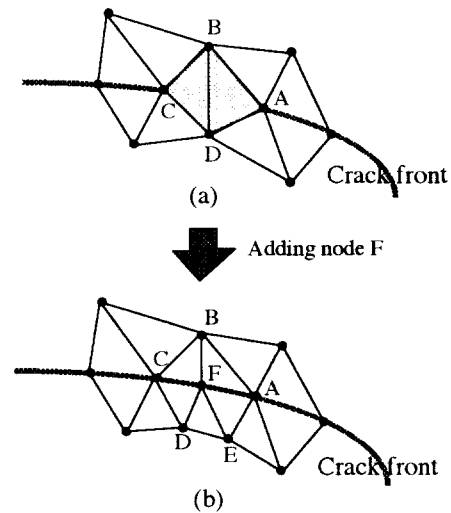


Fig. 1 Technique of avoiding mis-match elements

3. Calculation of SIF

3.1 Singular Element

When ordinary quadratic tetrahedral elements are employed to calculate the SIF, a very fine mesh is required near crack front to capture \sqrt{r} variation in displacements and $1/\sqrt{r}$ variation in stresses where r denotes the distance from crack front. To relax this situation, singular elements as shown in Fig. 2 are adopted[12]. In the singular element, the mid-point nodes near a crack front are shifted to the quarter-points. This conversion of ordinary tetrahedral elements along a front of 3D crack to the singular elements is automatically performed in the last stage of

the creation of a FE model.

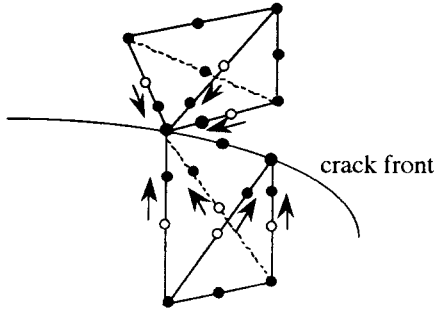


Fig. 2 Conversion of ordinary quadratic tetrahedral elements along crack front into singular elements

3.2 Calculation of SIFs

SIFs are computed using the displacement extrapolation method[2]. Nodal displacements calculated along the crack face are substituted in the following crack tip displacement equation :

$$K = \frac{E'}{4} \sqrt{2\pi} \lim_{r \rightarrow 0} \frac{w}{\sqrt{r}}, \quad E' = \begin{cases} \frac{E}{1-\nu^2} & \text{(for plane strain)} \\ E & \text{(for plane stress)} \end{cases} \quad (1)$$

where w is a nodal displacement, and E' is equal to E in the plane stress condition or $E/(1-\nu^2)$ in the plane strain condition. Only positions of free surface intersection are regarded as in the plane stress condition, while other positions are in the plane strain condition. Although there is no clear boundary between the plane strain region and the plane stress region, the point on the free surface, i.e. $\phi=0$, is in plane stress. Radial lines for the semi-elliptical surface crack front and those of the semi-circular one are defined as in Fig. 3.

For each radial line with an elliptic angle ϕ , the nodal displacements resulting from an FE calculation are inserted in the well-known equation (1) for the displacements near the crack tip. By this means, SIF values K for each radial line may be computed from the displacements of the nodes at distance r from the crack face. The first segment of the K curve where K de-

pends linearly on r is extrapolated to $r = 0$. The intersection with the K -axis yields the desired value for the SIF. The procedure is rather tedious and a distinct linear segment cannot be recognized in every load case. Therefore, the least square method is applied to evaluate the SIF. In this least square operation, the K value evaluated at the shifted quarter point is neglected. This displacement extrapolation method is popularly used to calculate the SIF. In the present study, this process is fully automated. When a crack is designated by a user in the definition process of a geometry model, radial lines for the crack front are automatically determined. After the stress analysis using MARC, displacement distributions are interpolated along the radial lines, on each of which the SIF is calculated by the least square method.

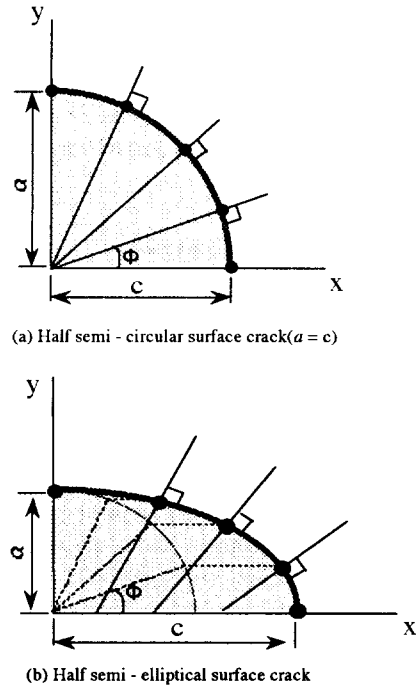


Fig. 3 Definition of radial lines to calculate the stress intensity factors

4. Result and Discussion

In order to examine efficiency and accuracy of the present system, a surface cracked plate of width $2b$, thickness t and height $2h$ subjected to uniform tension

as shown in Fig. 4 was analyzed. A semi-elliptical surface crack is assumed here.

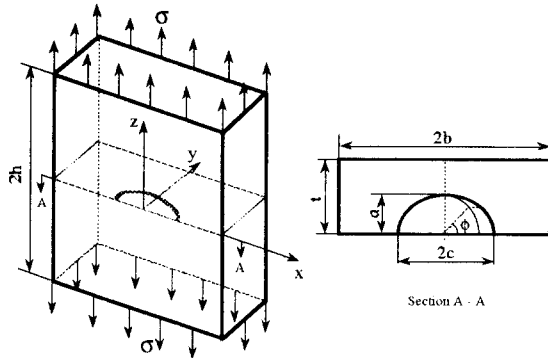


Fig. 4 A plate with a semi-elliptical surface crack

Fig. 5 shows a typical FE mesh of a quarter portion of a plate with a semi-elliptical surface crack generated by the present system. The mesh consists of 2,982 quadratic tetrahedral elements and 5,842 nodes. Nodes and elements are generated in about 90 seconds and in about 25 seconds, respectively. This is measured on a popular engineering workstation (EWS), SUN SparcStation 10. To complete this mesh, the following three node patterns are utilized ; (a) the base node pattern in which nodes are generated with uniform spacing over a whole analysis domain, (b) a special node pattern for the semi-elliptical surface crack, and (c) a special node pattern in which node density is getting coarser departing from the bottom face including the surface crack and the ligament section.

The analyses were performed for three aspect ratios of $a/c = 0.4, 0.6$ and 1.0 , and two crack depths of $a/t = 0.2$ and 0.4 . Young's modulus E and Poisson's ratio were assumed to be $205,800\text{MPa}$ and 0.3 , respectively. For example, Fig. 6 shows the comparison between the present solutions and Newman-Raju's solutions[5,7] for single crack configuration ($a/c=0.6, a/t=0.4$).

It can be seen from the figures that the present results using the singular elements agree well with Raju-Newman's solutions within 2 to 3% difference.

Fig. 7 shows the measured processing time of a whole process plotted against the total number of nodes. These are also measured on a popular engineering workstation, SUN SparcStation 10. Among a

whole process, the interactive operations to be done by a user, i.e. the definition of a geometry model, the designation of local node patterns and the assignment of material properties and boundary conditions are performed in about 2 minutes. The other processes are fully automatically performed.

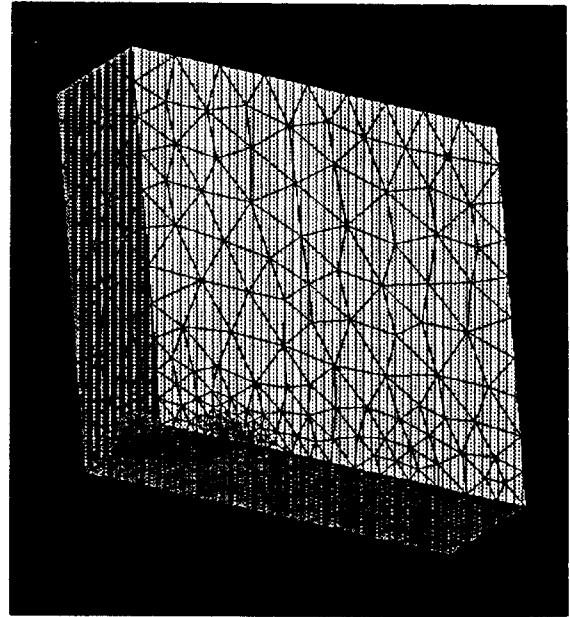


Fig. 5 A typical mesh of a quarter portion

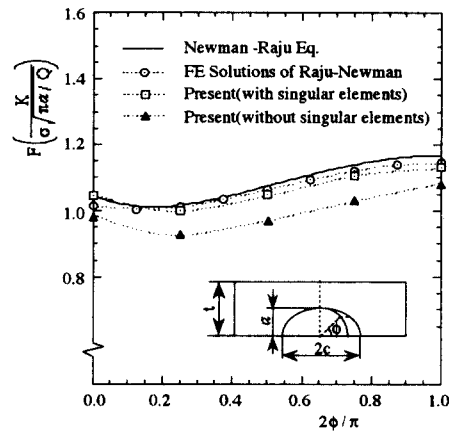


Fig. 6 Comparison of present SIF with Raju-Newman solutions

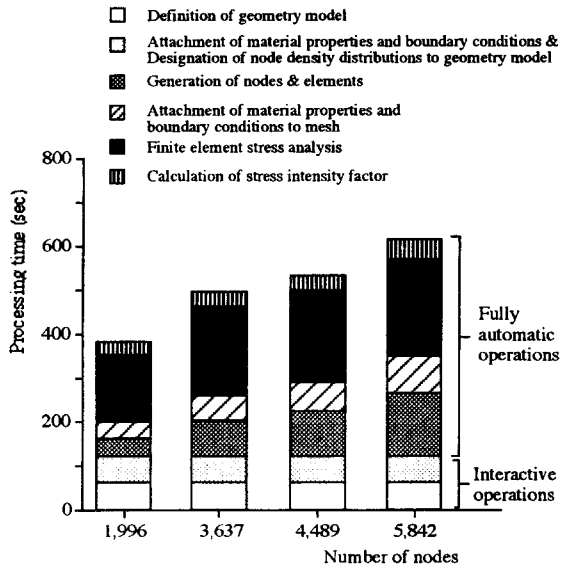


Fig. 7 Processing time vs. number of nodes

5. Conclusion

A new automated system for analyzing the SIFs of 3D cracks was developed in the present study. The automatic FE mesh generation technique based on the fuzzy knowledge processing and the computational geometry techniques were integrated in the system, together with one of commercial FE programs. Here interactive operations to be done by a user can be performed in a few minutes even for complicated problems. The other processes are fully automated being able to be performed in several minutes in a popular engineering workstation environment. It can be seen that the results using the present system agree well with Raju-Newman's solutions.

References

[1] Wilson, W. K. and Thompson, D. G., "On the Finite Element Method for Calculating Stress Intensity Factors for Cracked Plates in Bending", *Engineering Fracture Mechanics*, Vol. 3, pp. 97 - 102, 1971.
 [2] Tracey, D. M., "Finite Elements for Three-Dimensional Elastic Crack Analysis", *Nuclear Engineer-*

ing and Design, Vol. 26, ppl. 282-290, 1974.

[3] Chow, C. L. and Lau, K. J., "On the Finite Element Method for Calculating Stress Intensity Factors with a Modified Elliptical Model", *International Journal of Fracture*, Vol. 12, pp. 59 - 69, 1976.
 [4] Blackburn, W. S. and Hellen, T. K., "Calculation of Stress Intensity Factors in Three Dimensions by Finite Element Methods", *International Journal for Numerical Methods in Engineering*, Vol. 11, pp. 211 - 229, 1977.
 [5] Raju, I. S. and Newman, J. C., "Stress-Intensity Factors for a Wide Range of Semi-Elliptical Surface Cracks in Finite-Thickness Plates", *Engineering Fracture Mechanics*, Vol. 11, pp. 817 - 829, 1979.
 [6] Joon-Seong Lee et al., "Automatic Mesh Generation for Three-Dimensional Structures Consisting of Free-Form Surfaces", *The Korea Society of CAD/CAM Engineers*, Vol. 1, pp. 65-75, 1996.
 [7] Newman, J. C. and Raju, I. S., "An Empirical Stress-Intensity Factor Equation for the Surface Crack", *Engineering Fracture Mechanics*, Vol. 15, pp. 185-192, 1981.
 [8] Chiyokura, H., "Solid Modeling with Designbase : Theory and Implementation", Addison - Wesley, 1988.
 [9] Joon-Seong Lee, "Automated CAE System for Three-Dimensional Complex Geometry", The Doctoral Thesis, The University of Tokyo, 1995.
 [10] Asano, T., "Practical Use of Bucketing Techniques in Computational Geometry", *Computational Geometry*, North-Holland, pp. 153 - 195, 1985.
 [11] Watson, D. F., "Computing the n-Dimensional Delaunay Tessellation with Application to Voronoi Polytopes", *The Computer Journal*, Vol. 24, pp. 162 - 172, 1981.
 [12] Barsoum, R. S., "Application of Quadratic Isoparametric Finite Elements in Linear Fracture Mechanics", *International Journal of Fracture*, Vol. 10, pp. 603 - 605, 1974.