Proceedings of the Korean Nuclear Society Spring Meeting
Kwangju, Korea, May 1997

Stability Analysis of an Accelerator-Driven Fluid-Fueled Subcritical Reactor System

Do Sam Kim and Nam Zin Cho

Korea Advanced Institute of Science and Technology
Department of Nuclear Engineering
373-1 Kusong-dong, Yusong-gu
Taejon, Korea 305-701

Abstract

In this work, linear dynamics of a circulating fluid-fueled subcritical reactor system with temperature feedback and external neutron source was modeled and examined. In a circulating fluid-fuel system, the stable region is slightly moved by a circulation fluid effect. The effects of subcriticality and temperature feedback coefficient on the reactor stability were tested by calculating frequency response of neutron density originated from reactivity perturbation or external source oscillation of the system. The amplitude transfer function has a different shape near subcritical region due to the exponential term in the transfer function. The results of the study show that at a slightly subcritical region, low frequency oscillation in accelerator current or reactivity can be amplified depending on the temperature feedback.. However, as the subcriticality increases, the oscillation becomes negligible regardless of the magnitude of the temperature feedback coefficient.

I. Introduction

There has been a recent growth in interest in accelerator-driven subcritical systems. Such a system can be used for numerous purposes (energy production, accelerator transmutation of waste(ATW), and accelerator-based conversion(ABC)). In particular, ATW and ABC are accelerator-driven subcritical fluid-fueled systems. A salient dynamic characteristic of the system is that the neutron population (power) is very sensitive to the level of subcritical reactivity, which can depend on poisoning, depletion, and thermal feedback over short operational time scales. Although the interest in the accelerator-driven subcritical nuclear system[1] is growing lately, the detailed dynamic analysis has not been done so far. In this paper, we investigated the stability of slightly subcritical region of a subcritical reactor system, using a simple model which includes one-group delayed neutrons and temperature feedback of the flowing fuel system with external neutron source. Also, we investigated the reactor response to external source perturbation or reactivity oscillation in operational respect.

II. Theory and Methodology

In general, the prompt neutron equation with external neutron source and temperature feedback can be written as Eq. (1). For delayed neutrons, one-group model which includes a modification to accounts for the loss of delayed neutron precursors from the core vessel exit and a modification to account for re-entry of delayed neutron precursors at the core entrance was used as in Eq. (2)[2]. Eq. (3) represents the temperature feedback equation of the reactor. To simplify the model, we assume that the core inlet temperature is constant :

$$\frac{dN(t)}{dt} = \frac{1}{l}(\rho - \beta + \alpha\delta T)N(t) + \lambda C(t) + q \tag{1}$$

$$\frac{dC(t)}{dt} = \frac{\beta}{l}N(t) - \lambda C(t) - \frac{1}{\tau_c}\{C(t) - e^{-\lambda\tau}C(t-\tau)\} \tag{2}$$

$$\frac{dT(t)}{dt} = KN(t) - \frac{1}{\tau_c}(T(t) - T_{in}) \ , \tag{3}$$

where $$\rho = \frac{k_{eff}-1}{k_{eff}} \ , \quad \frac{1}{\tau_c} = \frac{\dot{m}}{M}, \quad K = \frac{b}{C_pM},$$

τ : residence time in the external loop.

The equations can be linearized by assuming small variations around the steady state solution as in Eq. (4). After linearization, we take Laplace transform of the equations to obtain transfer function. Here, we assume small reactivity and external source perturbation as the input variables.

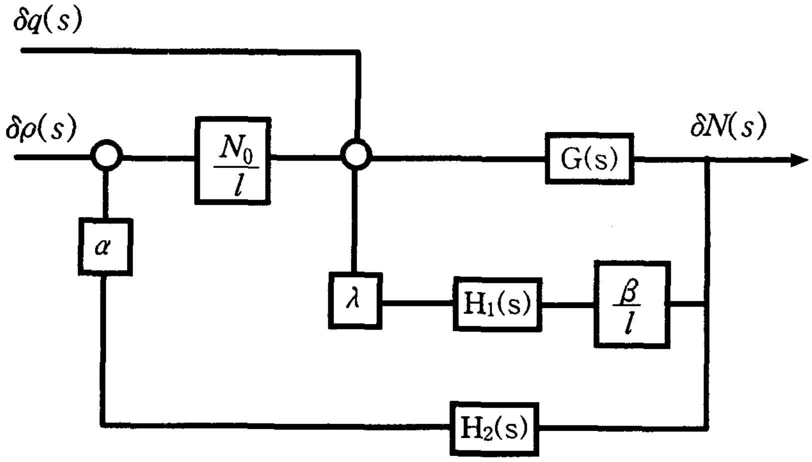
$$\begin{aligned} N(t) &= N_0 + \delta N(t) \ , \quad \rho(t) = \rho_0 + \delta\rho(t) \\ C(t) &= C_0 + \delta C(t) \ , \quad q(t) = q_0 + \delta q(t) \\ T(t) &= T_0 + \delta T(t) \end{aligned} \tag{4}$$

Letting $\delta N(s), \delta C(s), \delta\rho(s), \delta T(s)$ and $\delta q(s)$ be the Laplace transforms of $\delta N(t)$, $\delta C(t), \delta\rho(t), \delta T(t)$ and $\delta q(t)$, respectively, the transformed linearized kinetic system can be represented in block diagram form as in Fig. 1. Finally, we can obtain the transfer function F(s) as in Eq. (6) which represents the response from reactivity or external source perturbation to the variation of neutron density.

$$N(s) = F(s)\left(\delta\rho(s) + \frac{l}{N_0}\delta q(s)\right) \tag{5}$$

$$F(s) = \frac{N_0G(s)}{l - \alpha N_0G(s)H_2(s) - \lambda G(s)\beta H_1(s)} \tag{6}$$

We note that $H_1(s)$ has exponential terms of s which originate from the flowing delayed neutron precursor equation. Eq. (6) shows that the effect of external source on



$$G(s)=\frac{l}{sl+\alpha\beta-\rho_0}$$

단, $$H_1(s)=\frac{1}{s+\lambda+\frac{1}{\tau_c}(1-e^{-\tau s}e^{-\lambda\tau})}$$

$$H_2(s)=\frac{K}{s+\gamma}$$

neutron density is similar to that of reactivity.

$$a=\frac{\lambda\tau_c}{\lambda\tau_c+1-e^{-\lambda\tau}}\ ,\quad \gamma=\frac{1}{\tau_c}$$

Fig. 1. Model of a circulating fuel reactor with temperature feedback.

III. Results and Discussion

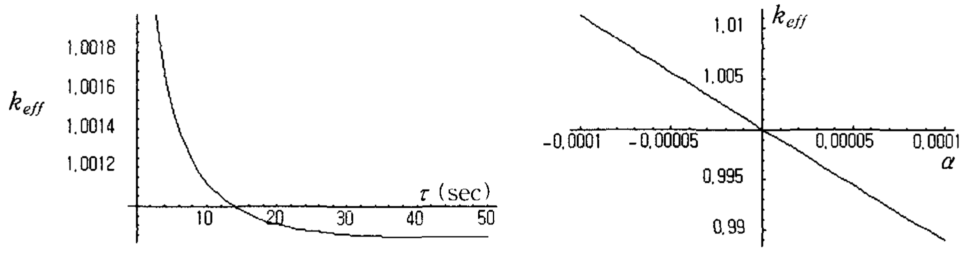
As a simple criterion for the safety region, we employed the Routh table by examining the characteristic equation of the transfer function F(s). Also, we can investigate the system response to the external source or reactivity perturbation using the frequency response of the unapproximated transfer function. The subcriticality ($1-k_{eff}$), temperature reactivity coefficient (α), and residence time in the external loop (τ) are selected as the variables. For core parameters, the results of Woosley and Rydin[3] who treated the Los Alamos ABC (accelerator based conversion) system were used.

Table 1. Parameters used for ABC kinetics equation

β	l	λ	b	C_p	M	Pth	$\dot{m}$	τ
3.91E-3	1.0E-4s	0.1/s	5.86E-8/MW	2300J/kg℃	5100	700MW	2150kg/s	10.26s

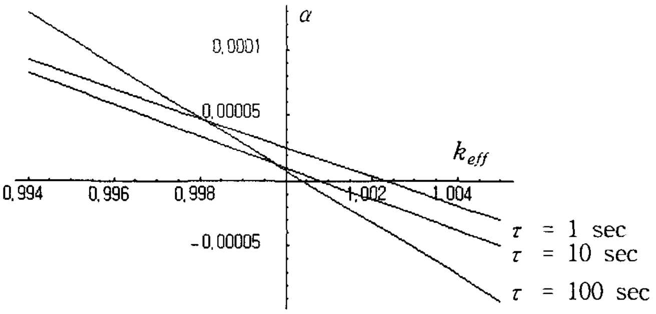
1. Routh Table

To make the characteristic equation of the transfer function $F(s)$ a polynomial, we approximate $e^{-\tau s}$ to $1-\tau s$ near the s=0 point, because the pole crosses 0 along the real axis from negative to positive. Table 1 shows the parameters used. We can obtain three conditions for stability according to the Routh table, which is shown in Fig. 2.



a) condition (i)

b) condition (ii)



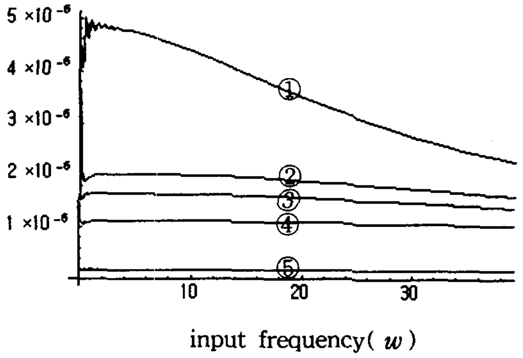
c) condition (iii)

Fig. 2. Stability region of temperature feedback coefficient and subcriticality

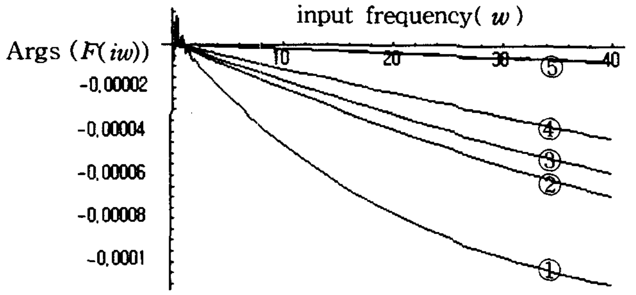
The first condition sets the upper limit of k_{eff} versus τ. Although the flowing fluid fuel decreases the upper limit of the multiplication factor, the value of the multiplication factor could be larger than 1.0. The second and third conditions set the boundaries of the multiplication factor and temperature feedback coefficient. In the range of α ($-1.0\times10^{-4}\sim1.0\times10^{-4}$), the flow effect almost does not affect the second condition. But the third condition shows that the boundary of the stability region is affected by the flowing fluid fuel.

2. Frequency Response

Figs. 3. shows the amplitude and phase response of the transfer function when the temperature feedback coefficient is $-1.0\times10^{-5}/\,^{\circ}K$. The case of $\tau=10.26$sec shows much larger amplitude response than the case of no flow($\tau = 0$ sec) in the low frequency region. But the phase difference from input perturbation to neutron density is only slightly different. It means that small oscillation of the accelerator current can result in large reactor power fluctuation. However, the flow effect disappears when the subcriticality increases (when k_{eff}=0.95, there is no difference between $\tau = 10.26$ sec and $\tau = 0.0$ sec).



a) Amplitude response of F(s)



b) Phase response of F(s)

Fig. 3. Amplitude and phase response of ABC system ($\alpha = -1.0 \times 10^{-5}$).

In summary, the flowing fuel increases the amplitude of the response at low frequency and generates oscillation in a slightly subcritical region with a slightly negative temperature feedback coefficient. Although the fluid fuel does not make significant change in the boundary of stability region, in some cases, small oscillation in the accelerator current or reactivity can make a significant change in the power. Therefore, the subcriticality must be determined by considering both the temperature feedback coefficient and the characteristics of the incident accelerator proton current, in conjunction with the reactor power level.

References

[1] C.D. Bowman, et al., "Nuclear Energy Generation and Waste Transmutation Using Accelerator-Driven Intense Thermal Neutron Source," *Nuclear Instruments & Methods in Physics Research*, A320, 336-367, 1992.

[2] J. MacPhee, "The Kinetics of Circulating Fuel Reactors," *Nuclear Science and Engineering*, 4, 588-597, 1958.

[3] M.L. Woosley, Jr. and R.A. Rydin, "Dynamic Analysis of an Accelerator-Driven Fluid-Fueled Subcritical Radioactive Waste Burning System," submitted to *Nuclear Science and Engineering*.