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A Numerical Study of Stiffness in Point Reactor Kinetics

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Abstract

A stiffness in a dynamical system is numerically studied to investigate a sensitivity of a reactor to the delayed neutron spectra with the Doppler feedback effect. To test numerical procedure, we adopted a case of a reactivity accident in a point reactor model. We found that the stiffness is sensitive to a reactivity insertion rate and the delayed neutron spectra in the Doppler feedback phase. Our numerical results show that global reactor characteristics are not very sensitive to the delayed neutron spectra even though their instantaneous ones are sensitive. We present the time evolution of each precursor group explicitly.

I. Introduction

Recently a series of studies of a point reactor kinetics have been performed by Das[1,2] to investigate a sensitivity of a nuclear reactor to the delayed neutron spectra. Since a stiff differential equation is involved in this problem, a sophisticated algorithm is required to handle the stiffness properly and to find a stable numerical result. The algorithm developed in Reference [2] requires four sets of initial conditions that are composed of up to the third order derivatives at t=0 including the zeroth order one. This algorithm can only be justified in a special problem such as a case for a point kinetics, because precursor populations vary in conjunction with a neutron population in a steady state.

In mathematical point of view, however, it can be argued that the problem must be over-specified, since a set of the first order differential equations can be solved with only one set of initial conditions. In this paper we re-examined a dynamical system for a point nuclear reactor with a set of initial conditions by making use of an algorithm developed for a stiff equation.[3,4] Finally, we discuss a role of each precursor group, based on numerical results, in a response of the Doppler feedback reactivity.

II. A Model for Point Reactor Kinetics

In a nuclear reactor having an external neutron source $s_{eff}(t)$, a change of the neutron, $n(t)$, and precursor, $c_i(t)$, populations can be written in a vectorial form

$$\frac{d}{dt}\vec{y}(t) = \boldsymbol{A}(t)\vec{y}(t) + \vec{s}(t) \tag{1}$$

with the initial condition $\vec{y}(t_o) = \vec{y_o}$ at $t = t_o$, where the column vectors and the coefficient

matrix are defined as $\vec{y}(t) \equiv \{n(t), c_1(t), \ldots, c_m(t)\}^T$, $\vec{s}(t) \equiv \{s_{eff}(t), 0, \ldots, 0\}^T$, respectively,

and $$A(t) \equiv \begin{pmatrix} l_{eff}^{-1}[(1-\beta_{eff})k(t)-1] & \lambda_1 & \cdots & \lambda_m \\ l_{eff}^{-1}\beta_{1eff}k(t) & -\lambda_1 & 0 & 0 \\ l_{eff}^{-1}\beta_{2eff}k(t) & 0 & -\lambda_2 & 0 \\ \vdots & 0 & \ddots & 0 \\ l_{eff}^{-1}\beta_{meff}k(t) & 0 & \cdots & -\lambda_m \end{pmatrix}.$$

Since the $k(t)$ in the matrix element represents an instantaneous effective multiplication factor, the reactivity can be read $\Delta k(t) = k(t) - 1$. The l_{eff} denotes effective mean prompt neutron lifetime. The effective fraction of delayed neutrons and the decay constant of precursors of the i^{th} group are β_{ieff} and λ_i, respectively. Assuming a *six-group* of the delayed neutrons, the total effective fraction of the delayed neutrons becomes $\beta_{eff} = \sum_{i=1}^{m=6} \beta_{ieff}$.

The general solution to the Eq. (1) reads

$$\vec{y}(t) = \Psi(t)\,\vec{y}_o + \Psi(t)\int_{t_o}^{t} \Psi^{-1}(\tau)\,\vec{s}(\tau)\,d\tau \tag{2}$$

where the operator $\Psi(t)$ satisfies an auxiliary differential equation of

$$\frac{d}{dt}\Psi(t) = A(t)\,\Psi(t) \tag{3}$$

with an initial condition $\Psi(t_o) = I$, i.e. an identity matrix.

Substituting a solution to Eq. (3), $\Psi(t) = e^{\int_{t_o}^{t} A(\tau)\,d\tau}$, into Eq. (2) yields

$$\vec{y}(t) = e^{\int_{t_o}^{t} A(\tau)\,d\tau}\left[\vec{y}_o + \int_{t_o}^{t} e^{-\int_{t_o}^{\tau} A(\xi)\,d\xi}\,\vec{s}(\tau)\,d\tau\right] \tag{4}$$

We deal with a case that there is no external neutron source, $s_{eff}(t) = 0$. In a general case of a stiff equation, note that the Eq. (4) can not be separated with the eigenvalues of the matrix $A(t)$.

In order to investigate sensitivities of the dynamic behavior of a reactor to the delayed neutron energy spectra, a linear time-dependence in a fraction of the delayed neutron has been assumed

$$\beta_{eff}(t) = K_\beta(\beta_{eff})_o + Q_\beta t \tag{5}$$

$$\beta_{ieff}(t) = K_\beta(\beta_{ieff})_o + Q_{\beta i}\,t, \qquad \text{with } Q_{\beta i} = Q_\beta/6 \tag{6}$$

We also assume a linear time-dependence in the reactivity $\Delta k(t)$, that can be splitted into two terms; the reactivity insertion term and the Doppler feedback term

$$\Delta k(t) = \Delta k_i(t) + \theta(t - t_m)\Delta k_f(t) \tag{7}$$

where the reactivity insertion term is $\Delta k_i(t) = (\textit{insertion rate})\ t$, the step function represents that the Doppler feedback occurs at the peak power, i.e., at t_m. Introducing the Doppler coefficient(D_θ) and the heat capacity(C_p) of the core, we write the Doppler reactivity

$$\Delta k_f(t) = -\frac{|D_\theta|}{C_p}\int_0^t P(t)dt \propto \int_0^t n(t)dt. \tag{8}$$

III. Numerical Computation and Discussion

Since stiff differential equations evolve on more than one scale, a variable step size should be chosen in a discretization process, i.e., the step size should be made as small as is needed to achieve a desired accuracy, but it should be increased whenever possible to reduce the total CPU time needed.

To test numerical procedures, we adopted an 1250MW_{th} fast reactor model. We assumed that the core absorbs all fission energies adiabatically soon after a reactivity accident occurs at t=0 and that initial conditions, $n(0)=P_o l_{eff}\bar{\nu}_p C$, $c_i(0)=((\beta_{ieff})_o n(0))/(\lambda_i l_{eff})$. The delayed neutron data, the reactor parameters presented in Table 1 are adopted to find the peak power, the energy released, the neutron and precursor populations, the time to reach the peak power and melting points.

Numerical computations have been splitted into two stages, the first one is for the time of $0\leq t\leq t_m$ and the second one is for the time of $t>t_m$, because of the step function in the Doppler feedback term. Note that the first stage determines the time to reach a peak power and the populations of neutrons and precursors at that moment, and that these informations are used as initial conditions for the second stage computation.

The time to reach the peak power can be found by computing the time for the system to reach the released energy of 1061.55MJ equivalent to the temperature 573K. The calculated results are presented in Table 2 and Fig. 1.

The equations governing in the second stage are a form of integro-differential equations because of the Doppler feedback term in Eq. (8). Numerical computations for this stage are carried out by dividing the time intervals into much finer sets to convert the integro-differential equations into ordinary differential equations. Consequently this procedure is very similar to the step reactivity changes[5] and enables one to use the same numerical procedure adopted in the first stage. To obtain accurate results, however, the differential equations should be solved for each time interval recursively, i.e., redirecting the end point data into the initial data for next time interval.

The melting points of the cladding(1700K) and fuel(2758K) can be determined by the released energies equivalent to 3987.89MJ and 7731.62MJ, respectively. The calculated results are shown in Table 3 and Fig. 2.

For studying a reactor kinetics, one can use an algorithm[2] demanding initial conditions of vanishing higher order derivatives at t=0 as shown in Figs. 1 and 3. However, it is not applicable to a dynamical system that changes its character at a specific moment such as a case of a step function involved. Also, Fig. 3 shows that the time rates of precursor reponses are closely related to their mean life time.

References

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Table 1. Input constants and parameters for numerical computations.

Initial reactor thermal power, P_o			1250 MW
Mean prompt neutron lifetime, l_{eff}			0.346 μs
Average number of fission neutrons emitted, $\bar{\nu}_t$			2.89
Total effective delayed neutron fraction, $(\beta_{eff})_o$			$3.6\ 10^{-3}$
Conversion factor from joules to integrated fissions, C			$3.1\ 10^{10}$
Doppler reactivity feedback coefficient, D_δ			-0.002 $\Delta k/K$
Mass of fuel			10.11 Ton
Fuel heat capacity, C_p			0.35 J/gK
Melting point of fuel			2758 K
Melting point of cladding			1700 K
Ramp reactivity insertion rates, $\Delta k/s$			0.01~0.8
Time variation of β_{eff} during transients, Q_β			$5\ 10^{-4}\ \Delta k/s$
$(\beta_{1eff})_o$=8.3845 10^{-5}	$(\beta_{4eff})_o$=1.3100 10^{-3}	λ_1 = 0.0129	λ_4 = 0.3310
$(\beta_{2eff})_o$=7.6095 10^{-4}	$(\beta_{5eff})_o$=5.9860 10^{-4}	λ_2 = 0.0311	λ_5 = 1.2600
$(\beta_{3eff})_o$=6.7000 10^{-4}	$(\beta_{6eff})_o$=1.7640 10^{-4}	λ_3 = 0.1340	λ_6 = 3.2100

Table 2. Calculated dynamical parameters of a reactor core.

Reactivity Insertion Rate ($\Delta k/s$)	K_β	Max. Reactivity Reached (10^{-3})	Peak Power (MW)	Time to Reach Peak Power (ms)	K_β	Max. Reactivity Reached (10^{-3})	Peak Power (MW)	Time to Reach Peak Power (ms)
0.01	0.8	2.803	$1.941\ 10^4$	280.30	1.2	3.746	$8.530\ 10^3$	374.64
0.05		3.178	$5.111\ 10^5$	63.55		4.582	$3.484\ 10^5$	91.63
0.20		3.638	$1.829\ 10^6$	18.19		5.040	$1.617\ 10^6$	25.20
0.80		4.656	$4.756\ 10^6$	5.82		6.032	$4.460\ 10^6$	7.54
0.01	0.9	3.067	$1.491\ 10^4$	306.70	1.3	3.939	$7.452\ 10^3$	393.90
0.05		3.529	$4.599\ 10^5$	70.57		4.933	$3.242\ 10^5$	98.66
0.20		3.986	$1.754\ 10^6$	19.93		5.392	$1.513\ 10^6$	26.96
0.80		5.000	$4.662\ 10^6$	6.25		6.384	$4.364\ 10^6$	7.98
0.01	1.0	3.315	$1.208\ 10^4$	331.50	1.4	4.118	$6.632\ 10^3$	411.80
0.05		3.858	$4.231\ 10^5$	77.16		5.284	$2.903\ 10^5$	105.67
0.20		4.336	$1.711\ 10^6$	21.68		5.746	$1.449\ 10^6$	28.73
0.80		5.344	$4.591\ 10^6$	6.68		6.728	$4.335\ 10^6$	8.41
0.01	1.1	3.537	$9.822\ 10^3$	353.70				
0.05		4.231	$3.815\ 10^5$	84.61				
0.20		4.688	$1.666\ 10^6$	23.44				
0.80		5.688	$4.523\ 10^6$	7.11				

Table 3. The calculated time to reach the melting point of fuel and cladding.
Note: The abbreviations (C) and (F) stand for the fuel and cladding.

Ramp rate	K_β						
$\Delta k/s$	0.8	0.9	1.0	1.1	1.2	1.3	1.4
0.01 (C)	224.57s	224.57s	224.57s	224.57s	224.57s	224.57s	224.57s
0.01 (F)	436.19s	436.19s	436.20s	436.20s	436.20s	436.21s	436.21s
0.05 (C)	45.135s	45.138s	45.122s	45.116s	45.110s	45.104s	45.097s
0.05 (F)	87.511s	87.509s	87.509s	87.505s	87.503s	87.502s	87.500s
0.20 (C)	11.197s	11.196s	11.196s	11.195s	11.194s	11.194s	11.189s
0.20 (F)	21.805s	21.805s	21.779s	21.778s	21.778s	21.778s	21.778s
0.80 (C)	2.801s	2.798s	2.800s	2.801s	2.800s	2.801s	2.802s
0.80 (F)	5.455s	5.434s	5.436s	5.447s	5.447s	5.447s	5.447s

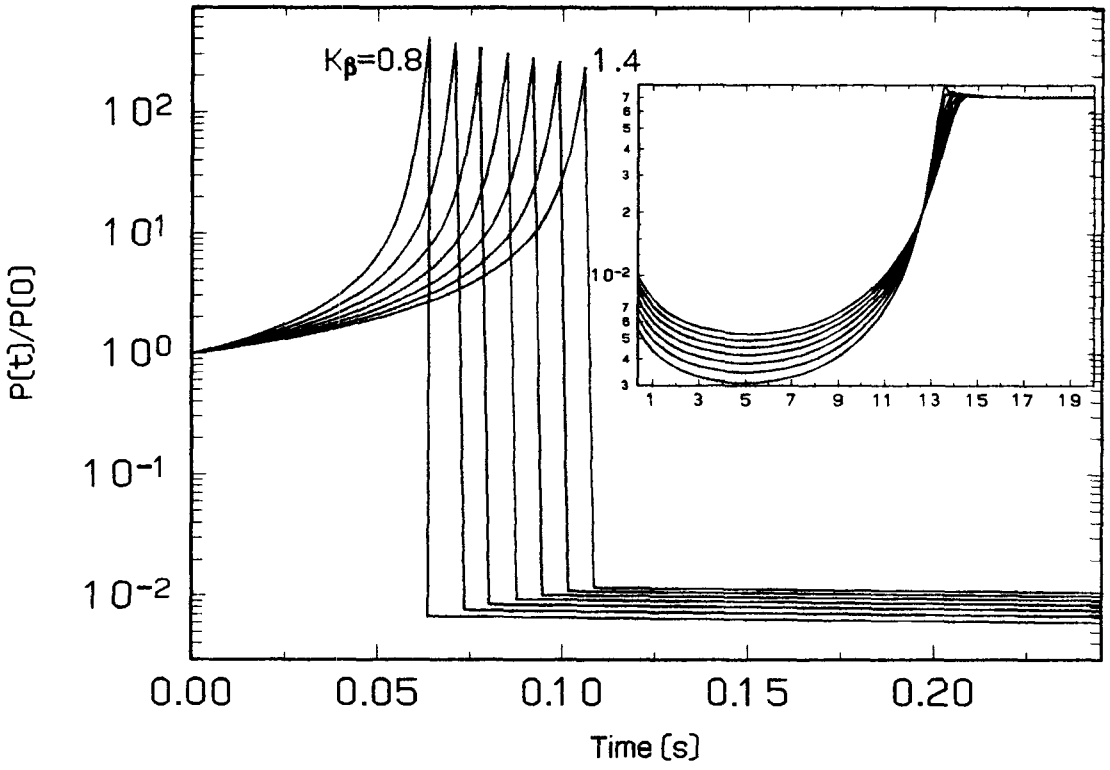

Fig. 1 Plot of $P(t)/P(0)$, equivalently $n(t)/n(0)$, for $K_\beta = 0.8, \ldots, 1.4$ at a ramp rate of 0.05.

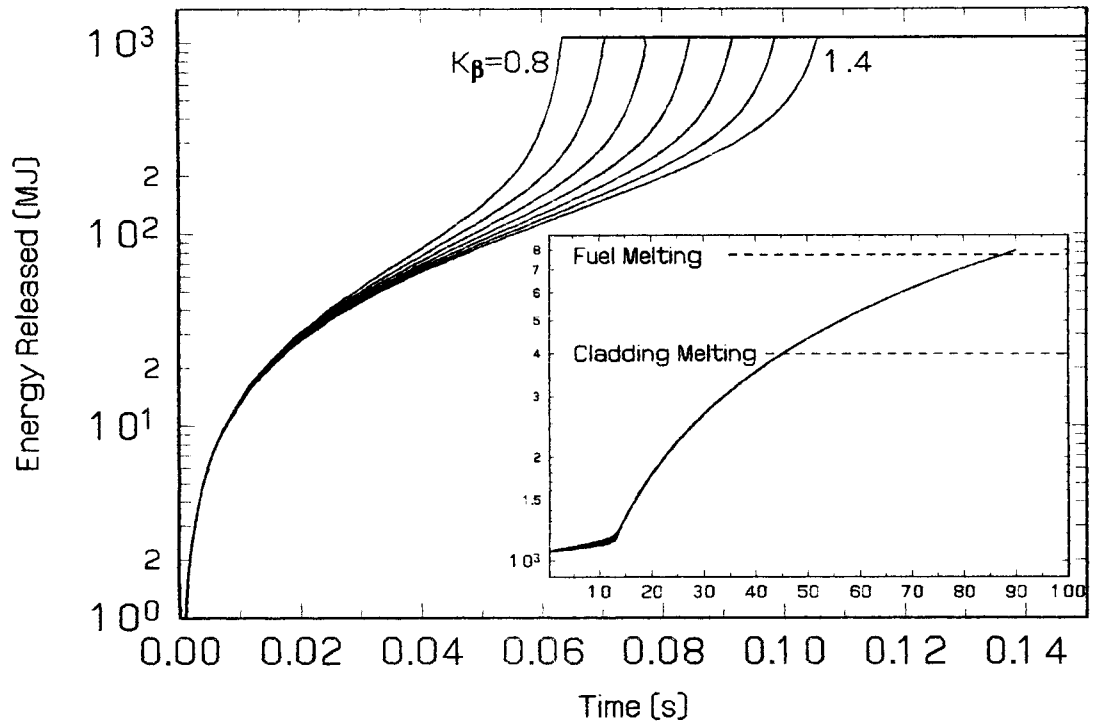

Fig. 2 Plot of the energy released for $K_\beta = 0.8, \ldots, 1.4$ at a ramp rate of 0.05.

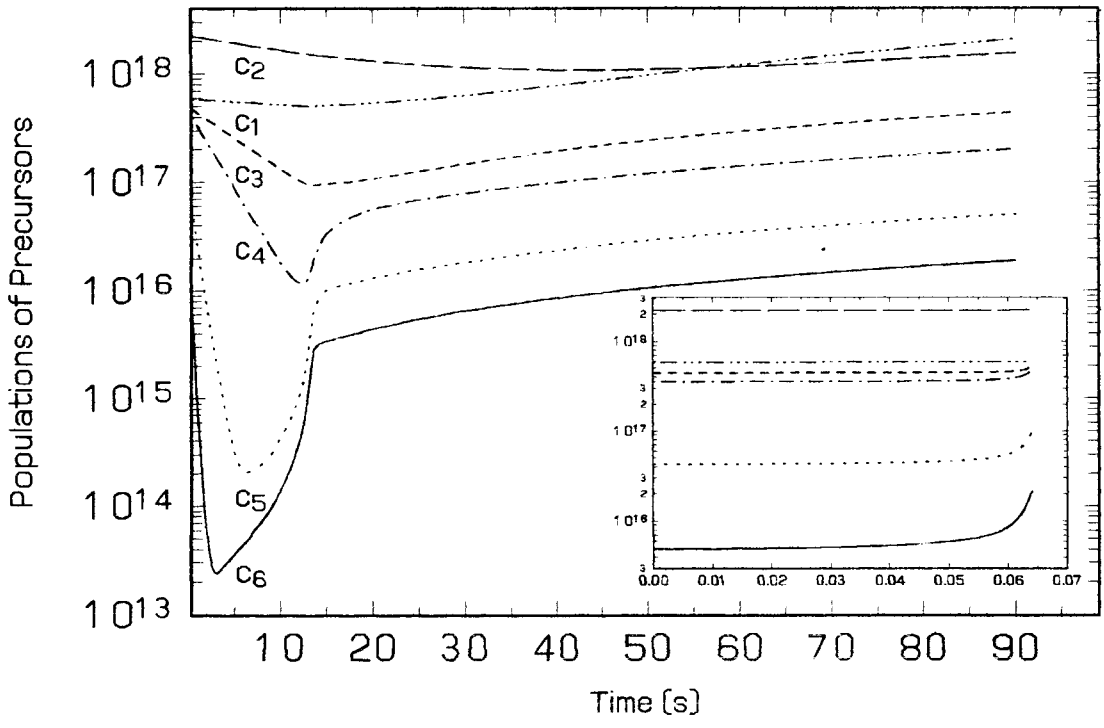

Fig. 3 Plot of population of each precursor group, $c_i(t)$, for $K_\beta = 0.8$ at a ramp rate of 0.05