선형 시변 시스템에 대한 주기 예측 제어기의 시불변 안정성

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Uniform Stability of Intervalwise Receding Horizon Controls for Linear Time-Varying Systems

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Abstract

In this paper, intervalwise receding horizon controls (IRHCs) are proposed for linear time systems subject to H_2 and H_∞ problems. Uniform stability conditions are provided for those systems. Under given conditions stability is proved without using an adjoint system. It is also shown that under proposed stability conditions for H_∞ problem, H_∞-norm bound is satisfied. The results in this paper are also applicable to periodic systems which belong to the class of time systems.

1 Introduction

The pointwise receding horizon control (PRHC), also known as receding horizon control (RHC) has received much attention as a powerful tool for the control of industrial process systems since it has many advantages such as simple computation [3], I/O (input/output) constraint handling [6], and tracking performance [4], [5]. Disturbance rejection [7], robustness property [6], and stability property [2], have been also investigated for both linear or nonlinear plants.

In PRHC, more than one control input is calculated as the terminal point of a fixed-length cost N-horizon recedes continuously at every time instant. However, only the first one is implemented. It is noted that if we use these ignored control inputs besides the first one, we have a much lower computation cost, and may have a better tracking performance and lower gain of control input though it requires more memory than PRHC. This control will be called an intervalwise receding horizon control (IRHC). In this case, after T period the terminal point of the cost horizon moves by one N-horizon and is fixed for the next T period. In addition, compared with PRHC, IRHC may have different charateristics for several control issues such as I/O constraint handing, disturbance rejection, robustness property, and stability property.

IRHC has been developed only for periodic systems including time-invariant systems subject to H_2 problem [1], and H_∞ problem [8], while PRHC have been developed for time-invariant and time systems including periodic systems subject to H_2 problem [2], [4], and H_∞ problem [7]. In addition, terminal inequality condition for continuous systems has not been investigated in literature which is more flexible than terminal equality condition [4] and enables us to handle I/O constraint[6].

In this paper, we investigate IRHC for linear time systems subject to H_2 and H_∞ problems which may include previous results as special case. Uniform stability conditions are proposed under which closed-loop stability of those systems is guaranteed with IRHC. It is also shown that under the stability conditions proposed for H_∞ problem, H_∞-norm bound is satisfied.

In Section 2, uniformly stabilizing intervalwise receding horizon (IRH) H_2 and H_∞ controls are proposed and it is shown that H_∞-norm bound is satisfied for continuous time systems. In Section 3, uniformly stabilizing intervalwise receding horizon (IRH) H_2 and H_∞ controls are proposed and it is also shown that H_∞-norm bound is satisfied for discrete time systems. Finally, conclusions are presented in Section 4.

2 IRHC for continuous time systems

In this Section, we suggest IRH H_2 and H_∞ controls and stabilizing conditions for continuous time systems.

First, we investigate an IRH H_2 control. Consider the following continuous time system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^p$. Consider also a cost function with $Q_2 > 0$

$$\begin{aligned} J_{t_1,t_2}(U_{t_1,t_2}) &= x^T(t_2)Q(t_2)x(t_2) + \int_{t_1}^{t_2} (x^T(t)C^T(t) \\ &\quad C(t)x(t) + u^T(t,t_2)R(t)u(t,t_2))dt \end{aligned}$$

where $R(t)$ are piecewise continuous and symmetric.

We assume there exist fixed positive real numbers α_1, α_2, such that

$$\alpha_1 I \le R(t) \le \alpha_2 I \quad for \ \forall \ t. \tag{2.2}$$

Define $t_b = t_0 + kT$, $t_d = t_b + T$, and $t_e = t_b + N$ for $T > 0$, $\infty > N \ge T + \delta$ and $\delta > 0$, and $k = 0, 1, \cdots$, then we propose an IRH H_2 control and stabilizing conditions.

$$u^*(t,t_e) = -R^{-1}(t)B^T(t)P(t,t_e)x(t) \tag{2.3}$$

$$\begin{aligned} -\frac{\partial P(\tau,t_e)}{\partial \tau} &= A^T(\tau)P(\tau,t_e) + P(\tau,t_e)A(\tau) + \\ &\quad C^T(\tau)C(\tau) - P(\tau,t_e)B(\tau) \\ &\quad R^{-1}(\tau)B^T(\tau)P(\tau,t_e) \end{aligned} \tag{2.4}$$

where $P(t,t_e)$ is obtained by integrating (2.4) backward from t_e to $\tau = t$ for $t \in [t_b, t_d)$.

THEOREM *1 :* Assume (2.2), $0 \le C^T(t)C(t) \le \alpha_3 I$ for $\forall t$, and

$$\begin{aligned} Q(t_e) &\ge \Phi_F(t_e)Q(t_e+T)\Phi_F^T(t_e) \\ &+ \int_{\tau=t_e}^{t_e+T} \Psi_F(t_e,\tau)[C(\tau)^T C(\tau) + \\ &H(\tau)^T R(\tau)H(\tau)]\Psi_F^T(t_e,\tau)d\tau \quad (2.5) \end{aligned}$$

where

$$\begin{aligned} \Phi_F(\sigma) &= \Psi_F(\sigma, \sigma+T) \\ \Psi_F(\sigma,\tau), &\quad \text{where } x(\tau) = \Psi_F^T(\sigma,\tau)x(\sigma) \\ F(\tau) &= A(\tau) + B(\tau)H(\tau) \\ &\quad \text{for some } H(\tau) \in R^{m*n}. \end{aligned}$$

Then if $(A(t), B(t))$ is uniformly completely controllable for some $\delta_c > 0$, the system (2.1) with (2.3) for $\delta \ge \delta_c$ is uniformly asymptotically stable for δ_c defined in [2].

proof: Since $P(t_d^+, t_e)$-$P(t_d^-, t_e) \le 0$ by (2.5), the remaining proof procedure is parallel to that of [2]. ■

The proof of the following **Theorem** is shown directly without using an adjoint system [2].

THEOREM *2 :* Assume (2.2), $0 < C^T(t)C(t) \le \alpha_3 I$ for $\forall t$, and (2.5) is satisfied. Then if $(A(t), B(t))$ is uniformly completely controllable for some $\delta_c > 0$, the system (2.1) with (2.3) for $\delta \ge \delta_c$ is uniformly asymptotically stable.

proof: Consider the system of (2.1) with the control (2.3) and a Lyapunov function

$$V(t, x(t)) = x^T(t)P(t, t_e)x(t). \quad (2.6)$$

Since there exist positive scalars α_4 and α_5 such that $\alpha_4 I \le P(t, t+\delta') \le \alpha_5 I$ for each δ' satisfying $\delta \le \delta' < \infty$ from Lemma 2.1 of [2],

$$\alpha_6 x^T(t)x(t) \le V(t,x(t)) \le \alpha_7 x^T(t)x(t)\frac{\partial P(\tau,\sigma)}{\partial \sigma}|_{\sigma=t_e} \le 0. \quad (2.7)$$

Then from (2.6) and (2.7), $V(t, x(t; t_s, x(t_s))) - V(t_s, x(t_s))$

$$\begin{aligned} &= \int_{t_s}^{t} \dot V(\tau, x(\tau))d\tau + \\ &\quad \sum_{t_d \in [t_s, t)} x^T(t_d)[P(t_d^+, t_e) - P(t_d^-, t_e)]x(t_d) \\ &\le \int_{t_s}^{t} \dot V(\tau, x(\tau))d\tau \\ &\le \int_{t_s}^{t} x^T(\tau)[-C^T(\tau)C(\tau) - P(\tau)B(\tau)R^{-1}(\tau) \\ &\quad B^T(\tau)P(\tau)]x(\tau)d\tau \\ &= -\int_{t_s}^{t}[x^T(\tau)C^T(\tau)C(\tau)x(\tau) + u^{*T}(\tau)R(\tau)u^*(\tau)]d\tau \\ &\le -\alpha_9 x^T(t_s)x(t_s), \quad t > t_s. \quad (2.8) \end{aligned}$$

where $F(t) = (A(t) - B(t)R^{-1}(t)B(t)^T P(t, t_e))$ and $\alpha_9 > 0$. From (2.7) and (2.8), the system (2.1) with (2.3) is uniformly asymptotically stable. ■

Second, we investigate an IRH H_∞ control.

$$\begin{aligned} \dot x(t) &= A(t)x(t) + B(t)u(t) + D(t)w(t) \\ z(t) &= C(t)x(t) + E(t)u(t) \quad (2.9) \end{aligned}$$

where $E^T(t)E(t) = I$ and $C^T(t)E(t) = 0$, $x(t) \in R^n$, $u(t) \in R^m$, and $w(t) \in R^l$. Consider also the cost index with $Q(t_2) > 0$:

$$J_{t_1,t_2}(U_{t_1,t_2}, W_{t_1,t_2}) = x^T(t_2)Q(t_2)x(t_2) + \int_{t_1}^{t_2}(\|z(t)\|^2 - \gamma^2\|w(t)\|^2)dt$$

Assume that there exist positive scalars α_{10} and α_{11} such that

$$\alpha_{10} I \le P(t, t+\delta') \le \alpha_{11} I \quad for \ \delta' \ge \delta. \quad (2.10)$$

We also propose an IRH H_∞ control and prove the stability of that control.

$$\begin{aligned} u^*(t,t_e) &= -B^T(t)P(t,t_e)x(t) \quad (2.11) \\ w^*(t,t_e) &= \gamma^{-2}D^T(t)P(t,t_e)x(t) \\ -\frac{\partial P(\tau,t_e)}{\partial \tau} &= A^T(\tau)P(\tau,t_e) + P(\tau,t_e)A(\tau) \\ &+C^T(\tau)C(\tau) - P(\tau,t_e)(B(\tau)B^T(\tau) \\ &-\gamma^{-2}D(\tau)D^T(\tau))P(\tau,t_e). \quad (2.12) \end{aligned}$$

THEOREM *3 :* Assume (2.10), $0 \le C^T(t)C(t) \le \alpha_{12} I$, $\frac{\partial P(t,\sigma)}{\partial \sigma}|_{\sigma=t_e} \le 0$ for $\forall\ t \in [t_b, t_d)$, and

$$\begin{aligned} Q(t_e) &\ge \Phi_F(t_e)Q(t_e+T)\Phi_F^T(t_e) \\ &+ \int_{\tau=t_e}^{t_e+T} \Psi_F(t_e,\tau)[C(\tau)^T C(\tau) \\ &+H(\tau)^T H(\tau)]\Psi_F^T(t_e,\tau)d\tau \quad (2.13) \end{aligned}$$

for some $H(\tau) \in R^{m*n}$. Then if $(A(t), B(t))$ is uniformly completely controllable for some $\delta_c > 0$, the system (2.9) with (2.11) is uniformly asymptotically stable and H_∞-norm bound is satisfied.

proof: The proof procedure is parallel to that of **Theorem 1**. ■

THEOREM *4 :* Assume (2.10), $0 < C^T(t)C(t) \le \alpha_{12} I$ for $\forall t$, and (2.13) is satisfied. Then the system (2.9) with (2.11) for $\delta \ge \delta_c$ is uniformly asymptotically stable.

proof: Consider a Lyapunov function with closed-loop system $\dot{\hat x}(t) = A(t)\hat x(t) + B(t)u^*(t)$. The proof procedure is parallel to that of **Theorem 2.** ■

H_∞-norm bound is shown in [9]. ■

3 IRHC for discrete time systems

Here we suggest IRH H_2 and H_∞ controls and stabilizing conditions for discrete time systems.

First, we investigate an IRH H_2 control. Consider the following discrete time system

$$\begin{aligned} x_{i+1} &= A_i x_i + B_i u_i \\ y_i &= C_i x_i \quad (3.1) \end{aligned}$$

where $x_i \in R^n$, $u_i \in R^m$, and $y_i \in R^p$. Consider also a cost function

$$J_{i_1,i_2}(U_{i_1,i_2}) = x_{i_2}^T Q_{i_2} x_{i_2} + \sum_{i=i_1}^{i_2-1}[x_i^T C_i^T C_i x_i + u_{i,i_2}^T R_i u_{i,i_2}]$$

where $Q_{i_2} > 0$.

We assume there exist fixed positive real numbers β_1, β_2, such that

$$\beta_1 I \le R_i \le \beta_2 I \quad for\ \forall\ i. \quad (3.2)$$

Define i_b=$i_0 + kT$, i_d=$i_b + T$, and i_e=$i_b + N$ for

$N \geq T + \delta$, $T \geq 1$, $\delta \geq 1$, and $k = 0, 1, \cdots$, where i_0 is an initial point of control. Now we propose an IRH H_2 control and stabilizing conditions for the system (3.1).

$$\begin{aligned} u^*_{i,i_e} &= -[R_i^{-1} + B_i^T P^{-1}_{i+1,i_e} B_i]^{-1} B_i^T P_{i+1,i_e} A_i \quad (3.3) \\ P_{i,i_2} &= A_i^T P_{i+1,i_2} A_i + C_i^T C_i \\ &\quad - A_i^T P_{i+1,i_2} B_i [R_i^{-1} + B_i^T P_{i+1,i_2} B_i]^{-1} B_i^T \\ &\quad P_{i+1,i_2} A_i; \quad P_{i_2,i_2} = Q_{i_2} \quad (3.4) \\ &= F_i^{*T} P_{i+1} F_i^* + C_i^T C_i + K_{i,i_2}^T K_{i,i_2} \\ F_i^* &= A_i + B_i K_{i,i_2}, \\ K_{i,i_2} &= -[R_i^{-1} + B_i^T P^{-1}_{i+1,i_2} B_i]^{-1} B_i^T P_{i+1,i_2} A_i. \end{aligned}$$

The IRH control uses only the first T-horizon controls among N-horizon controls u^*_{i,i_e}. After T period, the terminal point of the cost horizon moves by one N-horizon and is fixed for the next T period.

Before proposing the stabilizing conditions, define :

$$\begin{aligned} L_{l,n}^T(M) L_{l,n}(M) &= \sum_{i=l}^{n-1} \Psi_{l,i}(M) C_i^T C_i \Psi_{l,i}^T(M), \\ \Psi_{l,n}(M) &= M_l^T M_{l+1}^T \cdots M_{n-2}^T M_{n-1}^T \\ \Phi_l(M) &= \Psi_{l,l+T}(M), \end{aligned}$$

Corollary *1 :* Assume (3.2), $0 \leq C_i^T C_i \leq \beta_3 I$ for $\forall i$, and

$$\begin{aligned} Q_{i_e} \geq \; & \Phi_{F_{i_e}} Q_{i_e+T} \Phi_{F_{i_e}}^T + \sum_{j=i_e}^{i_e+T-1} \\ & \Psi_{F_{i_e},j} [C_j^T C_j + H_j^T R_j H_j] \Psi_{F_{i_e},j}^T \quad (3.5) \end{aligned}$$

where

$$\begin{aligned} \Phi_{F_l} &= \Psi_{F_{l,l+T}} \\ \Psi_{F_{l,j}} &= F_l^T F_{l+1}^T \cdots F_{j-2}^T F_{j-1}^T \\ F_l &= A_l + B_l H_l \quad for \;\; some \;\; H_l \in R^{m*n}. \end{aligned}$$

Then if (A_i, B_i) is uniformly completely controllable for some $\delta_c > 0$, the system (3.1) with (3.3) for $\delta \geq \delta_c$ is uniformly asymptotically stable.

Corollary *2 :* Assume (3.2), $0 < C_i^T C_i \leq \beta_3 I$ for $\forall i$, and (3.5) is satisfied. Then if (A_i, B_i) is uniformly completely controllable for some $\delta_c > 0$, the system (3.1) with (3.3) for $\delta \geq \delta_c$ is uniformly asymptotically stable.

Second, we investigate an IRH H_∞ control for discrete time systems.

$$\begin{aligned} x_{i+1} &= A_i x_i + B_i u_i + D_i w_i \\ z_i &= C_i x_i + E_i u_i \quad (3.6) \end{aligned}$$

Consider (3.6) where $E_i^T E_i = I$ and $C_i^T E_i = 0$ and a cost function

$$J_{i_1,i_2}(U_{i_1,i_2}, W_{i_1,i_2}) = x_{i_2}^T Q_{i_2} x_{i_2} + \sum_{i=i_1}^{i_2-1} [\|z_i\|^2 - \gamma^2 \|w_i\|^2]$$

where $Q_{i_2} > 0$.

Using the result of [12], if and only if $[I - \gamma^{-2} D_i^T M_{i+1,i_2} D_i] > 0$ over $i \in [i_b, i_e]$,

$$\begin{aligned} u^*_{i,i_e} &= -[I + B_i^T P_{i+1,i_e} B_i]^{-1} B_i^T P_{i+1,i_e} A_i x_i \\ P_{i,i_2} &= A_i^T P_{i+1,i_2} A_i - A_i^T P_{i+1,i_2} B_i [I + B_i^T P_{i+1,i_2} \\ &\quad B_i]^{-1} B_i^T P_{i+1,i_2} A_i + C_i^T C_i + \gamma^{-2} P_{i,i_2} D_{i-1} \\ &\quad [I + \gamma^{-2} D_{i-1}^T P_{i,i_2} D_{i-1}]^{-1} D_{i-1}^T P_{i,i_2}. \quad (3.7) \end{aligned}$$

where $P_{i,i_2} = M_{i,i_2}[I - \gamma^{-2} D_{i-1} D_{i-1}^T M_{i,i_2}]^{-1}$.

Assume that there exist positive scalars β_4 and β_5 such that

$$\beta_4 I \leq P_{i,i+\delta'} \leq \beta_5 I \quad for \;\; \delta' \geq \delta. \quad (3.8)$$

The stability result is parallel to that of the IRH H_2 control for discrete time systems.

4 Conclusion

In this paper, intervalwise receding horizon controls (IRHCs) are proposed for linear time systems subject to H_2 and H_∞ problems each other. Uniform stability is proved under some conditions including terminal inequality condition. It is also shown that under the proposed stabilizing conditions for H_∞ problem, H_∞-norm bound is satisfied.

Our results enable us to take advatages of IRHC for linear time systems. In the same way, an intervalwise receding horizon concept can be applied to various systems such as linear or nonlinear systems with plant uncertainty and delay subject to H_2 and H_∞ problems with I/O constraints. Especially if we use the proposed terminal inequality condition together with LMI (linear matrix inequality) technique, we can handle I/O constraints.

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