Use of Normal Forms Technique In Control Design
Part I: General Theory

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Abstract: This is Part I of a two part paper dealing with control design in power systems using the method of normal forms. In stressed power systems, due to the presence of increased nonlinearity and the existence of nonlinear modal interactions, there exist some limitation to the use of conventional linear control design techniques. The objective of this work is to understand the effect of the nonlinear modal interaction on control performance and to develop a procedure to design controls incorporating the nonlinear information. Part II presents the numerical results dealing with the design procedure.

1. Introduction

Large stressed interconnected power systems exhibit complicated dynamic behavior when subjected to disturbances. A complete theoretical analysis of this behavior is not feasible in large systems. A disturbance excites numerous modes of oscillation. Only a few of these modes are of primary interest to the system designer. These include the poorly damped, low frequency inter-area modes, in which generators that are geographically far away from each other participate, and the control modes that represent the influence of the controllers on the system. In particular, excitation control is called upon to increase the damping of poorly damped inertial modes. In large systems, design of the exciter constants usually follows from a linear systems analysis, neglecting the possible nonlinear interaction between modes. Recently, a number of studies have been published that address the nonlinear behavior of large power systems. Among them are the analysis of the stability boundary of a stable equilibrium point (SEP) [1], stability assessment using the TEF method [2], and analysis of auto parametric resonances [3, 4]. A series of papers by the authors[1,5-7] shows that second order nonlinear modal interaction obtained via normal forms of the system dynamics [8], allows insight into the nonlinear behavior of a power system (including AC/DC systems) and can be used to predict inter-area separation [5]. For controlled power systems, [6] shows, by analyzing a four-generator test system [9], that second order nonlinear interactions between low frequency inertial modes and control modes are crucial to understand the dynamic behavior of these systems. For power systems equipped with fast exciters, the exciter gains have crucial influence on the system dynamic behavior. In order to be able to tune the exciter gains for optimal system performance, one has to understand, how the system response changes with different gain settings. In linear analysis, this consists of determining the eigenvalues for various gains, and computing the sensitivity of the eigenvalues under gain variations. If one takes into account the influence of the second order normal forms on the system response, then the corresponding interaction coefficients and their sensitivity with respect to gain variations has to be studied as well. This is the topic of the study presented here. As we will see using the 50-generator IEEE test system [10] in a companion paper [11], the second order interaction coefficients and their sensitivity with respect to varying exciter gains yield substantial insight into controlled power systems.

2. Approach

2.1 The Power System Model

The generators without excitation control are represented by the classical model [12]. Generators with excitation control are described by the two-axis model. The block diagram of the static exciter model [13] is shown in Figure 1.

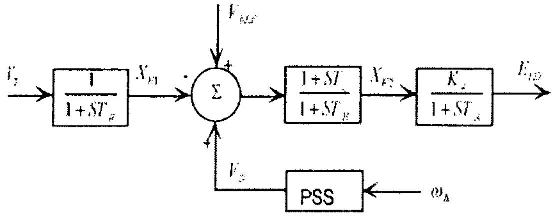

Figure 1: Static Exciter Model

In what follows, we concentrate on varying the exciter gains Ka. The network is represented classically: quasi state-steady network parameters, constant impedance loads, etc. Assuming the generator internal reactance to be constant, a network representation at the internal generator nodes can be obtained. The procedure described in (Chapter 9 of [12]) yields the direct and quadrature axes currents for the generators represented in detail. The currents for the classically represented machines can also be obtained. We assume that in an m-generator system there are l generators represented by the two-axis model and equipped with exciters, the remaining m-l generators are presented by the classical model. Then the dynamic equations governing the generators and the excitation system have the general form

$$\dot{X} = F(X) \qquad (1)$$

where,

$$X = [E'_{q1}, E'_{d1}, \omega_1, \delta_1, E_{FD1}, x_{E1_1}, x_{E2_1}, \ldots,$$
$$E'_{ql}, E'_{dl}, \omega_{l}, \ldots, \omega_m, \delta_m]^T$$

and F is an analytic vector field.

2.2 The Normal Form Technique

We expand (1) as a Taylor series about a stable equilibrium point X_{SEP} and obtain using again X and x_i as the state variables.

$$\dot{x}_i = A_i x + \tfrac{1}{2} X^T H^i X + H.O.T. \quad (2)$$

where,

A_i=I_{th} row of Jacobian A which is equal to

$$[\partial F / \partial X]_{X_{SEP}}$$

H_i= $\partial^2 F_i / \partial x_j \partial x_k]_{X_{SEP}}$ =Hessian matrix

Denote by J the (complex) Jordan form of A, and by U the matrix of the right eigenvalues of A. Then the transformation X=UY yields for the linear and the second order terms of (2) the equivalent system

$$\dot{y}_i = \lambda_i y_j + \sum_{k=1}^{N}\sum_{l=1}^{N} C^j_{kl} y_k y_l \quad (3a)$$

where,

$$C^j = \tfrac{1}{2}\sum_{p=1}^{N} V^T_{jp}[U^T H^p U] = [C^j_{kl}] \quad (3b)$$

and V denotes the matrix of associated left eigenvectors. If the second order non-resonance condition holds, i.e. if $\lambda_j \neq \lambda_k + \lambda_l$ for all three tuples of eigenvalues of A, then the normal form transformation of (3a) is defined by

$$Y=Z+h2(Z) \quad (4a)$$

where,

$$h2^j(Z) = \sum_{j=1}^{N}\sum_{l=1}^{N} h2^j_{kl} z_k z_l \quad (4b)$$

$$h2^j_{kl} = \frac{C^j_{kl}}{\lambda_k + \lambda_l - \lambda_j} \quad (4c)$$

In Z-coordinates, the system (3a) takes on the form

$$\dot{z}_j = \lambda_j z_j \quad (5)$$

Equations (2)-(5) allow us to obtain explicit second order solutions for the system in the different coordinate systems:

$$z_j(t) = z_{jo} e^{\lambda_j t} \quad (6)$$

$$y_j(t) = z_{jo} e^{\lambda_j t} + \sum_{k=1}^{N}\sum_{l=1}^{N} h2^j_{kl} z_{ko} z_{lo} e^{(\lambda_k+\lambda_l)t} \quad (7)$$

$$x_i(t) = \sum_{j=1}^{N} u_{ij} z_{jo} e^{\lambda_j t} + \sum_{j=1}^{N} u_{ij} [\sum_{k=1}^{N}\sum_{l=1}^{N} h2^j_{kl} z_{ko} z_{lo} e^{(\lambda_k+\lambda_l)t}] \quad (8)$$

Furthermore, the solution of the linear part of (3a) is

$$\bar{y}_j(t) = y_{jo} e^{\lambda_j t} \quad (9)$$

The comparison of (6), (7), and (9) leads to the definition of the nonlinear interaction index I1 for mode j as in [5].

$$I1(j) = |(y_{jo} - z_{jo}) + \max_{k,l} h2^j_{kl} z_{ko} z_{lo}| \quad (10)$$

Comparing the linear part of (7) with (9) leads to the nonlinearity index I2 which is a measure of the relative size of the nonlinearity in the initial value, defined as

$$I2(j) = \frac{\max_{k,l} h2^j_{kl} z_{ko} z_{lo}}{z_{jo}} \quad (11)$$

The indices I1 and I2 will be used in the subsequent analysis, as well as the second order interaction coefficients of mode j, $h2^j_{kl} z_{ko} z_{lo}$ as they appear in equations (7) and (8).

Linear participation factors are a well-known method to find out mode-machine interactions [14]. The participation factor p_{ki} represents a measure of the participation of the k-th machine state in the trajectory of the i-th mode. It is given by

$$p_{ki} = u_{ki} * v_{ik} \quad (12)$$

Since linear participation factors are functions of both the left and right eigenvectors, they are independent of eigenvector scaling. Using normal forms we apply the concept of nonlinear participation factors [15]. The normal form initial conditions, using the second order approximation of the inverse transformation in (4a), can be used to express the solution for k-th machine state variable as

$$x_k(t) = \sum_{i=1}^{N} u_{ki}(v_{ik} + v2_{ikk}) e^{\lambda_i t} \quad (13)$$
$$+ \sum_{p=1}^{N}\sum_{q=p}^{N} u2_{kpq}(v_{pk} + v2_{pkk})(v_{qk} + v2_{qkk}) e^{(\lambda_p+\lambda_q)t}$$

where,

$$v2_{ipp} = -\sum_{k=1}^{N}\sum_{l=1}^{N} h2^i_{kl} v_{kp} v_{lp}$$

$$u2_{jkl} = \sum_{j=1}^{N} u_{ij} h2^j_{kl}$$

Using the approach given in [15], one can define second order participation factors according to

$$x_k(t) = \sum_{i=1}^{N} p2_{ki} e^{\lambda_i t} + \sum_{p=1}^{N} \sum_{q=p}^{N} p2_{kpq} e^{(\lambda_p + \lambda_q)t} \quad (14)$$

where,

$$p2_{ki} = u_{ki}(v_{ik} + v2_{ikk})$$

$$p2_{kpq} = u2_{kpq}(v_{pk} + v2_{pkk})(v_{qk} + v2_{qkk})$$

Note that there are two types of second order participation factors. The $p2_{ki}$ represents the second order participation of the k-th machine state in the i-th single-eigenvalue mode. In fact, the linear participation factor, p_{ki}, is one term in the expression for $p2_{ki}$ which includes the second order corrections. The $p2_{kpq}$ represents the second order participation of the k-th machine state in the 'mode' formed by the combination of the p and q modes. This is used in Table 7 in the part II to provide the participating states at the combination modes which have strong interactions with the critical modes.

2.3 Sensitivity Analysis

- Eigenvalue Sensitivity

In the power system (1) the vector field F depends on the gain constants Ka of the exciters present in the system, as do all the terms of the Taylor expansion in (2). Hence the eigenvalues of the system matrix A in (2) depend on Ka. Using again U_i as the i-th column vector of the eigenvector matrix U, and V^T_j as the j-th row vector of the left eigenvector matrix V^T, we obtain

$$\frac{\partial \lambda_i}{\partial Ka} = \frac{V_i^T (\frac{\partial A}{\partial Ka}) U_i}{V_i^T U_i} \quad (15)$$

as the sensitivity of the i-th eigenvalue with respect to Ka.

- Sensitivity of the Normal Form Coefficient

The normal form coefficient in (4c) provides the coefficient second order terms in the system. Using equations (3b) and (4c) we obtain for its sensitivity with respect to the exciter gain Ka as follows.

$$\frac{\partial h2^j_{kl}}{\partial Ka} = \frac{\frac{\partial C^j_{kl}}{\partial Ka}(\lambda_k + \lambda_l - \lambda_j) - C^j_{kl}(\frac{\partial \lambda_k}{\partial Ka} + \frac{\partial \lambda_l}{\partial Ka} - \frac{\partial \lambda_j}{\partial Ka})}{(\lambda_k + \lambda_l - \lambda_j)^2} \quad (16)$$

Both sensitivity quantities will be used in the sequel to analyze the dependence of the behavior of the critical modes on varying exciter gain settings.

2.4 Linear Gain Tuning

Linear gain tuning algorithms adhere roughly to the following procedure: The critical (low frequency interarea) inertial modes with poor damping are identified. For the corresponding eigenvalues the sensitivity with respect to the exciter gains is computed according to equation (15). The exciter gain is shifted so that the real part of the critical eigenvalues becomes more negative. As a measure of the appropriate shift the linear approximation of the eigenvalues as functions of the exciter gains is used. The linear approximation can be computed from the eigenvalue and from (15). A technique based on the linear analysis of the system neglects potentially important terms in the system response, compare (7) and (8). Therefore, it cannot always predict system behavior correctly (see [5] for e.g., analysis of system separation). Furthermore, such a technique does not always provide the mechanism by which control settings influence the inertial modes as we will see below. The choice of the most influential exciter and of the amount of gain shifting may lead to wrong settings, when based on linear analysis alone. Finally, optimal gain values for the linear system and for the nonlinear system may be different, due to differences in the linear and the nonlinear response behavior. Therefore we propose a control tuning technique based on second order normal forms. This method includes the effect on the second order terms. The procedure developed is general and can be extended to include higher order terms.

2.5 Procedure to Tune Gains

A. Determination of Critical Modes

1. Develop the system equations at the post disturbance Stable Equilibrium Point (SEP), conduct the disturbance simulation, perform the eigenanalysis and the normal forms calculation.
2. Identify the critical inertial modes and the control modes using the results of the eigenanalysis, and linear participation factors.
3. Apply the nonlinear interaction index (11) developed, and among the critical modes, identify those with the highest index Ii.
4. For the inertial modes identified by the nonlinear interaction index Ii, determine the control modes with the largest second order interaction. This is done by determining the magnitude of the second order interaction coefficient.
5. For the control modes identified, use nonlinear participation factors and identify the control states (in our case exciter states), which participate in these interacting modes.

B. Control Tuning Procedure.

6. Compute the sensitivity of the second order interaction coefficients of the identified inertial modes with respect to the control states (in our case exciter states) identified in step 5.
7. For the selected inertial modes and the corresponding interacting control modes, compute the linear eigenvalue sensitivity with respect to the gain of the exciter identified in step 5. If this sensitivity is positive (negative), adjust the

corresponding gain setting to lower (higher) values to obtain a more stable system. Also verify from the sensitivity of the second order terms that the nonlinearity is reduced by the change in control setting.

8. Among different exciter gain settings that result in similar stability behavior of the critical inertial modes, choose the one with lower nonlinearity index 12 for the critical inertial modes.

3. Conclusions and Discussions

This paper develops the analytical basis for tuning controls (exciter settings) in power systems using the nonlinear information provided by the method of normal forms. The technique developed is based on using indices developed in previous work [5], to identify control modes which interact nonlinearly with inertial modes. The concept of nonlinear participation factors, and sensitivity of the normal forms coefficient together with linear participation factors and eigenvalue sensitivity are used to vary control settings. The control settings are varied to obtain improved stability and to reduce the nonlinearity in the system. Detailed results on tuning the controller settings are provided in a companion paper [11].

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