

# 비선형 PSS 을 위한 NFL-H<sub>∞</sub>/SMC 의 설계 : Part B

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## NFL-H<sub>∞</sub>/SMC Design for Nonlinear PSS : Part B

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**[Abstract]** In this paper, the standard Dole, Glover, Khargoneker, and Francis (abbr. : DGKF 1989) H<sub>∞</sub> controller (H<sub>∞</sub>C) is extended to the nonlinear feedback linearization-H<sub>∞</sub>/sliding mode controller (NFL-H<sub>∞</sub>/SMC) to solve the problem associated with the full state feedback for the unmeasurable state variables in the conventional SMC, to obtain the smooth control as the linearized controller for a linear system (or to cancel the nonlinearity for the nonlinear system), and to improve the time-domain performance under worst case.

**Keywords** : nonlinear feedback linearization-H<sub>∞</sub>/sliding mode controller, power system stabilizer

### 1. Introduction

Under worst situation, the standard Dole, Glover, Khargoneker, and Francis (abbr. : DGKF 1989) H<sub>∞</sub> controller (H<sub>∞</sub>C) [1] is extended to the nonlinear feedback linearization-H<sub>∞</sub>/sliding mode controller (NFL-H<sub>∞</sub>/SMC). The proposed controller is obtained by combining the H<sub>∞</sub> estimator [1] with the nonlinear feedback linearization-sliding mode controller (NFL-SMC) and eliminates the need to measure all the state variables in the conventional SMC [2-18]. The effectiveness of the proposed controller is verified by the simulations in case of 3-cycle line-ground fault and in case of parameter variations.

### 2. Proposed NFL-H<sub>∞</sub>/SMC

The state equations under worst case based on

nonlinear feedback linearization (NFL) [20] are

$$z(t) = T(x(t)) \quad (1)$$

$$\dot{z}(t) = Az(t) + B_1 w_{\text{worst}}(t) + B_2 u(t) \quad (2)$$

$$p(t) = C_1 z(t) + D_{11} w_{\text{worst}}(t) + D_{12} u(t) \quad (3)$$

$$y(t) = C_2 z(t) + D_{21} w_{\text{worst}}(t) + D_{22} u(t) \quad (4)$$

where  $x \in R^n$ ,  $z \in R^n$ ,  $w_{\text{worst}} \in R^{m_1}$ ,  $u \in R^{m_2}$ ,  $p \in R^{p_1}$ ,  $y \in R^{p_2}$ ,  $A$  is the  $n \times n$  system matrix,  $B_1$  is the  $n \times m_1$  exogenous input matrix,  $B_2$  is the  $n \times m_2$  control matrix,  $C_1$  is the  $p_1 \times n$  regulated output matrix,  $C_2$  is the  $p_2 \times n$  output or measurement matrix,  $D_{11}$  is the  $p_1 \times m_1$  regulated direct feed-forward matrix,  $D_{12}$  is the  $p_1 \times m_2$  regulated direct feed-forward matrix,  $D_{21}$  is the  $p_2 \times m_1$  output direct feed-forward matrix, and  $D_{22}$  is the  $p_2 \times m_2$  output direct feed-forward matrix.

The H<sub>∞</sub> estimator state equation based on NFL is [1]

$$\dot{\hat{z}}(t) = A\hat{z}(t) + B_2 u(t) + B_1 \hat{w}_{\text{worst}}(t) + Z_w K_e (y(t) - \hat{y}(t)) \quad (5)$$

$$\text{where } \hat{w}_{\text{worst}}(t) = \gamma^{-2} B_1^T X_w \hat{z}(t) \quad (6)$$

$$\hat{y}(t) = [C_2 + \gamma^{-2} D_{21} B_1^T X_w] \hat{z}(t) \quad (7)$$

The controller gain  $K_c$  is given by

$$K_c = \tilde{D}_{12} (B_1^T X_w + D_{12}^T C_1) \quad (8)$$

$$\text{where } \tilde{D}_{12} = (D_{12}^T D_{12})^{-1} \quad (9)$$

The estimator gain  $K_e$  is given by

$$K_e = (Y_w C_1^T + B_1 D_{11}^T) \tilde{D}_{21} \quad (10)$$

$$\text{where } \tilde{D}_{21} = (D_{21} D_{21}^T)^{-1} \quad (11)$$

The term  $Z_w$  is given by

$$Z_w = (I - \gamma^{-2} Y_w X_w)^{-1} \quad (12)$$

The controller Riccati equation term  $X_w$  is

$$X_w = Ric \begin{bmatrix} A - B_1 \tilde{D}_{11} D_{11}^T C_1 & \gamma^{-1} B_1 B_1^T - B_1 \tilde{D}_{11} B_1^T \\ -\tilde{C}_1^T \tilde{C}_1 & -(A - B_1 \tilde{D}_{11} D_{11}^T C_1)^T \end{bmatrix} \quad (13)$$

$$\text{where } \tilde{C}_1 = (I - D_{12} \tilde{D}_{12} D_{12}^T) C_1 \quad (14)$$

The estimator Riccati equation term is

$$Y_w = Ric \begin{bmatrix} (A - B_1 \tilde{D}_{11} D_{11}^T C_1)^T & \gamma^{-1} C_1 C_1^T - C_1^T \tilde{D}_{11} C_1 \\ -\tilde{B}_1 \tilde{B}_1^T & -(A - B_1 \tilde{D}_{11} D_{11}^T C_1) \end{bmatrix} \quad (15)$$

$$\text{where } \tilde{B}_1 = B_1 (I - D_{21}^T \tilde{D}_{21} D_{21}) \quad (16)$$

$$\text{The estimated control input based on NFL is } u_c(t) = -K_c \hat{z}(t) \quad (17)$$

The internally stabilizing control gain is

$$K_{H_{sc}}(s) = \begin{bmatrix} A_1 & Z_w K_c \\ -K_c & 0 \end{bmatrix} \quad (18)$$

$$\text{where } A_1 := A - B_1 K_c - Z_w K_c C_1 + \gamma^{-2} (B_1 B_1^T - Z_w K_c D_{21} B_1^T) X_w \quad (19)$$

The closed loop system can be expressed as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & -B_1 K_c \\ Z_w K_c C_1 & A_2 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ Z_w K_c D_{21} \end{bmatrix} w_{worst}(t) \quad (20)$$

$$\begin{bmatrix} p(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_1 & -D_{12} K_c \\ C_2 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} w_{worst}(t) \quad (21)$$

$$\text{where } A_2 := A - B_1 K_c + \gamma^{-2} B_1 B_1^T X_w - Z_w K_c (C_2 + \gamma^{-2} D_{21} B_1^T X_w) \quad (22)$$

From equations (2) and (6), the state equation based on NFL can be expressed as

$$\begin{aligned} \dot{z}(t) &= Az(t) + B_1 w_{worst}(t) + B_2 u(t) \\ &= (A + B_1 (\gamma^{-2} B_1^T X_w)) z(t) + B_2 u(t) \end{aligned} \quad (23)$$

The switching surface vector and the differential switching surface vector can be expressed as

$$\sigma(z(t)) = G^T z(t) \quad (24)$$

$$\dot{\sigma}(z(t)) = G^T \dot{z}(t) \quad (25)$$

where  $G^T$  is the sliding surface gain [2-5,11].

$$\text{The Lyapunov's function candidate is chosen by } V(z(t)) = \sigma^2(z(t)) / 2 \quad (26)$$

The time derivative of  $V(z(t))$  can be expressed as

$$\begin{aligned} \dot{V}(z(t)) &= \sigma(z(t)) \dot{\sigma}(z(t)) \\ &= G^T z(t) G^T \dot{z}(t) = G^T z(t) G^T \end{aligned} \quad (27)$$

$$\left[ (A + B_1 (\gamma^{-2} B_1^T X_w)) z(t) + B_2 u_{NFL-H_{sc}}(t) \right] \leq 0 \quad (28)$$

The control inputs with switching function are

$$\begin{aligned} u_{NFL-H_{sc}}^+(t) &\geq -(G^T B_2)^{-1} \left[ G^T (A + B_1 (\gamma^{-2} B_1^T X_w)) z(t) \right] \\ &\text{for } G^T z(t) > 0 \end{aligned} \quad (29)$$

$$\begin{aligned} u_{NFL-H_{sc}}^-(t) &\leq -(G^T B_2)^{-1} \left[ G^T (A + B_1 (\gamma^{-2} B_1^T X_w)) z(t) \right] \\ &\text{for } G^T z(t) < 0 \end{aligned} \quad (30)$$

The control input with sign function is

$$u_{NFL-H_{sc}}^{sm}(t) = -(G^T B_2)^{-1} \left[ G^T (A + B_1 (\gamma^{-2} B_1^T X_w)) z(t) \right] \text{sign}(\sigma(z(t))) \quad (31)$$

$$\text{The above equation (31) can be reform as}$$

$$u_{NFL-H_{sc}}^{sm}(t) = -K_{w-SMC} z(t) \text{sign}(\sigma(z(t))) \quad (32)$$

$$K_{w-SMC} := (G^T B_2)^{-1} \left[ G^T (A + B_1 (\gamma^{-2} B_1^T X_w)) \right] \quad (33)$$

Finally, the estimated control input vector is

$$u_{NFL-H_{sc}/SMC}^{sm}(t) = -K_{w-SMC} \hat{z}(t) \text{sign}(\sigma(\hat{z}(t))) \quad (34)$$

The internally stabilizing control gain is

$$K_{H_{sc}/SMC}(s) = \begin{bmatrix} A_1 & Z_w K_c \\ -K_{w-SMC} \text{sign}(\sigma(\hat{z}(t))) & 0 \end{bmatrix} \quad (35)$$

$$\begin{aligned} \text{where } A_1 := & A - B_1 K_{w-SMC} \text{sign}(\sigma(\hat{z}(t))) - Z_w K_c C_1 \\ & + \gamma^{-2} (B_1 B_1^T - Z_w K_c D_{21} B_1^T) X_w \end{aligned} \quad (36)$$

The closed loop system can be expressed as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & -B_1 K_{w-SMC} \text{sign}(\sigma(\hat{z}(t))) \\ Z_w K_c C_1 & A_2 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ Z_w K_c D_{21} \end{bmatrix} w_{worst}(t) \quad (37)$$

$$\begin{bmatrix} p(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_1 & -D_{12} K_{w-SMC} \text{sign}(\sigma(\hat{z}(t))) \\ C_2 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} w_{worst}(t) \quad (38)$$

$$\begin{aligned} \text{where } A_2 := & A - B_1 K_{w-SMC} \text{sign}(\sigma(\hat{z}(t))) + \gamma^{-2} B_1 B_1^T X_w \\ & - Z_w K_c (C_2 + \gamma^{-2} D_{21} B_1^T X_w) \end{aligned} \quad (39)$$

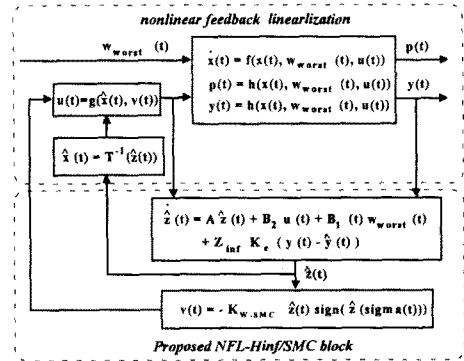


Fig. 1 Proposed NFL-H<sub>sc</sub>/SMC.

### 3. Nonlinear power system model

The d-axis current and the q-axis current are [19]

$$i_d(t) = c_1 e_q'(t) - c_2 (R_1 \sin \delta(t) + X_1 \cos \delta(t)) \quad (40)$$

$$i_q(t) = c_3 e_d'(t) - c_4 (-X_2 \sin \delta(t) + R_1 \cos \delta(t)) \quad (41)$$

$$c_1 = \frac{(C_1 X_1 - C_2 R_2)}{(R_1 R_2 + X_1 X_2)}, \quad c_2 = \frac{V_{inf}}{(R_1 R_2 + X_1 X_2)}$$

$$c_3 = \frac{(C_1 R_1 + C_2 X_2)}{(R_1 R_2 + X_1 X_2)}, \quad c_4 = \frac{V_{inf}}{(R_1 R_2 + X_1 X_2)}$$

$$Z_1 = R_1 + jX_1, \quad Z_2 = R_2 + jX_2, \quad Y = G + jB$$

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}, \quad 1 + Z_T Y = C_1 + jC_2, \quad C_1 = 1 + RG - XB$$

$$C_2 = XG + RB, \quad R_1 = R - C_2 x_d', \quad R_2 = R - C_2 x_q$$

$$X_1 = X + C_1 x_q, \quad X_2 = X + C_1 x_d'$$

$$v_d(t) = x_q i_q(t) \quad (42)$$

$$v_q(t) = e_q'(t) - x_d' i_d(t) \quad (43)$$

$$v_r^2(t) = v_d^2(t) + v_q^2(t) \quad (44)$$

$$T_r(t) \approx P_r(t) = i_d(t)v_d(t) + i_q(t)v_q(t)$$

$$= e_q'(t)i_q(t) + (x_q - x_d')i_d(t)i_q(t) \quad (45)$$

The nonlinear 4-th order state equations are [19]

$$\dot{\omega}(t) = \frac{1}{M} T_m - \frac{1}{M} T_r(t) \quad (46)$$

$$\delta(t) = \omega_o(\omega(t) - 1) \quad (47)$$

$$\dot{e}_q'(t) = -\frac{1}{T_{do}} e_q'(t) - \frac{(x_q - x_d')}{T_{do}} i_d(t) + \frac{1}{T_{do}} e_{fd}(t) \quad (48)$$

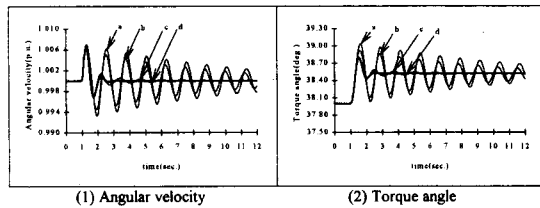
$$\dot{e}_{fd}(t) = -\frac{1}{T_A} e_{fd}(t) + \frac{K_A}{T_A} (V_{ref} - v_r(t) + u_E(t)) \quad (49)$$

$$e_{fd \min} \leq e_{fd} \leq e_{fd \max} \quad \text{and} \quad u_{E \min} \leq u_E \leq u_{E \max} \quad (50)$$

$$e_{fd \max} = 6.0 \quad e_{fd \min} = -6.0, \quad \text{and} \quad u_{E \min} = +0.2 \quad u_{E \max} = -0.2$$

## 4. Simulation

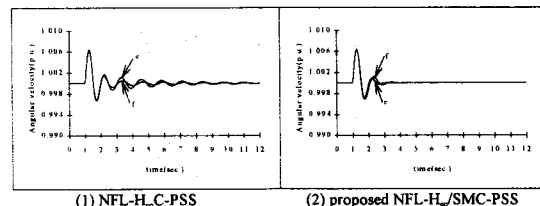
The proposed NFL-H<sub>∞</sub>/SMC-PSS in Fig. 2 exhibits better damping properties.



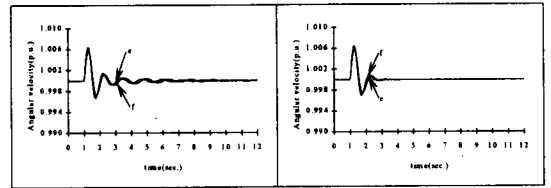
**Fig. 2** Normal load operation. (a : no control b : conventional PSS c : NFL-H<sub>∞</sub>-C-PSS d : proposed NFL-H<sub>∞</sub>/SMC-PSS)

### 2. Parameter variation test

The proposed NFL-H<sub>∞</sub>/SMC-PSS in Fig. 3 (2) and in Fig. 4 (2) exhibits better damping properties and is less sensitive to variations of AVR gain as compared to the NFL-H<sub>∞</sub>-C-PSS in Fig. 3 (1) and in Fig. 4 (1).



**Fig. 3** Angular velocity waveforms for AVR gain. (e : normal f : parameter variation)



**Fig. 4** Angular velocity waveforms for inertia moment. (e : normal f : parameter variation)

## 5. Conclusion

The effectiveness of the proposed controller has been verified by the nonlinear time-domain simulations in case of 3-cycle line-ground fault and in case of parameter variations for AVR gain  $K_A$  and for inertia moment  $M$ .

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