NFL-O/SMMFC 의 안정도 증명 : Part 3

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Stability Proof of NFL-O/SMMFC : Part 3

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[Abstract] This paper presents a stability proof for the nonlinear feedback linearization-observer/sliding mode model following controller (NFL-O/SMMFC). The separation principle is derived, and the closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.
Keywords : nonlinear feedback linearization-observer/sliding mode model following controller, Lyapunov function, separation principle, stability proof

1. Introduction

In this paper, to tackle the problem associated with the full state feedback [1-17], the nonlinear feedback linearization-observer/sliding mode model following controller (NFL-O/SMMFC) for unmeasurable plant state variables is developed. By the separation principle, the proposed NFL-O/SMMFC is obtained by combining the observer with the nonlinear feedback linearization-sliding mode model following controller (NFL-SMMFC). The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

2. NFL-O/SMMFC design

The NFL-based reference model state equation is [19]

$$z_m(t) = T(x_m(t)) \tag{1}$$

$$\dot{z}_m(t) = A_m z_m(t) + B_m u_m(t) \tag{2}$$

where $x_m \in R^n$ is the state vector for model, $z_m \in R^n$ is the transformed state vector for model, $u_m \in R^p$ is the control input for model, A_m is the $n \times n$ system matrix for model, and B_m is the $n \times p$ control vector for model.

The control input for a reference model is

$$u_m(t) = -K_m z_m(t) \tag{3}$$

$$K_m = R_m^{-1} B_m^T P_m \tag{4}$$

$$P_m A_m + A_m^T P_m - P_m B_m R_m^{-1} B_m^T P_m + Q_m = 0 \tag{5}$$

where K_m is a $p \times n$ optimal feedback gain for model, and P_m is the algebraic matrix Riccati equation.

The closed loop feedback system is

$$\dot{z}_m(t) = (A_m - B_m K_m) z_m(t) \tag{6}$$

$$A_{km} := A_m - B_m K_m \tag{7}$$

The NFL-based state equation for the reference model including CLF is reformed as

$$\dot{z}_m(t) = A_{km} z_m(t) \tag{8}$$

The control input for a controlled plant is

$$u_p(t) = -K_p z_p(t) \tag{9}$$

$$K_p = R_p^{-1} B_p^T P_p \tag{10}$$

$$P_p A_p + A_p^T P_p - P_p B_p R_p^{-1} B_p^T P_p + Q_p = 0 \tag{11}$$

The NFL-based state equation for the controlled plant and the output equation are formed as

$$z_p(t) = T(x_p(t)) \tag{12}$$

$$\dot{z}_p(t) = A_p z_p(t) + B_p u_p(t) \tag{13}$$

$$y_p(t) = C_p z_p(t) \tag{14}$$

where $x_p \in R^n$ is the state vector for plant, $z_p \in R^n$ is the transformed state vector for plant, $u_p \in R^p$ is the control input for plant, $y_p \in R^p$ is the available output measured for plant, A_p is the $n \times n$ system matrix for plant, B_p is the $n \times p$ control matrix for plant, and C_p is the $p \times n$ output matrix for plant.

The NFL-based observer equation is expressed as [18]

$$\dot{\hat{z}}_p(t) = A_p\hat{z}_p(t) + B_pu_p(t) + L_p(y_p(t) - C_p\hat{z}_p(t)) = (A_p - L_pC_p)\hat{z}_p(t) + B_pu_p(t) + L_py_p(t) \quad (15)$$

$$L_p = P_pC_p^TR_p^{-1} \quad (16)$$

$$A_pP_p + P_pA_p^T - P_pC_p^TR_p^{-1}C_pP_p + Q_p = 0 \quad (17)$$

where $\hat{z}_p \in R^n$ is the estimated state for plant based on NFL, L_p is the $n \times p$ output injection matrix for plant, P_p is the symmetric positive definite solution, and, Q_p and R_p are positive definite matrices.

The NFL-based state equation for the controlled plant including CLF is expressed as

$$\dot{z}_p(t) = (A_p - B_pK_p)z_p(t) \quad (18)$$

$$A_{kp} := A_p - B_pK_p \quad (19)$$

The NFL-based state equation for the controlled plant including CLF is reformed as

$$\dot{z}_p(t) = A_{kp}z_p(t) + B_pu_{cp}(t) \quad (20)$$

The error and the differential error equations are

$$e(t) = z_m(t) - z_p(t) \quad (21)$$

$$\dot{e}(t) = \dot{z}_m(t) - \dot{z}_p(t) \quad (22)$$

From equations (8), (20) and (22), we get

$$\dot{e}(t) = \dot{z}_m(t) - \dot{z}_p(t) = A_{km}z_m(t) - A_{kp}z_p(t) - B_pu_{cp}(t) \quad (23)$$

$$z_m(t) = e(t) + z_p(t) \quad (24)$$

Let us write the motion equation with respect to the error vector

$$\dot{e}(t) = A_{km}e(t) + [A_{km} - A_{kp}]z_p(t) - B_pu_{cp}(t) \quad (25)$$

The sliding surface vector and the differential sliding surface vector are expressed as

$$\sigma(e(t)) = G_{SS}^Te(t) = G_{SS}^Tz_m(t) - G_{SS}^Tz_p(t) \Rightarrow 0 \quad (26)$$

$$\dot{\sigma}(e(t)) = G_{SS}^T\dot{e}(t) = G_{SS}^TA_{km}e(t) + G_{SS}^T[A_{km} - A_{kp}]z_p(t) - G_{SS}^TB_pu_{cp}(t) \Rightarrow 0 \quad (27)$$

where G_{SS}^T is the sliding surface gain [1-4,10].

The Lyapunov's function candidate is chosen by

$$V(e(t)) = \sigma^2(e(t))/2 \quad (28)$$

The time derivative of equation (28) is given by

$$\dot{V}(e(t)) = \sigma(e(t))\dot{\sigma}(e(t)) = G_{SS}^Te(t)G_{SS}^T[A_{Km}e(t) + [A_{Km} - A_{kp}]z_p(t) - B_pu_{SMMFC}(t)] \le 0 \quad (29)$$

The equation (29) is represented as the control input with switching function

$$u_{SMMFC}^+(t) \ge (G_{SS}^TB_p)^{-1}G_{SS}^T[A_{km}e(t) + [A_{km} - A_{kp}]z_p(t)]$$

$$\text{for } G_{SS}^Te(t) > 0 \quad (30)$$

$$u_{SMMFC}^-(t) \le (G_{SS}^TB_p)^{-1}G_{SS}^T[A_{km}e(t) + [A_{km} - A_{kp}]z_p(t)]$$

$$\text{for } G_{SS}^Te(t) < 0 \quad (31)$$

The control input vector with sign function is simplified as follows:

$$u_{SMMFC}^{sign}(t) = [E_{SMMFC}^{equal}e(t) + P_{SMMFC}^{equal}z_p(t)]sign(\sigma(e(t))) \quad (32)$$

subject to $sign(\sigma(e(t))) = 1$ for $\sigma(e(t)) > 0$

$sign(\sigma(e(t))) = 0$ for $\sigma(e(t)) = 0$

$sign(\sigma(e(t))) = -1$ for $\sigma(e(t)) < 0$

where

$$E_{SMMFC}^{equal} := (G_{SS}^TB_p)^{-1}G_{SS}^TA_{Km} \quad (33)$$

$$P_{SMMFC}^{equal} := (G_{SS}^TB_p)^{-1}G_{SS}^T(A_{Km} - A_{kp}) \quad (34)$$

where E_{SMMFC}^{equal} is an sliding mode-model following control-equal error feedback gain, and P_{SMMFC}^{equal} is a sliding mode-model following control-equal plant feedback gain.

Finally, the *estimated control input vector* with sign function is simplified as follows:

$$\hat{u}_{O/SMMFC}^{sign}(t) = [E_{SMMFC}^{equal}e(t) + P_{SMMFC}^{equal}\hat{z}_p(t)]sign(\sigma(e(t))) \quad (35)$$

Theorem 1: Consider the state equations of the reference model and of the controlled plant based on NFL for the regulation problem and the observer state equation based on NFL

$$\dot{z}_m = A_{km}z_m \text{ and } y_m = C_mz_m$$

$$\dot{z}_p = A_{kp}z_p + B_p\hat{u}_{O/SMMFC}^{sign} \text{ and } y_p = C_pz_p$$

$$\dot{\hat{z}}_p = A_p\hat{z}_p + B_p\hat{u}_{O/SMMFC}^{sign} + L_p(y_p - C_p\hat{z}_p)$$

Consider $G_{SS}^TB_p(G_{SS}^TB_p)^{-1} = I$, $y_p = C_pz_p$, $e = z_m - \hat{z}_p$, and $z_p = e_p + \hat{z}_p$. Suppose that (A_p, C_p) is detectable and $(A_p - L_pC_p)$ is Hurwitz. The estimated sliding mode model following control law with sign function based on NFL is guaranteed an *asymptotically stable* for the system (13)

$$\hat{u}_{O/SMMFC}^{sign} = [E_{SMMFC}^{equal}e + P_{SMMFC}^{equal}\hat{z}_p]sign(\sigma(e))$$

$$E_{SMMFC}^{equal} := (G_{SS}^TB_p)^{-1}G_{SS}^TA_{Km}$$

$$P_{SMMFC}^{equal} := (G_{SS}^TB_p)^{-1}G_{SS}^T(A_{Km} - A_{kp})$$

subject to $sign(\sigma(e)) = 1$ for $\sigma(e) > 0$

$sign(\sigma(e)) = 0$ for $\sigma(e) = 0$

$sign(\sigma(e)) = -1$ for $\sigma(e) < 0$

Proof. Let us define the error equation and the differential error equation

$$e_p = z_p - \hat{z}_p$$

$$\dot{e}_p = \dot{z}_p - \dot{\hat{z}}_p$$

$$= A_{kp}z_p + B_p\hat{u}^{sign}_{O/SMMFC} - A_{kp}\hat{z}_p - B_p\hat{u}^{sign}_{O/SMMFC} - L_pC_pz_p + L_pC_p\hat{z}_p$$
$$= A_{kp}z_p - A_p\hat{z}_p - L_pC_pz_p + L_pC_p\hat{z}_p = \left(A_{kp} - L_pC_p\right)e_p$$

Lyapunov's function candidate using the addition form of the plant siding surface gain σ_p, and the plant error e_p is chosen by

$$V = \frac{1}{2}\sigma^T(e)\sigma(e) + \frac{1}{2}e_p^Te_p$$

The derivative of a Lyapunov's function candidate is obtained by

$$\dot{V} = \sigma^T(e)\dot{\sigma}(e) + e_p^T\dot{e}_p$$
$$= \sigma^T(e)\left(G_{SS}^TA_{km}e + G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p - G_{SS}^TB_p\hat{u}_{O/SMMFC}\right)$$
$$+e_p^T\left(A_{kp} - L_pC_p\right)e_p$$
$$= \sigma^T(e)\left(G_{SS}^TA_{km}e + G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p\right.$$
$$\left.-\left(G_{SS}^TB_pE^{equal}_{SMMFC}e + G_{SS}^TB_pP^{equal}_{SMMFC}\hat{z}_p\right)sign(\sigma(e))\right)$$
$$+e_p^T\left(A_{kp} - L_pC_p\right)e_p$$

Let us define $E^{equal}_{SMMF} := \left(G_{SS}^TB_p\right)^{-1}G_{SS}^TA_{Km}$, and

$$P^{equal}_{SMMF} := \left(G_{SS}^TB_p\right)^{-1}G_{SS}^T\left(A_{km} - A_{kp}\right)$$

Therefore,

$$\dot{V} = \sigma^T(e)\left(G_{SS}^TA_{km}e + G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p\right.$$
$$-G_{SS}^TB_p\left(\left(G_{SS}^TB_p\right)^{-1}G_{SS}^TA_{Km}\right)e \;\; sign(\sigma(e))$$
$$\left.-G_{SS}^TB_p\left(\left(G_{SS}^TB_p\right)^{-1}G_{SS}^T\left(A_{km} - A_{kp}\right)\right)\hat{z}_p sign(\sigma(e))\right)$$
$$+e_p^T\left(A_{kp} - L_pC_p\right)e_p$$

Consider $G_{SS}^TB_p\left(G_{SS}^TB_p\right)^{-1} = I$, $y_p = C_pz_p$, $e = z_m - \hat{z}_p$, and $z_p = e_p + \hat{z}_p$

$$\dot{V} = \sigma^T(e)\left(G_{SS}^TA_{km}e + G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p \;\; -G_{SS}^TA_{Km}e \;\; sign(\sigma(e))\right.$$
$$\left.-G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p sign(\sigma(e))\right) + e_p^T\left(A_{kp} - L_pC_p\right)e_p$$
$$= \sigma^T(e)\left(G_{SS}^TA_{km}e + G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p \;\; -G_{SS}^TA_{Km}e \;\; sign(\sigma(e))\right.$$
$$\left.-G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p sign(\sigma(e))\right) + e_p^T\left(A_{kp} - L_pC_p\right)e_p$$
$$= \sigma^T(e)G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p \;\; -\sigma^T(e)G_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p sign(\sigma(e))$$
$$+\sigma^T(e)G_{SS}^TA_{km}e \;\; -\sigma^T(e)G_{SS}^TA_{Km}e \;\; sign(\sigma(e))$$
$$+e_p^T\left(A_{kp} - L_pC_p\right)e_p$$
$$= \sigma^T(e)G_{SS}^T\left(A_{km} - A_{kp}\right)\left(1 - sign(\sigma(e))\right)\hat{z}_p \;\; +\sigma^T(e)G_{SS}^TA_{km}e$$
$$-\sigma^T(e)G_{SS}^TA_{Km}e \;\; sign(\sigma(e)) \;\; +e_p^T\left(A_{kp} - L_pC_p\right)e_p$$

If $\left(A_{kp} - L_pC_p\right)$ is stable, the error is $e_p \to 0$, and $e \to 0$ as $t \to 0$.

$$\dot{V} = \sigma^T(e)G_{SS}^T\left(A_{km} - A_{kp}\right)\left(1 - sign(\sigma(e))\right)\hat{z}_p \le 0$$

subject to if $\sigma(e) > 0$, $\dot{V} = 0$

if $\sigma(e) = 0$, $\dot{V} = 0$

if $\sigma(e) < 0$, $\dot{V} = -2kG_{SS}^T\left(A_{km} - A_{kp}\right)\hat{z}_p < 0$

, k is positive constant.

i.e., $\dot{V} < 0$ and so the system is *asymptotically stable*. This completes the proof of this theorem.□

3. Conclusion

A separation principle and a stability proof of a nonlinear feedback linearization-observer/sliding mode model following controller (NFL-O/SMMFC) have been done.

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