Auxiliary Problem Principle 알고리즘에 기초한 최적 조류 계산의 분산 처리 기법에 관한 연구

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Distributed Implementation of Optimal Power Flow (OPF) Based on Auxiliary Problem Principle

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Abstract - We present an approach to parallelizing optimal power flow (OPF) that is suitable for distributed implementation and is applicable to very large interconnected power systems. The objective of this paper is to find a set of control parameters with which the Auxiliary Problem Principle (Algorithm - APP) can be best implemented in solving optimal power flow (OPF) problems. We employed several IEEE Reliability Test Systems to demonstrate the alternative parameter sets.

1. Introduction

The OPF problem seeks to find an optimal profile of active and reactive generations along with voltage magnitudes in such a manner as to minimize a given objective function, while satisfying network security constraints.

Recently, forces such as increasing competition, interests in deregulation, and the advent of new planning strategies have put pressure on electric utilities to become more efficient. So, the requirement for faster and more accurate solutions has encouraged the consideration of parallel implementations using decentralized processors, which can potentially greatly increase the available computational capacity.

In this paper, we apply a mathematical decomposition coordination method - Auxiliary Problem Principle - to implement distributed optimal power flow. We also propose the Dummy Generator - Dummy Generator (DGDG) scheme [1].

In DGDG scheme, dummy generators are put on the duplicated border buses in one region and the other region to represent the power flow between a pair of interconnected regions. We interpret a positive output as injected power to the border bus, while negative output is interpreted as a demand at the border bus where the dummy generator is connected. It is noted that the dummy generators in one region and the other region are not both expected to produce or consume power at the same time: if a dummy generator in one region produces power, then the corresponding dummy generator in the other region is supposed to consume positive power in equal amount at the same production cost so that the optimal solution is not affected by the dummy generators.

In our scheme, there is no need for a uniform implementation of OPF across all utilities, nor any need for all the utilities in the system to run full OPFs, so long as each region can represent dummy generators in its OPF or economic dispatch (ED). The parallel OPF can be implemented across a multi-utility system without major disruption to existing OPF or ED investments by individual utilities.

We present the basic mathematical concepts of parallelizing optimal power flow (OPF), and demonstrate the results on several IEEE Reliability Test Systems.

2. Regional Decomposition

2.1 Illustration

To illustrate the regional decomposition, we will consider dividing a power system into two overlapping regions.

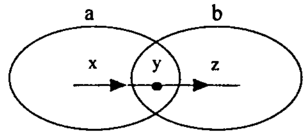

Fig.1 Decomposition of a power system

Consider Figure 1, which shows the case of a single tie-line joining regions a and b. Between and common to the two regions there is an overlap region, with a vector of variables denoted by y.

For each tie-line we must include a bus in the border region. If there is no bus already there, we create a "dummy bus". Associated with each dummy bus are the real and reactive power flows through the bus and the voltage and angle at the bus. That is, the vector y has four entries for each tie-line.

In summary, in Figure 1 region a has state vector (x, y), while region b has state vector (y, z). The y variables are the border variables, while x and z can be thought of as core variables for regions a and b. In typical systems, the vector y will be much smaller than the vectors x and z.

2.1.1 Objective

To analyze the decomposed system, we assume that the production costs for the whole system can be written formally as :

$$\min \{ C_a(x) + C_b(z) \} \tag{1}$$

2.1.2 Constraints

We assume that the constraints on the system involve x and y or y and z, but not x and z nor x, y and z. This assumption is reasonable for the power equations, since the bus admittance matrix couples only those variables pertaining to buses that are connected by a line [3].

With this assumption, we can write the power flow constraints for region a in the form $F_a(x, y)=0$ and for region b in the form $F_b(y, z)=0$.

Similarly, we can write the inequality constraints for region a in the form $G_a(x, y)\le 0$ and for region b in the form $G_b(y, z)\le 0$. The functions G_a and G_b represent the line flow, and voltage constraints in the individual regions a and b, respectively.

2.2 Auxiliary Problem Principle (APP)

First, define the copies of y to be y_a and y_b, assigned to the regions a and b, respectively. Then, for $\gamma\ge 0$, (1) is equivalent to :

$$\min \{ C_a(x) + C_b(z)+ \frac{\gamma}{2}\|y_a - y_b\|^2 : y_a - y_b = 0 \} \tag{2}$$

The quadratic term added to the objective does not affect the solution since the constraint $y_a - y_b = 0$ will make the quadratic term equal to zero at any solution. Next we apply the "auxiliary problem principle" [4]. Under certain conditions, we can solve (2) by solving a sequence of problems of the form :

$$(x^{k+1}, y_a^{k+1}, y_b^{k+1}, z^{k+1}) = \operatorname{argmin} \left(\begin{array}{l} C_a(x)+C_b(z)+ \\ \frac{\beta}{2}\|y_a - y_a^k\|^2 + \frac{\beta}{2}\|y_b - y_b^k\|^2 + \\ \gamma(y_a - y_b)^{\dagger}(y_a^k - y_b^k) + \\ \lambda^{k'}(y_a - y_b) \end{array} \right), \tag{3}$$

$$\lambda^{k+1} = \lambda^k + \alpha(y_a^k - y_b^k) \tag{4}$$

where the superscript k indicates the iteration index, α and β are positive constants, and the superscript † denotes transpose.

For the purpose of distributing computations, the important thing to note is that problem (3) separates into smaller problems for regions a and b, respectively, as follows :

$$(x^{k+1}, y_a^{k+1}) = \operatorname{argmin}\left(C_a(x) + \frac{\beta}{2}\|y_a - y_a^k\|^2 + \gamma y_a^{\dagger}(y_a^k - y_b^k) + \lambda^{k'} y_a \right) \tag{5}$$

$$(y_b^{k+1}, z^{k+1}) = \operatorname{argmin}\left(C_b(z) + \frac{\beta}{2}\|y_b - y_b^k\|^2 - \gamma y_b^{\dagger}(y_a^k - y_b^k) - \lambda^{k'} y_b \right) \tag{6}$$

which are essentially OPF problems for regions a and b including their own border. The second through fourth terms in the objective of (5) constitute the cost function of the dummy generators in region a. The costs are quadratic and depend on the values of the Lagrange multipliers as well as on previous values of the iterates. A similar interpretation applies for the terms in (6) for region b.

Then, the problems for regions a and b can be solved by two decentralized processors. The Lagrange multipliers in (4) are the price coordination signals between the regions.

3. Results

3.1 Case Study

We implemented the parallel AC OPF using GAMS 2.25 with the MINOS package [2]. Data from IEEE Reliability Test Systems were used to demonstrate the performance of the algorithm. Table 1 summaries the test systems.

Buses	Regions	Core Buses	Ties	Lines
50	2	24,24	2	80
78	3	24,24,24	6	126
108	4	24,24,24,24	12	186

Table 1 Case Study Systems

3.2 Test Results

The rate of convergence is dependent on the system and on the parameters α, β and γ. These parameters were tuned for each system to minimize the number of iterations required to satisfy the stopping criterion. We chose the maximum mismatch between the border variables as the stopping criterion. Typically, the mismatches on most buses were much smaller than 0.01 per unit.

The relationship between parameters and the number of iterations is shown in Figures 2, 3 and 4. Figure 2 shows the case for 2-regions, Figure 3 shows the case for 3-regions, and Figure 4 shows the case for 4-regions.

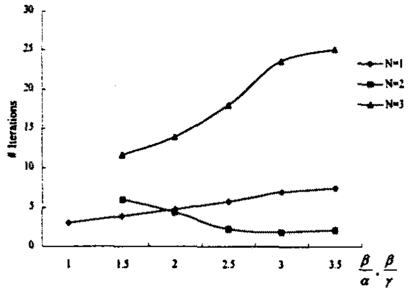

Fig.2 Number of iterations versus parameters for 2-regions (N : Number of tie-lines)

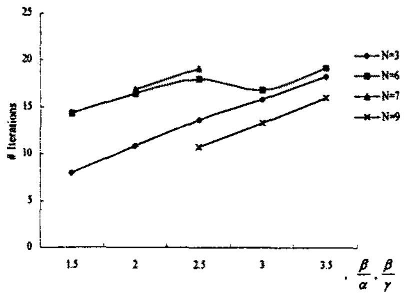

Fig.3 Number of iterations versus parameters for 3-regions (N : Number of tie-lines)

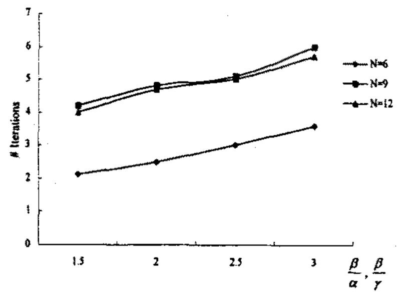

Fig.4 Number of iterations versus parameters for 4-regions (N : Number of tie-lines)

In case of 3-regions, we can see that for the purpose of enhancing the convergence rate, the values of β/α and β/γ must be larger with increasing the number of tie-lines as can be observed in Figure 3. In case of 4-regions, we can again note from Figure 4 that the values of β/α and β/γ must be 1.5 to improve the convergence rate, which is independent of the number of tie-lines.

In Algorithm - APP, our simulations show that the choice of $\alpha=\gamma$ seems the best among other possible combinations.

Table 2 summaries the results on several IEEE Reliability Test Systems.

# area	2 - regions			3 - regions				4 - regions		
# ties	1	2	3	3	6	7	9	6	9	12
β/α	1.0	3.0	1.5	1.5	1.5	2.0	2.5	1.5	1.5	1.5
β/γ	1.0	3.0	1.5	1.5	1.5	2.0	2.5	1.5	1.5	1.5

Table 2 Relationship with parameters versus the number of tie-lines, and regions

4. Conclusion & Future Study

We have tuned parameters versus the number of tie-lines and regions to improve the convergence rate of the Algorithm - APP.

Our next step is to derive the general rule by which we can determine the relationship with parameters as a function of the number of tie-lines and/or as a function of the number of regions and core buses. The relevance of the general rule will be further discussed after the experimental verifications and the mathematical derivations.

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