새로운 스위칭 평면을 이용한 강인한 최적 제어기의 설계

박 승 규, 안 호 균, 김 태 원
창원대학교 전기공학과

Robust Controller Design with Novel Sliding Mode Surface - Linear Optimal Control Case

SEUNG KYU PARK, HO KYUN AHN, TAE WON KIM
Department of Electrical Engineering, Changwon National University

Abstract - In this paper, a novel sliding surface is proposed by introducing a virtual state. This sliding surface has nominal dynamics of an original system and makes it possible that the Sliding Mode Control(SMC) technique is used with the various types of controllers. Its design is based on the augmented system whose dynamics have one higher order than that of the original system. The reaching phase is removed by using an initial virtual state which makes the initial switching function equal to zero.

1. Introduction

The SMC is a popular robust control method which has many good results and its applications[1], however it has the reaching phase problem and the input chattering problem[2][3]. Besides these two problems, the SMC is very conservative to be used with other controller design methods because the state trajectories of SMC system is determined by the sliding mode dynamics which can not have the same order dynamics of the original system. To overcome this conservatism and remove the reaching phase, a novel virtual state is defined based on the controllable canonical form of the nominal system. With this virtual state, an augmented system is constructed and a novel sliding mode surface is proposed. This makes it possible that the new sliding mode has the dynamics of the nominal system which is the desired dynamics controlled by various types of control strategies. In this paper, an optimal controller is considered as a nominal one and sliding mode has the dynamics of the nominal optimal control system. By using an initial virtual state which makes the initial switching function equal zero, the reaching phase is removed.

2. Problem Statement

Consider the n-th order system described by

$$\dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)u(t) + Df(t) \quad (1)$$

where $x \in R^n$, $u \in R$, $f \in R^r$ and the bounded uncertainties $\Delta A, \Delta B$ and the disturbance matrix D satisfy he following matching condition.

$$rank([B \vdots \Delta A \vdots \Delta B \vdots D]) = rank B \quad (2)$$

The existing sliding mode surfaces have the following form[4].

$$s(x,t) = c_n(t)x_n + c_{(n-1)}(t)x_{(n-1)} + \cdots c_1(t)x_1 + c_0(t) = 0 \quad (3)$$

where $c_0(t), c_1(t) \cdots c_n(t)$, are given so that sliding mode dynamics can be stable.

The above sliding surface has (n-1)-th order dynamics which are not the same as the n-th order dynamics of the original system. The reaching phase exists when the initial $s(x,t)$ is not zero.

The following condition guarantees the sliding mode[1].

$$s(x,t)\,\dot{s}(x,t) < 0 \quad (4)$$

Using a number of existing techniques[2], the above condition can be satisfied by solving for the functionals $u^+(\cdot)$ and $u^-(\cdot)$ of the following feedback control which is discontinuous on the surface defined by $s(x,t)$

$$u(\cdot) = \begin{cases} u^+(\cdot), & \text{for } s>0 \\ u^-(\cdot), & \text{for } s<0 \end{cases} \quad (5)$$

The problems to be solved in this paper is as follows.

- to overcome the conservatism of SMC by using a novel sliding mode surface which has the same dynamics of the nominal original system controlled by a nominal controller.
- to remove reaching phase.

3. SMC with New sliding surface

Various types of sliding surfaces have been proposed including time-varying sliding mode surface[5]. These existing sliding mode surface can not have the dynamics of the original system controlled by other type of controller. This makes the conventional SMC very conservative to be combined with the other types of control strategies. To overcome this conservatism completely, a new SMC, with a novel sliding mode surface which has the dynamic of the nominal original system controlled by nominal controller, is proposed. The novel sliding surface is designed based on the augmented system which has a virtual state. The virtual state is defined from the controllable canonical form of the nominal system.

Let's consider the following nominal system for the original system of Eq.(1).

$$\dot{x}_o(t) = Ax_o(t) + Bu_o(t) \quad (6)$$

where $u_o(x_o(t),t)$ is a nominal regulating control input and differentiable.

The novel virtual state is defined based on the following controllable canonical form for the above system.

$$\dot{z}_o(t) = A_c z_o(t) + B_c u_o(t) \quad (7)$$

$$\text{with } A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \vdots & \\ -a_1 & a_2 & & \cdots & -a_n \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

A novel virtual state z_{ov}, which is proposed in this paper, is defined as a derivative of z_{on} and its dynamic is

$$\dot{z}_{ov}(t) = -a_n z_{ov}(t) \cdots - a_2 z_{o3}(t) - a_1 z_{o2}(t) + \dot{u}_o(x_o, t) \quad (8)$$

This is the differential form of the last equation of Eq.(7). From the above equation, the following novel virtual state z_v will be defined by replacing nominal state z_o with original state z.

A novel virtual state is defined to have the following dynamic.

$$\dot{z}_v(t) = -a_n z_v(t) \cdots - a_2 z_3(t) - a_1 z_2(t) + \dot{u}_o(x,t) \quad (9)$$

where $u_o(x,t)$ and $\dot{u}_o(x,t)$ are obtained from $u_o(x_o,t)$ and $\dot{u}_o(x_o,t)$ respectively by replacing nominal state x_o with original state x.

Any uncertainty must not be considered in this

procedure.
With the novel virtual state, the augmented system is constructed as follows.

$$\dot{x}(t)=(A+\Delta A)x(t)+(B+\Delta B)u(t)+Df(t)$$
$$\dot{z}_v(t)=-a_n z_v(t)\cdots-a_2 z_3(t)-a_1 z_2(t)+\dot{u}_o(x,t) \quad (10)$$

where $u(t)$ guarantees the sliding mode on the sliding mode surface.
For the above augmented system, the new sliding mode surface is defined as

$$s(z,z_v)=z_v(t)+a_n z_n(t)\cdots+a_1 z_1(t)-u_o(x,t)=0 \quad (11)$$

The following initial virtual state makes the intial value of $s(z,z_v)$ equal to zero.

$$z_v(t_0)=-a_n z_n(t_0)\cdots-a_1 z_1(t_0)+u_o(t_0) \quad (12)$$

This removes the reaching phase completely.
Now the following theorem is obtained.

Theorem 1. The novel sliding mode surface $s(x,z_v)$ of Eq.(11) has the same dynamics as those of the nominal system of Eq.(6) and Eq.(7) controlled by nominal control input.

Proof) Suppose $z_{o1}, z_{o2} \ldots z_{on}, z_{ov}$ are on the sliding surface where $\begin{bmatrix} z_{o1} \\ z_{o2} \\ \vdots \\ z_{on} \end{bmatrix}=P\begin{bmatrix} x_{o1} \\ x_{o2} \\ \vdots \\ x_{on} \end{bmatrix}$.

Then the following equation is satisfied.

$$z_{ov}(t)+a_n z_{on}(t)\cdots+a_1 z_{o1}(t)-u_o(x_o,t)=0 \quad (13)$$

Set $z_{o2}=\dot{z}_{o1},\cdots,z_{on}=\dot{z}_{o(n-1)}$. (14)

By differentiating Eq.(13),

$$\dot{z}_{ov}(t)+a_n\dot{z}_{on}(t)+a_{n-1}\dot{z}_{o(n-1)}(t)\cdots+a_1\dot{z}_{o1}(t)-\dot{u}_o(x_o,t)$$
$$\dot{z}_{ov}(t)+a_n\dot{z}_{on}(t)+a_{n-1}z_{on}(t)\cdots+a_1 z_{o2}(t)-\dot{u}_o(x_o,t)=0 \quad (15)$$

is obtained.
According to Eq.(10), z_{ov} has the following dynamic.

$$\dot{z}_{ov}(t)=-a_n z_{ov}(t)\cdots-a_2 z_{o3}(t)-a_1 z_{o2}(t)+\dot{u}_o(x_o,t) \quad (16)$$

From Eq.(15) and Eq.(16),

$$z_{ov}=\dot{z}_{on} \quad (17)$$

Now the following is obtained from the Eq.(13).

$$\dot{z}_{on}(t)=-a_n z_{on}(t)\cdots-a_2 z_{o2}(t)-a_1 z_{o1}(t)+u_o(x_o,t) \quad (18)$$

Eq.(18) and Eq.(14) are the canonical form of the nominal system. It is transformed to the Eq.(6) by the transformation $x_o(t)=P^{-1}z_o(t)$. Therefore the novel sliding mode surface $s(x,z_v)$ has the same dynamics as that of the nominal system.
End of Proof

From Theorem 1 mentioned above and SMC theory, the following result is obtained.

Theorem 2. If SMC input $u(t)$ is designed to force the states of the system onto the sliding surface $s(z,z_v)$, then the states $x(t)$ follow the trajectories of the nominal system controlled by $u_o(x,t)$.

Proof) It is obvious from the Theorem 1 and SMC theory.

Note that the nominal control input $u_o(x,t)$ can be any type of control input and this makes it possible that the SMC is used with the various type of controllers. This means that the conservatism of the SMC is removed.

4. Robust Optimal Control using the Nov Mode Surface

Using an optimal controller as a nominal control input $u_o(x,t)$, a robust optimal controller, which makes the states follow the optimal trajectories in spite of parameter uncertainties, can be defined. For a clear explanation, let's consider the following second order system. This can be extended to the n-th order system without loss of generality.

$$\dot{x}(t)=(A+\Delta A)x(t)+Bu(t) \quad (19)$$

where $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $\Delta A=\begin{bmatrix} \Delta a_{11} & \Delta a_{12} \\ \Delta a_{21} & \Delta a_{22} \end{bmatrix}$
and $\Delta a_{ij}<\Delta_{ij}$ (constant).
Its nominal system is as follows.

$$\dot{x}_o(t)=Ax_o(t)+Bu_o(x_o,t) \quad (20)$$

Performance index for the above system is given by

$$J=\int_{t_0}^{\infty}\frac{1}{2}(x_o^T Q x_o+ru_o^2)dt \quad (21)$$

The optimal control input for the nominal system is [5].

$$u_o^*(x_o)=-\frac{1}{r}[\,b_1\ \ b_2\,]Sx_o(t)=-[\,k_1\ \ k_2\,]x_o(t) \quad (22)$$

where S is the solution of the following Riccati equation.

$$-SA-A^TS-Q+\frac{1}{r}SB^TBS=0 \quad (23)$$

The nominal system with this optimal control input is

$$\dot{x}_o(t)=Ax_o(t)+Bu_o^*(x_o) \quad (24)$$

$\dot{u}_o^*(x_o)$ is calculated as follows.

$$\dot{u}_o^*=-K(Ax_o+Bu^*(t))$$
$$=k_3x_{o1}+k_4x_{o2} \quad (25)$$

where $k_3=-k_1(a_{11}-b_1k_1)-k_2(a_{21}-b_2k_1)$
$k_4=-k_1(a_{12}-b_1k_2)-k_2(a_{22}-b_2k_2)$
The following canonical system is obtained by a state transformation $z_o=\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}x_o$.

$$\dot{z}_o(t)=\begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}z_o(t)+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u_n^*(x_o) \quad (26)$$

According to Eq.(9), the virtual state z_v is defined as

$$\dot{z}_v=-a_1z_2-a_2z_v+\dot{u}_o^*(x)$$
$$=(-a_1p_{21}+k_3)x_1+(-a_1p_{22}+k_4)x_2-a_2z_v \quad (27)$$

The augmented system is constructed as follows.

$$\dot{x}(t)=(A+\Delta A)x(t)+Bu(x)$$
$$\dot{z}_v(t)=-a_1p_{21}x_1(t)-a_1p_{22}x_2(t)-a_2z_v(t)+\dot{u}_o^*(x) \quad (28)$$

For the above system, the proposed novel sliding surface is given by

$$s=z_v+a_1z_1+a_2z_2-u_o^*(x)$$
$$=z_v+k_5x_1+k_6x_2=0 \quad (29)$$

where $k_5=-a_1p_{11}-a_2p_{21}+k_1$
$k_6=-a_1p_{12}-a_2p_{22}+k_2$
Sliding mode control input $u(t)$ is given by

$$u(t)=k_9x_1+k_{10}x_2+k_{11}z_v \quad (30)$$

where k_9, k_{10}, k_{11} are variable gains which guarantee the sliding mode.

5. Numerical Examples and Simulation Results

Consider the following second order system.

$$\dot{x}_1(t)=(-1+\Delta a_1)x_1(t)+u(t)$$
$$\dot{x}_2(t)=\Delta a_1x_1(t)-2x_2(t)+u(t)$$

where $|\Delta a_1|<3$
Performance index is given as follows.

$$J=\int_{t_0}^{\infty}\frac{1}{2}(x_o^T\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}x_o+u_o^2)dt$$

The optimal control input for the nominal system is

$$u_o^*(x_o)=-2.2845x_{o1}-0.1392x_{o2}$$

From Eq.(27) z_v is defined by

$$\dot{z}_v=9.8215x_1-3.3843x_2-3z_v$$

SMC control input is obtained as follows.

$$u=(-1.5742x_1-4.0005sgn(x_1s))x_1+2.3777x_2+0.5532z_v$$

where $s = z_v + 1.2845x_1 + 4.1392x_2$

The simulation results are shown in the following figures. Fig.1 shows that the reaching phase is removed. The optimal trajectories of x_1 and x_2 without parameter uncertainties are presented in Fig.2. Fig.3 shows the state trajectories of x_1 and x_2 controlled by optimal controller with uncertainties. They are not optimal trajectories any more. The state trajectories of x_1 and x_2 controlled by new SMC with parameter uncertainties are shown in Fig.4. In Fig.2, Fig.4, it is shown that the proposed sliding mode has the nominal dynamics of the optimal control system. This means that the trajectories of the system controlled by new SMC follow the nominal l trajectories in spite of parameter uncertainties. Fig.5 shows the trajectory of x_1 and x_2. Fig.6 is for the control input. Fig.7 is the value of switching function $s(x, z_v)$.

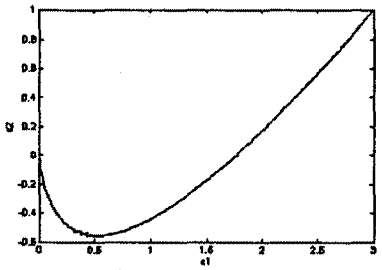

Fig.1 Phase trajectory of sliding mode design using the novel sliding mode surface

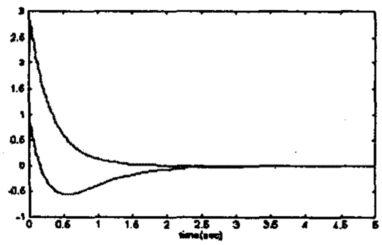

Fig.2 The optimal trajectories of x_1 and x_2 without uncertainties

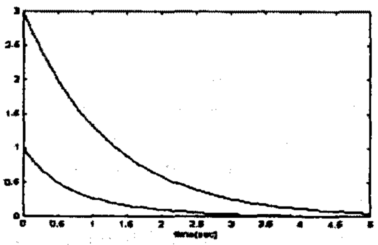

Fig.3 The state trajectories of x_1 and x_2 controlled by the optimal controller with uncertainties

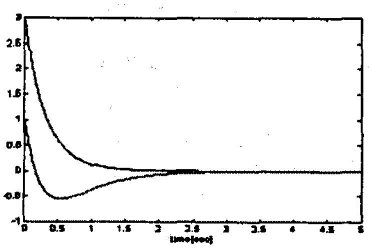

Fig.4 The state trajectories of x_1 and x_2 controlled by the new SMC with uncertainties

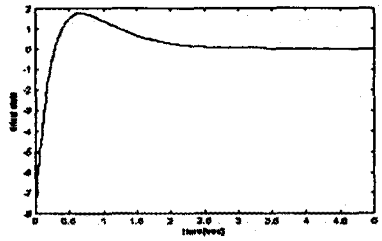

Fig.5 The trajectories of virtual state z_v

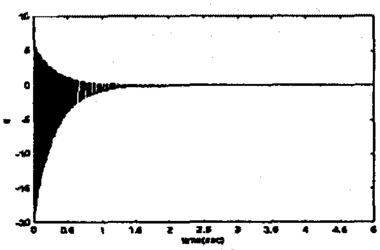

Fig.6 The SMC input $u(t)$

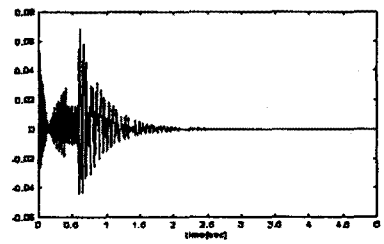

Fig.7 The value of $s(x, z_v)$

6. Conclusions

A novel design method of sliding mode surface has been proposed. With this sliding mode surface, a new SMC, which makes the states of the system follow the nominal trajectory controlled by a nominal controller, can be designed. Any type of controller which is differentiable, can be a nominal controller. It has shown that the robust optimal controller with the novel sliding mode is designed using optimal controller as a nominal one. The reaching phase is easily removed by setting initial appropriately. The result of this paper opens up very attractive area that various type of controller can be combined with the novel sliding mode surface.

References

[1] HUNG, J.Y., GAO, W., HUNG, J.C. : 'Variable structure control : A survey,' IEEE Trans. on Industrial Electronics, 1993, 40(1), pp. 2-22

[2] UTKIN, V.I. : 'Sliding modes and their application in variable structure systems'(Mir Publishers, Moscow, 1978)

[3] ITKIS,U. : 'Control systems of variable structure' (JOHNWILLY & SONS, New York,1976)

[4] ROY, R.G., OLGAC, N. : 'Robust nonlinear control via moving sliding surfaces - n-th order Case', CDC'97, December 1997

[5] KIRK, D.E. : 'Optimal control theory' (Prentice -Hall,1970)