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음향파 공명 산란의 새로운 해석방법

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A new method for extracting resonance information in acoustic wave resonance scattering

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ABSTRACT

A new method is proposed for the isolation of resonances from scattered waves for acoustic wave resonance scattering problems. The resonance scattering function consisting purely of resonance information is defined. Acoustic wave scattering from a variety of submerged bodies is numerically analyzed. The classical resonance scattering theory (RST) and the new method compute identical magnitude of the resonance from each scattered partial wave, however, the phases are significantly different. The exact π - radians phase shifts through the resonance and anti-resonance show that the proposed method properly extracts the vibrational resonance information of the scatterer. Due to the difference in the phase of each partial wave, the new method and RST generate different total resonance spectra.

INTRODUCTION

Since the formalism of classical resonance theory of nuclear reactions was applied to the problem of acoustic wave scattering from submerged elastic circular cylinders and spheres, RST[1] has been the foundation of numerous studies on acoustic and elastic wave scattering. RST demonstrates that the strongly fluctuating behavior of the cross section for sound scattering from elastic bodies is caused by a superposition of the scatterer's eigenvibration and a smoothly-varying geometric background. The resonance scattering formalism shows the total scattering is obtained as a sum of resonance terms and a background term. Utilizing RST, the resonance terms have been obtained by subtraction of a proper background from the individual partial wave in the Rayleigh normal mode series during last two decades [1-8]. The magnitude of the resonances could be obtained with confidence by this procedure. However, the phase information is not clearly explainable and thus has remained unclear although the phase information is as important as the magnitude information. In this paper, we propose a new method to isolate the resonance information from scattered waves so that both the magnitude and phase (or real and imaginary parts) of the isolated resonances are physically explainable and meaningful.

I. RESONANCE SCATTERING THEORY

Let us consider an infinite plane acoustic wave $p_0 \exp i(kX - \omega t)$ with a propagation constant $k = \omega / c$, incident along the X- axis on a solid elastic cylinder of radius a and density ρ_c whose axis coincides with the Z- axis.

At a point $P(r,\phi)$ located in the ouside fluid of density ρ_w, it produces scattered field P_{sc} :

$$P_{sc}(r,\phi) = p_0 \sum_{n=0}^{\infty} \varepsilon_n i^n A_n(x) H_n^{(1)}(kr) \cos n\phi , \quad (1a)$$

where

$$A_n = -\frac{J_n(x)F_n - xJ_n'(x)}{H_n^{(1)}(x)F_n - xH_n^{(1)'}(x)}. \quad (1b)$$

p_0 is the incident pressure magnitude, ε_n is the Neumann factor (ε_n=1 for n = 0, and ε_n=2 for $n > 0$), J_n and $H_n^{(1)}$ are the Bessel function and the Hankel function of the first kind, respectively, and F_n, related to the modal mechanical impedance of the cylinder, is the quotient of two 2 x 2 determinants

$$F_n = -\frac{\rho_w}{\rho_c} x_T^2 \frac{D_n^{(1)}(x)}{D_n^{(2)}(x)}, \quad (2a)$$

where

$$D_n^{(1)}(x) = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \quad D_n^{(2)}(x) = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}. \quad (2b)$$

The elements a_{lm} are given in standard texts [5]. The argument x of the Bessel and Hankel functions in Eq. (1b) is $x = ka = \omega a / c_w$, where c_w is the speed of sound in the ambient fluid. The prime denotes differentiation with respect to the argument. The elements a_{lm} of Eq. (2b) contain Bessel functions with arguments $x_L = k_L a = \omega a / c_L$ and $x_T = k_T a = \omega a / c_T$ where c_L and c_T are, respectively, the speeds of longitudinal and transversal waves in the cylinder

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material. If the asymptotic form of Hankel function is introduced in the far-field where $r >> a$, the far-field scattered pressure becomes

$$P_{sc}(\phi) = p_0 e^{ikr}\sqrt{\frac{2}{\pi ikr}}\sum_{n=0}^{\infty}\varepsilon_n i^n A_n(x)\cos n\phi . \quad (3)$$

The far-field form function f_∞ is defined to give a non-dimensional representation of the scattered pressure

$$f_\infty(\phi) = \sqrt{\frac{2r}{a}}\frac{P_{sc}}{p_0}e^{-ikr} . \quad (4)$$

It can be given in other form by using Eq. (3),

$$f_\infty(\phi) = \frac{2}{\sqrt{\pi ix}}\sum_{n=0}^{\infty}\varepsilon_n A_n(x)\cos n\phi . \quad (5)$$

The individual normal modes or partial waves which make up the form functions are defined as

$$f_n(\phi) = \frac{2}{\sqrt{\pi ix}}\varepsilon_n A_n(x)\cos n\phi \quad (6)$$

so that

$$f_\infty(\phi) = \sum_{n=0}^{\infty} f_n(\phi) \cdot \quad (7)$$

There are two limiting cases of these results. If $\rho_c \to \infty$, the solution applies to scattering by a rigid cylinder:

$$A_n(x)^r = -\frac{J_n(x)}{H_n^{(1)}(x)} \cdot \quad (8)$$

If $\rho_c \to 0$, the solution applies to scattering by a soft cylinder:

$$A_n(x)^s = -\frac{J_n(x)}{H_n^{(1)}(x)} . \quad (9)$$

The scattered pressure of Eq. (1a) may be rewritten as the normal mode series

$$P_{sc}(r,\phi) = \frac{1}{2}\sum_{n=0}^{\infty}\varepsilon_n i^n (S_n - 1)H_n^{(1)}(kr)\cos n\phi , \quad (10)$$

where the scattering function of the n th mode with a constant unit magnitude, containing the scattering phase shifts δ_n is introduced as follows:

$$S_n = e^{2i\delta_n} . \quad (11)$$

In the present case,

$$S_n - 1 = 2A_n(x) . \quad (12)$$

For rigid and soft cylinders, the scattering functions are respectively

$$S_n^r = -\frac{H_n^{(2)'}}{H_n^{(1)'}} = e^{2i\delta_n^r} \quad (13a)$$

and

$$S_n^s = -\frac{H_n^{(2)}}{H_n^{(1)}} = e^{2i\delta_n^s} . \quad (13b)$$

The corresponding phase shifts can be shown to be the real quantities

$$\tan\delta_n^r = \frac{J_n'(x)}{Y_n'(x)}, \qquad \tan\delta_n^s = \frac{J_n(x)}{Y_n(x)} . \quad (14)$$

A rigid or soft scattering function may be factored out from the elastic scattering function as follows:

$$S_n = S_n^r (F_n^{-1} - z_n^{(2)-1}) / (F_n^{-1} - z_n^{(1)-1}) \quad (15a)$$

or

$$S_n = S_n^s (F_n - z_n^{(2)}) / (F_n - z_n^{(1)}) , \quad (15b)$$

where the quantities

$$z_n^{(i)} = xH_n^{(i)'}(x) / H_n^{(i)}(x), i = 1, 2 \quad (15c)$$

are related to the modal specific acoustic impedances.

Employing some mathematical manipulations, RST shows that the quantity that appears in Eq. (10) can be expressed as

$$S_n - 1 = 2ie^{2i\delta_n^r}\left[\sum_{l=1}^{\infty}\frac{\frac{1}{2}\Gamma_{nl}^r}{x_{nl}^r - x - \frac{1}{2}i\Gamma_{nl}^r} + e^{-i\delta_n^r}\sin\delta_n^r\right] \quad (16a)$$

or

$$S_n - 1 = 2ie^{2i\delta_n^s}\left[\sum_{l=1}^{\infty}\frac{\frac{1}{2}\Gamma_{nl}^s}{x_{nl}^s - x - \frac{1}{2}i\Gamma_{nl}^s} + e^{-i\delta_n^s}\sin\delta_n^s\right], \quad (16b)$$

where x_{nl}^r or x_{nl}^s is the l th resonance frequency of the n th mode, and Γ_{nl}^r or Γ_{nl}^s is the resonance width which is related to radiation damping. Based on the expressions (16a) and (16b), RST claims that the scattered field is a summation of two components, *i.e.*, the resonance component which is the first term of Eq. (16a) and (16b), and the smooth background component which is the second term of the same equations. Utilizing RST, numerous books and papers [1-8] have obtained the resonance information of the scatterer by just subtracting the proper background term from the total scattered wave field:

$$f_n^{res}(\phi) = \frac{2}{\sqrt{\pi ix}}\varepsilon_n (A_n - A_n^r)\cos n\phi = f_n - f_n^r \quad (17a)$$

or

$$f_n^{res}(\phi) = \frac{2}{\sqrt{\pi ix}}\varepsilon_n (A_n - A_n^s)\cos n\phi = f_n - f_n^s . \quad (17b)$$

The case of plane acoustic wave scattering from an elastic sphere can be treated analogously to that of a cylinder resulting in the following expressions

$$f_n^{res}(\phi) = -\frac{2}{x}i(2n+1)(A_n - A_n^r)P_n(\cos n\phi) = f_n - f_n^r \quad (18a)$$

or

$$f_n^{res}(\phi) = -\frac{2}{x}i(2n+1)(A_n - A_n^s)P_n(\cos n\phi) = f_n - f_n^s , \quad (18b)$$

where P_n is a Legendre function.

The case of cylindrical or spherical elastic ***shell*** structures rather than *solid* ones is basically the same as above with larger determinants $D_n^{(1)}$ and $D_n^{(2)}$ in Eqs. (2a) and (2b). The case of an insonified *fluid* cylinders or spheres instead of *elastic* bodies is also very similar, in which case F_n becomes the quotient of two cylindrical or spherical functions. However, the forms of expressions of Eqs. (17) and (18) still remain the same.

Eq. (17) or (18) seems to give the correct resonance magnitude information. However, the phase behavior of resonances obtained by this method is not physically explainable and, therefore, not useful although it is well known that the phase of a resonance term should shift by π radians as the frequency passes through the resonance frequency. The magnitude and phase calculated using the classical RST will be discussed further in section IV.

II. NEW RESONANCE FORMALISM

In this section, we attempt to establish a new resonance formalism[9] to extract the resonance information from scattered waves.

Restarting with Eq. (16a), the scattering function S_n can be expressed as follows:

$$S_n = S_n^r S_n^*, \tag{19}$$

where

$$S_n^* = (F_n^{-1} - z_n^{(2)-1}) / (F_n^{-1} - z_n^{(1)-1}).$$

In Eq. (19), S_n is the product of the rigid background S_n^r and the remaining term S_n^*, which includes resonance information. However, S_n^* is not a pure resonance form because it contains the natural frequency terms in both numerator and denominator. S_n^* has a constant term which hides resonances unless it is removed. S_n^* may be written

$$\begin{aligned} S_n^* &= (F_n^{-1} - z_n^{(2)-1}) / (F_n^{-1} - z_n^{(1)-1}) \\ &= (z_n^{(1)-1} - z_n^{(2)-1}) / (F_n^{-1} - z_n^{(1)-1}) + 1 \\ &= S_n^{res} + 1, \end{aligned} \tag{20}$$

where S_n^{res} is defined as the resonance scattering function (RSF) which consists purely of resonance information. By the definition in Eq. (19), the RSF can be expressed as

$$S_n^{res} = \frac{S_n}{S_n^r} - 1 = 2\frac{A_n - A_n^r}{1 + 2A_n^r}, \tag{21}$$

where $A_n = \frac{1}{2}(S_n - 1)$ and $A_n^r = \frac{1}{2}(S_n^r - 1)$.

$S_n - 1$ in Eq. (10) contributes to the total response including both the background and the resonance. S_n^{res} is the only resonance-related component of S_n. The subtraction of a constant real number from a complex quantity results in a new complex quantity different from the original complex quantity. Therefore, from Eq. (10), the scattered wave due to resonance can be obtained by replacing $S_n - 1$ with S_n^{res}:

$$\begin{aligned} P_{sc}^{res}(r,\phi) &= \frac{1}{2}\sum_{n=0}^{\infty} \varepsilon_n i^n S_n^{res} H_n^{(1)}(kr)\cos n\phi \\ &= \sum_{n=0}^{\infty} \varepsilon_n i^n \frac{A_n - A_n^r}{1 + 2A_n^r} H_n^{(1)}(kr)\cos n\phi \cdot \end{aligned} \tag{22}$$

Then, the individual normal mode for resonances in the far-field can be expressed as

$$f_n^{res}(\phi) = \frac{2}{\sqrt{\pi i x}} \varepsilon_n \frac{A_n - A_n^r}{1 + 2A_n^r} \cos n\phi \cdot \tag{23}$$

The expressions involving the corresponding soft background parameters are analogous:

$$\begin{aligned} S_n^* &= (F_n - z_n^{(2)}) / (F_n - z_n^{(1)}) \\ &= (z_n^{(1)} - z_n^{(2)}) / (F_n - z_n^{(1)}) + 1 \\ &= S_n^{res} + 1, \end{aligned} \tag{24}$$

$$S_n^{res} = \frac{S_n}{S_n^s} - 1 = 2\frac{A_n - A_n^s}{1 + 2A_n^s}, \tag{25}$$

and

$$f_n^{res}(\phi) = \frac{2}{\sqrt{\pi i x}} \varepsilon_n \frac{A_n - A_n^s}{1 + 2A_n^s} \cos n\phi \cdot \tag{26}$$

In case of a sphere instead of a cylinder, S_n^{res} contains spherical functions rather than cylindrical functions. The expressions corresponding to Eqs. (23) and (26) are respectively

$$f_n^{res}(\phi) = -\frac{2}{x} i(2n+1)\frac{(A_n - A_n^r)}{(1 + 2A_n^r)} P_n(\cos n\phi), \tag{27}$$

and

$$f_n^{res}(\phi) = -\frac{2}{x} i(2n+1)\frac{(A_n - A_n^s)}{(1 + 2A_n^s)} P_n(\cos n\phi) \cdot \tag{28}$$

The expressions in Eqs. (23), (26), (27) and (28) are also applicable to acoustic wave scattering from elastic cylindrical/spherical shell structures or fluid spheres /cylinders with the appropriate changes in F_n as explained in section I.

By using Eqs. (23), (26), (27) and (28), the resonances which are mixed with the background in the total scattered pressure field can be uncovered. The only difference between these new equations and the old equations (Eqs. (17) and (18)) is the existence of the denominator $1 + 2A_n^r$ or $1 + 2A_n^s$, which is respectively the scattering function S_n^r or S_n^s corresponding to the impenetrable targets in two extreme cases. In case of acoustic wave scattering, where no mode conversion occurs, and with no material damping, which is being studied in the present investigation, S_n^r and S_n^s are unitary. Therefore these new and old equations produce completely identical magnitudes. However, it should be noted that their phases are not the same because the scattering function S_n^r or S_n^s in the denominator has its own phase shift. If Nyquist plots rather than Bode type plots are examined, the new and old equations generate constrasting results. Moreover, in elastic wave scattering, where mode conversion occurs, both the magnitude and phase obtained by the two methods will be different because the unitarity condition applies to the scattering matrix and cannot be extended to the individual scattering functions. Therefore, the new resonance formalism has a more important meaning when we are dealing with elastic wave scattering. However, our discussion in this paper is limited to acoustic wave scattering case.

III. COMPARISON OF THE PROPOSED METHOD WITH RST

By considering Eq. (12) and following relationships

$$S_n^r = e^{2i\delta_n^r} \tag{29}$$

and

$$S_n^r - 1 = 2ie^{i\delta_n^r}\sin\delta_n^r \;, \tag{30}$$

Eq. (16a) can be expressed as

$$A_n = A_n^r + (1+2A_n^r)\sum_{l=1}^{\infty} i\frac{\frac{1}{2}\Gamma_{nl}^r}{x_{nl}^r - x - \frac{1}{2}i\Gamma_{nl}^r} \,. \tag{31}$$

Now, if we approximate S_n^{res} near resonance frequencies as

$$S_n^{res} = \sum_{l=1}^{\infty} i\frac{\Gamma_{nl}^r}{x_{nl}^r - x - \frac{1}{2}i\Gamma_{nl}^r}, \tag{32}$$

and substitute into Eq. (31), Eq. (21) of the resonance scattering function is obtained. In case of the soft background, Eq. (25) can be obtained by approximating

$$S_n^{res} = \sum_{l=1}^{\infty} i\frac{\Gamma_{nl}^s}{x_{nl}^s - x - \frac{1}{2}i\Gamma_{nl}^s}, \tag{33}$$

and by following the similar procedure used for rigid background case.

Therefore, it is revealed that the new resonance formalism in Eqs. (23), (26), (27) and (28) is consistent with RST formulation. One may argue that Eqs. (17) and (18) are merely a incorrect application of RST.

We note that Eqs. (32) and (33) are approximate expressions obtained by the linearization near resonance frequencies. Thus, these expressions are valid only near resonance frequencies because the interactions between resonances are not considered. Therefore, to compute the exact resonances, numerical solutions of Eqs. (23) and (26) for a cylinder (or Eqs. (27) and (28) for a sphere) are required.

IV. NUMERICAL ANALYSIS AND DISCUSSION

All examples in this section are performed for backscattering ($\phi = \pi$). The acoustic properties of materials used for numerical calculations are as follows: ρ_{water}=1000 kg m^{-3}, $c_{L,water}$= 1480 m sec^{-1}, $\rho_{aluminum}$=2800 kg m^{-3}, $c_{L,aluminum}$= 6370 m sec^{-1}, $c_{T,aluminum}$= 3070 m sec^{-1}, ρ_{air}=1.12 kg m^{-3}, $c_{L,air}$= 340 m sec^{-1}.

The background is assumed to be rigid for acoustic wave scattering from elastic bodies. Acoustic plane wave scattering from solid elastic spheres and cylinders in water is analyzed numerically in Fig. 1. Resonances in each partial wave are isolated and plotted separately by using the previous method (Eqs. (17) and (18)) and the new method (Eqs. (23) and (27)). As predicted in section II, while the magnitudes are perfectly identical, the phases are very different. The new method generates exact π-phase shifts through the resonance. The shift occurs abruptly or gradually depending on radiation damping. The almost constant phase through the first thick peak of Fig. 1 shows clearly that it is not related to the scatterer's resonance. The large size of the resonance-like peak may be partly due to the incorrectness of the rigid background especially in the low frequency region. The π- phase shift also occurs at the anti-resonance caused by the mutual interaction of adjacent resonances. RST calculates physically unexplainable phase information as can be seen by the dotted lines in Fig. 1.

If a Nyquist plot is constructed in the complex plane as a function of non-dimensionalized frequency ka, the new and previous methods produce contrasting trajectories because both the real and imaginary parts of the resonances are different. Fig. 2 shows the Nyquist plot for the resonance scattering function. Regardless of mode number n and the shape of scatterer (spherical or cylindrical), the trajectory of S_n^{res} makes a circle with unit radius centered at (-1,0). This can be explained as follows : By the definition of S_n^{res} in Eqs. (19) and (20) and by the fact that there is no mode conversion in acoustic wave scattering, $|S_n|=|S_n^r|=|S_n^*|= 1$, therefore, $|S_n^{res}|=|S_n^* - 1|$, which forms the circle in Fig.2 in the complex plane as the non-dimensionalized frequency ka varies. This unit circle is physically related to the energy conservation during acoustic wave scattering.

Fig. 3 are Nyquist plots of $S_n - S_n^r$ for the case of the sphere (n =2). Fig. 3 is a misleading information for the purpose of obtaining the true resonances.

Because of the difference in the phases of isolated resonances obtained by the two methods, one expects that the resonance spectrum, which is a total summation of isolated resonances, will not be the same. Fig. 4 explicitly show the differences in summed magnitude and phase for the cylinder. As can be seen in Fig. 4 efforts such as those of Maze[8] led to an incorrect resonance spectrum.

Aforementioned discussions are also applicable to spherical and cylindrical shell structures, of which inside is empty or filled with another material, instead of solid ones. Fig. 5 compares the resonances as calculated by two methods for an air-filled spherical shell. Fig. 6 compares the resonance spectrum up to ka = 40 calculated by the new method with that computed in the Ref. 4 for an air-filled aluminum cylindrical shell. The proposed method can also be applied to acoustic wave scattering from fluid spheres or cylinders. Fig. 7 show resonances for the air sphere (n =0). In this case the proper background is the soft background rather than the rigid background.

V. CONCLUSIONS

A new resonance formalism is proposed for the isolation of resonances from scattered waves for acoustic wave scattering. Plane compressive wave scattering from submerged bodies such as elastic solid spheres and cylinders, shell structures, and also fluid spheres and cylinders is analyzed by the new method and the classical RST. The exact π - phase shift through the resonance and anti-resonance shows that the proposed method properly extracts the resonances from each scattered partial wave. The total resonance spectra computed by the new method show different magnitude and phase from previous studies. This paper has shown that the physically unexplainable behavior of phase of the resonances extracted by RST for acoustic wave scattering was not due to the inexactness of the rigid or soft background, but due to the use of an incorrect resonance formalism.

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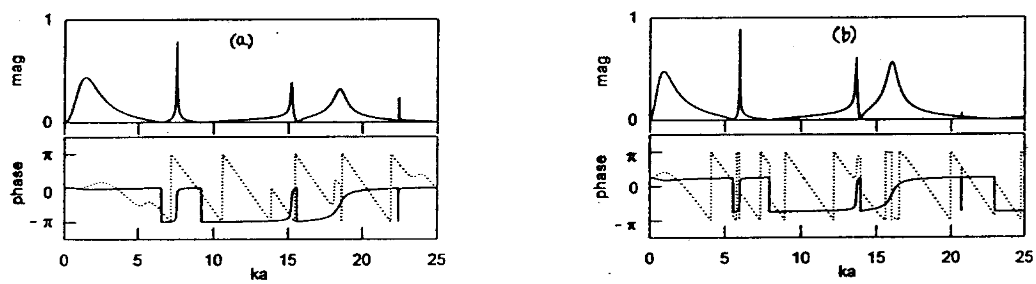

FIG. 1 Resonances isolated by the new (solid curve) and previous (dotted curve) methods for a submerged aluminum (a) sphere and (b) cylinder for n=1.

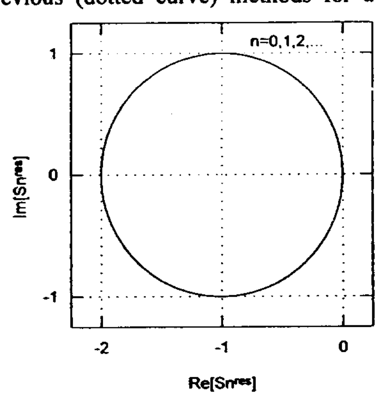

FIG. 2 Nyquist plot for the resonance scattering function for all frequency range for any type of scatterer immersed in water.

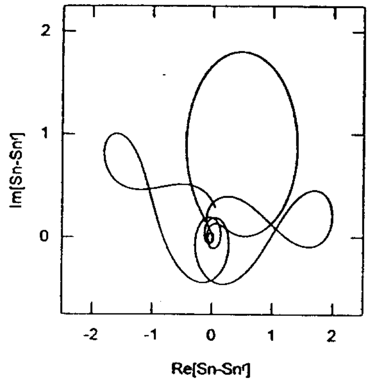

FIG. 3 Nyquist plot for $S_n - S_n^r$ for n th scattered partial wave for a submerged aluminum sphere for n =2 up to $ka = 25$.

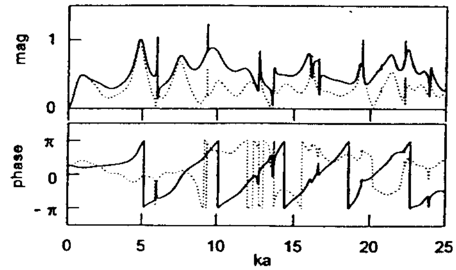

FIG. 4 Summed resonance spectra by the new (solid curve) and previous (dotted curve)[8] methods for a submerged aluminum cylinder.

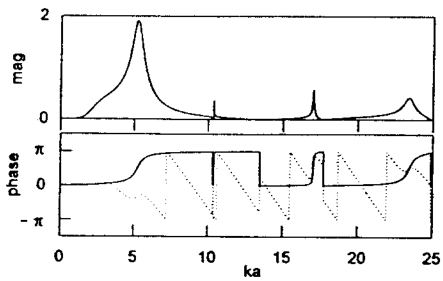

FIG. 5 Resonances idolated by the new (solid curve) and previous (dotted curve) methods for a submerged air-filled aluminum spherical shell (ratio of inner to outer radius = 0.2) for n =2.

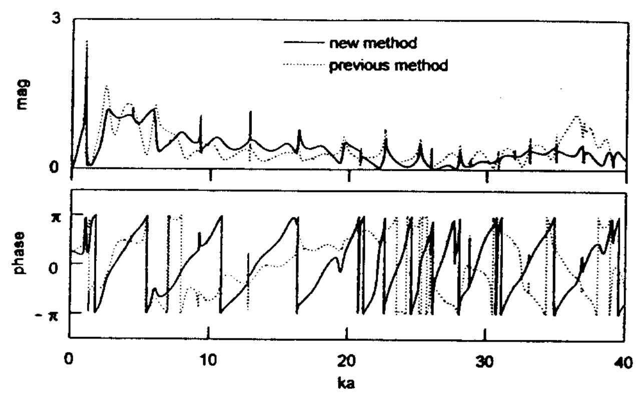

FIG. 6 Summed resonance spectra (for n = 0 to 47) by the new (solid curve) and previous (dotted curve) [4] methods for a submerged air-filled aluminum cylindrical shell (ratio of inner to outer radius = 2/3).

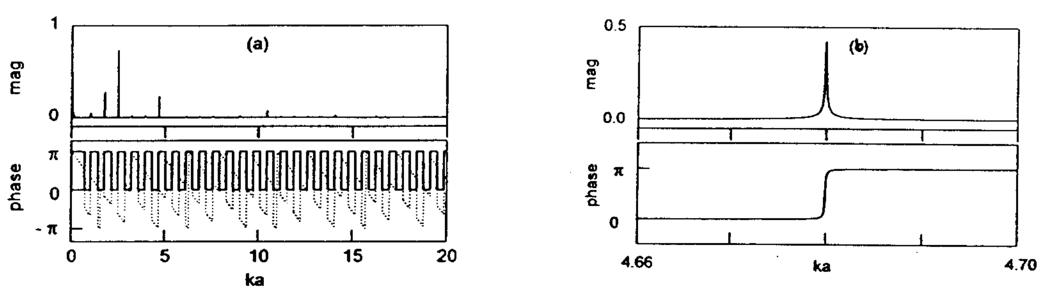

FIG. 7 Resonances isolated by the new (solid curve) and previous (dotted curve) methods for a submerged air sphere with (a) n =0 (b) n =0 detailed plot ka =4.66-4.70.