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ON FUZZY H-CONTINUOUS MAPPINGS

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ABSTRACT. We introduce the concept of a fuzzy H-continuity and find some properties.

1. Preliminaries.

In this section, we introduce some conception and notations. Let $I = [0,1]$. For a set X, let I^X denote the collection of all mapping from X into I. A member A of I^X is called a *fuzzy set* in X ($cf.$[6]).

Definition 1.1[5]. A fuzzy point x_λ in a set X is a fuzzy set in X denoted by for each $y \in x$.

$$x_\lambda(y) = \begin{cases} \lambda & \text{if} \quad y = x, \\ 0 & \text{if} \quad y \neq x, \end{cases} \qquad \text{where} \quad \lambda \in (0,1].$$

We will denote the family of all the fuzzy points in X as $F_P(X)$.

Definition 1.2[1]. A subfamily $\mathcal{T}$ of I^X is called a *fuzzy topology* on X if $\mathcal{T}$ satisfies the following conditions:

(1) $\emptyset, X \in \mathcal{T}$.

(2) If $\{U_\alpha : \alpha \in \Lambda\} \subset \mathcal{T}$, then $\bigcap_{\alpha\in\Lambda} U_\alpha \in \mathcal{T}$, where Λ is an index set.

(3) If $A, B \in \mathcal{T}$, then $A \cap B \in \mathcal{T}$.

Each member of $\mathcal{T}$ is called a *fuzzy open set* in X and its complement *fuzzy closed set* in X. The pair$(X, \mathcal{T})$ is called a *fuzzy topological space* (fts, in short).

Notation 1.A. *For a fts X, let:*

(a) $FO(X)$ denote the collection of all the fuzzy open set in X.

(b) $FC(X)$ denote the collection of all the fuzzy closed set in X

Definition 1.3[4]. Let $(X, \mathcal{T})$ be a fts, and let $Y \subset X$. Then $\mathcal{T}_Y = \{A|_Y : A \in \mathcal{T}\}$ is called the *relative fuzzy topology* and the fts $(Y, \mathcal{T}_Y)$ is called a *subspace* of $(X, \mathcal{T})$.

Definition 1.4[4]. A fts X is said to be *Hausdorff*, if for any two fuzzy points x_λ and y_μ such that $x \neq y$, there exists q-neighborhoods B and C of x_λ and y_μ, respectively, such that $B \cap C = \emptyset$.

Definition 1.5[1,2]. A mapping $f : X \longrightarrow Y$ is said to be:

(a) *fuzzy continuous* (*f-continuous*, in short) if for each $V \in FO(Y)$, $f^{-1}(V) \in FO(X)$.

(b) *fuzzy c-continuous* (*f-c-continuous*, in short) if for each $x_\lambda \in F_P(X)$ and each $V \in FO(Y)$ such that $f(x_\lambda) \in V$ and V^c is fuzzy compact inY, there exists a $U \in FO(X)$ such that $f(U) \subset V$.

Theorem 1.6[2]. *Let X and Y be fuzzy topological spaces and let $f : X \longrightarrow Y$ be a mapping. Then the following statements are equivalent:*

(a) f is f-c-continuous.

(b) If $V \in FO(Y)$ with compact complememt, then $f^{-1} \in FO(X)$.

2. Fundamental properties of fuzzy H-continuous mappings.

Definition 2.1. A fts X is said to be *fuzzy H-closed(f-H-closed,*in short) if each open cover $\{U_\alpha : \alpha \in \Lambda\}$ of X has a finite subfamily $\{U_{\alpha_i} : i = 1, \cdots, n\}$ such that $\bigcup_{i=1}^n \overline{U}_{\alpha_i} = X$.

A fuzzy set A in X is called a *fuzzy H-set(f-H-set,*in short) if for each family $\{U_\alpha : \alpha \in \Lambda\}$ of fuzzy open sets in X covering A (i.e. $A \subset \bigcup_{\alpha \in \Lambda} U_\alpha$), there exists a finite subfamily$\{U_{\alpha_i} : i = 1, \cdots, n\}$ such that $A \subset \bigcup_{i=1}^n \overline{U}_{\alpha_i}$.

Lemma 2.2. *If X is fuzzy Hausdorff and A is a f-H-set in X, then $A \in FC(X)$.*

Lemma 2.3. *If A and B are two f-H-sets in X,then $A \cup B$ is a f-H-set in X.*

Definition 2.4. A mapping $f : X \longrightarrow Y$ is said to be fuzzy H-continuous (f-H-continuous, in short) if for each $x_\lambda \in F_P(X)$ and each fuzzy open set V in Y containing $f(x_\lambda)$ having a f-H-set complement, there exists a fuzzy open set U in X containing x_λ such that $f(U) \subset V$.

Theorem 2.5. *Let $f : X \longrightarrow Y$ be a mapping. Then the following are equivalent:*
(a) f is f-H-continuous.
(b) if V is open in Y and has a f-H-set complement, then $f^{-1}(V) \in FO(X)$.
These statements are implied by
(c) If B is a f-H-set in Y, then $f^{-1}(B) \in FC(X)$.
Furthermore, if Y is Hausdorft, then all three statement are equivalent.

Corollary 2.51. *A mapping $f : X \longrightarrow Y$ is f-H-continuous if and only if for each closed f-H-set B in Y, $f^{-1}(B) \in FC(X)$.*

Lemma 2.6. *Let $\mathfrak{B}$ be the collection of all the fuzzy open sets in a fts $(X, \mathcal{T})$ having f-H-set complement(compact complement, resp.). Then there exists a fuzzy topology $\mathcal{T}^*$ ($\mathcal{T}^{**}$,resp.) on X for which $\mathfrak{B}$ is a base for $\mathcal{T}^*$($\mathcal{T}^{**}$, resp.). Furthermore, $\mathcal{T}^{**} \subset \mathcal{T}^* \subset \mathcal{T}$.*

Proposition 2.7. *(a) $f : X \longrightarrow (Y, \mathcal{T})$ is f-H-continuous if and only if $f : X \longrightarrow (Y, \mathcal{T}^*)$ is f-continuous.*
(b) $f : X \longrightarrow (Y, \mathcal{T})$ is f-c-continuous if and only if $f : X \longrightarrow (y, \mathcal{T}^{**})$ is f-continuous.
(c) $id : (Y, \mathcal{T}) \longrightarrow (Y, \mathcal{T}^*)$ and $id : (Y, \mathcal{T}^*) \longrightarrow (Y, \mathcal{T}^{**})$ are f-continuous.
(d) $id^{-1} : (Y, \mathcal{T}^*) \longrightarrow (Y, \mathcal{T})$ is f-H-continuous and $id^{-1} : (Y, \mathcal{T}^{**}) \longrightarrow (Y, \mathcal{T}^*)$ is f-c-continuous.

Theorem 2.8. *Let $f : X \longrightarrow (Y, \mathcal{T})$ be f-H-continuous. If $f : X \longrightarrow (Y, \mathcal{T}^*)$ is f-closed (f-open, resp.), then $f : X \longrightarrow (Y, \mathcal{T})$ is f-closed(f-open, resp.).*

3. Further results.

Theorem 3.1. *If $f : X \longrightarrow Y$ is f-H-continuous and $A \subset X$, then $f|_A : A \longrightarrow Y$ is f-H-continuous.*

Theorem 3.2. *If $f : x \longrightarrow Y$ is f-continuous and $g : Y \longrightarrow Z$ is f-H-continuous, then $g \circ f$ is f-H-continuous.*

Theorem 3.3. *Let $f : X \longrightarrow Y$ be f-continuous and bijective. If Y is fuzzy Hausdorff, then f^{-1} is f-H-continuous.*

Theorem 3.4. *Let $(Y, \mathcal{T})$ be a fts. If $(Y, \mathcal{T}^*)$ is Hausdorff, then $(Y, \mathcal{T})$ is H-closed.*

4. Problems.

(1) Let X be a fts and let $\{\mathcal{A}_\alpha : \alpha \in \Lambda\}$ a cover of X such that $\mathcal{S}(\mathcal{A}_\alpha) = \mathcal{A}_\alpha$ for each $\alpha \in \Lambda$ and
  (a) Each $\mathcal{A}_\alpha$ is open in X or
  (b) Each $\mathcal{A}_\alpha$ is closed in X and $\{\mathcal{A}_\alpha : \alpha \in \Lambda\}$ forms a neighborhood finitely family. If $f : X \longrightarrow Y$ is a mapping such that $f|_{\mathcal{A}_\alpha} : \mathcal{A}_\alpha \longrightarrow Y$ is f-H-continuous for all $\alpha \in \Lambda$, then f is f-H-continuous.

(2) Let $f : X \longrightarrow Y$ is f-open with closed graph. Then f is f-H-continuous.

(3) Let $\{X_\alpha : \alpha \in \Lambda\}$ and $\{Y_\alpha : \alpha \in \Lambda\}$ be fuzzy topological spaces and let
  (a) Each f_α is f-open and all but at most finitely many are surjective and
  (b) For at least one $\beta \in \Lambda$, f_β has a closed graph. Then $f : \prod_{\alpha\in\Lambda} X_\alpha \longrightarrow \prod_{\alpha\in\Lambda} Y_\alpha$, defined by $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ is f-H-continuous.

(4) Let $f : X \longrightarrow (Y, \mathcal{T})$ be a mapping, where $(Y, \mathcal{T})$ is f-semi-regular and $(Y, \mathcal{T}^*)$ Hausdorff. Then f is f-continuous if and only if f is f-H-continuous.

References

1. C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.
2. K.Hur, J.R.Moon and J.H.Ryou, *A note on fuzzy c-continuous mappings*, Proceding of KIFS Fall Conference '97 **7[2]** (1997), 214–217.
3. P.E. Long and T.R. Hamlett, *H-continuous functions*, Bull. U.M.I. **4** (1975), 552–558.
4. Pu Pao Ming and Liu Ying Ming, *Fuzzy topology I, Neighborhood structure of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl. **76** (1980), 531–599.
5. C.K.Wang, *Fuzzy points and local properties of fuzzy topology*, J. Math. Anal. Appl. **41** (1994), 316–328.
6. L.A.Zadeh, *Fuzzy set*, Inform and control **8** (1965), 338–353.