Multi-Objective Optimization of Rotor-Bearing System with Dynamic Constraints Using IGA

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ABSTRACT

An immune system has powerful abilities such as memory, recognition and learning how to respond to invading antigens, and has been applied to many engineering algorithms in recent year. In this paper, the combined optimization algorithm (Immune-Genetic Algorithm: IGA) is proposed for multi-optimization problems by introducing the capability of the immune system that controls the proliferation of clones to the genetic algorithm. The new combined algorithm is applied to minimize the total weight of the rotor shaft and the transmitted forces at the bearings in order to demonstrate the merit of the combined algorithm. The inner diameter of the shaft and the bearing stiffness are chosen as the design variables. The results show that the combined algorithm can reduce both the weight of the shaft and the transmitted forces at the bearing with dynamic constraints.

1. Introduction

In the design of modern rotating machinery, it is often necessary to increase the performance of rotor-bearing systems. This requires the design of more compact and lightweight systems, which will greatly save the fuel spent during the service life. Moreover, if the transmitted forces through the bearings can be reduced, the lifetime of the rotor-bearing system will be increased. The problem of weight minimization usually arises from the revision of an existing rotor-bearing system to increase the system performance. Many papers have shown that the system parameters, including the distribution of the mass and stiffness of the shaft and the coefficients of the bearings, have an influence on the dynamic characteristics of a rotor-bearing system[9-11].

In most case, many engineering problems can be formulated as the multi-objective function. It is not easy to solve such multi-objective functions by conventional optimization methods.

In the recent year, an immune system, which has powerful abilities such as memory, recogni- tion and learning how to respond to invading antigens, has been applied to many engineering algorithms[7].

This paper proposes the combined optimization algorithm for multi-objective problems by intro- ducing the capability of the immune system that controls the proliferation of clones to the genetic algorithm [3,5]. In order to demonstrate the merit of the combined algorithm, the combined algori- thm is applied to the design of a rotor-bearing system with minimum shaft weight and trans- mitted forces through the bearings under the requirements of dynamic behaviors such as critical speeds, dynamic stress, and steady-state unbalance response, to increase the performance of a rotor- bearing system in this paper.

The results show that the combined algorithm can reduce both the weight of the shaft and the forces at the bearing with dynamic behavior cons- traints.

2. Rotor Model

Figure 1 shows the analysis model of a rotor- bearing system as an example. The F(X Y Z); R(x y z) triad is a fixed, rotating reference with the X and x axes being collinear and coincident with the undeformed rotor centerline. R is defined relative to F by a single rotation ωt about X with ω denoting a whirl speed. In order to make an accurate analysis of the complex rotor-bearing systems, the vibrations are calculated using general finite element procedures[8] in this paper. Since the finite element discretization procedure is well documented in many literatures, the details will be omitted here and only the equations of motion are presented below.

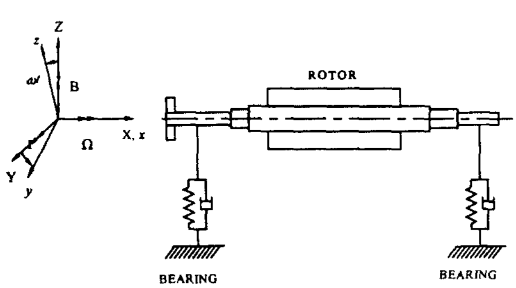

Fig. 1. Vibration analysis model.

The system equations that describe the behavior of the entire rotor-bearing system are formulated by taking into account the contributions from all the elements in the model. The assembled equation of motion with elements in the whirl frame coordinates is of the form

$$\boldsymbol{M}\ddot{\boldsymbol{p}} - i\Omega \boldsymbol{G}\dot{\boldsymbol{p}} + \boldsymbol{K}\boldsymbol{p} = \boldsymbol{Q}^u \quad (1)$$

where the matrices $\boldsymbol{M}$, $\boldsymbol{G}$ and $\boldsymbol{K}$ are the real symmetric matrices. $\boldsymbol{Q}^u$ is the force vector of mass unbalance.

2.1 Critical Speeds

For the isotropic and undamped system, the system response can be assumed to be of the form

$$\boldsymbol{p} = \boldsymbol{p}_0 e^{i\omega t} \quad (2)$$

Substituting equation (2) into the homogeneous part of equation (1) yields the characteristic equation

$$\{\boldsymbol{K} - \omega^2(\boldsymbol{M} - \lambda \boldsymbol{G})\}\boldsymbol{p}_0 = \boldsymbol{0} \quad (3)$$

where $\lambda = \Omega/\omega$ is the whirl ratio. By assigning a special whirl ratio and solving the eigenvalue problem of equation (3), one can obtain the whirl critical speeds associated with the whirl mode of a rotor-bearing system.

2.2 Steady-State Unbalance Response

The mass unbalance forces $\boldsymbol{Q}^u$ shown in equation (1) can usually be expressed as follows:

$$\boldsymbol{Q}^u = \boldsymbol{Q}_0 \Omega^2 e^{j\Omega t}$$

(4)

where the $\boldsymbol{Q}_0$ is independent of time and rota- ting speed. The steady-state response due to mass unbalance is assumed of the form

$$\boldsymbol{p} = \boldsymbol{p}_s e^{i\Omega t}$$

(5)

Substituting equations (4) and (5) into equation (1) for $\lambda=1$ yields

$$\{\boldsymbol{K}-\Omega^2(\boldsymbol{M}-\lambda\,\boldsymbol{G})\}\boldsymbol{p}_s=\Omega^2\boldsymbol{Q}_0 \tag{6}$$

Then the steady-state response can be obtained by solving equation (6) for $\boldsymbol{p}_s$.

2.3 Bending Stress Analysis

The bending stress resisted by the rotating flexible shaft is very important to the structural designer. It can be obtained by finding the external loads acting on the shaft elements. These loads can be calculated by solving equation (6) for the response. The bending stress at the nodal point can be obtained using equation (7) as follows[10]:

$$\sigma_l^{(i,m)}=\left[\sum_{j=1}^{4}K^{e^{(i)}}_{(2,j)}\cdot p^{(i,m)}_{s(j)}\right]r_{oi}/I_i$$
$$\sigma_r^{(i,m)}=\left[\sum_{j=1}^{4}K^{e^{(i)}}_{(4,j)}\cdot p^{(i,m)}_{s(j)}\right]r_{oi}/I_i \tag{7}$$

where the superscript i denotes the ith shaft element; the superscript m denotes when $\Omega=\Omega_m$, Ω_m is associated with the upper or the lower limit of the operating speed range; $\sigma_l^{(i,m)}$ and $\sigma_r^{(i,m)}$ represent the bending stress of a nodal point on the left-hand side and right-hand side of the shaft element; $p^{(i,m)}_{s(j)}$ represents the value of the $(4i-4+j)$ th entry of the steady-state response vector $\boldsymbol{p}_s$ as $\Omega=\Omega_m$ and $K^{e^{(i)}}_{(l,j)}$ denotes the value of the stiffness matrix $\boldsymbol{K}^e$ of the ith shaft element at the lth row and the jth column.

3. Optimization Approach

The multi-objective optimization problem usually requires the descriptions of the objective functions as well as the side and behavior of the constraint functions. In this study, the objective functions $F(\boldsymbol{d})$ are the weight of the shaft $W(\boldsymbol{d})$ and the jth bearing's transmitted force $F_j^b\|(\boldsymbol{d})\|$. The constraints on the critical speeds are taken as follows:

$$g_1(\boldsymbol{d})=\omega_i^c-\Omega_{low}/a_1\le 0$$
$$g_2(\boldsymbol{d})=a_2\cdot\Omega_{high}-\omega_{i+1}^c\le 0 \tag{8}$$

where Ω_{low} and Ω_{high} are the lower and upper bounds of the operating speed range of the existent rotor system. The ith and the $(i+1)$th critical speeds are denoted by ω_i^c and ω_{i+1}^c, respectively. The constraints a_1 and a_2 are some positive real numbers greater than one. Physically, equation (8) states that the ith critical speed of the new system has to be lower than Ω_{low} by a factor of a_1 and the $(i+1)$th critical speed has to be higher than Ω_{high} by a factor of a_2. This implies that the new system has a wider operating range and the performance is improved. If the limitations on bending stress and unbalance response are considered, the behavior constraints are as follows:

$$g_3(\boldsymbol{d})=|\sigma_{max}|-\sigma^*\le 0$$
$$g_4(\boldsymbol{d})=|\delta_{max}|-\delta^*\le 0 \tag{9}$$

where σ_{max} and δ_{max} denote the maximum bending stress and response in the steady-state. σ^* and δ^* represent the allowable stress and allowable steady-state response, respectively.

The inner raidus of the shaft elements and the bearing stiffnesses are primarily taken as the design variables since they play a very important role to determine the critical speeds and transmitted forces through the bearings. The optimum design problem can be expressed as follows:

$$Minimize:\ F(\boldsymbol{d})=W(\boldsymbol{d})\left(=\sum_{i=1}^{N_e}\rho_i l_i A_i\right)\ \text{and}$$

$$\| F_j^b(\boldsymbol{d}) \| \quad (10)$$

$$Subject\ to : g_1(\boldsymbol{d}) = \omega_i^c - \Omega_{low}/a_1 \le 0,$$

$$g_2(\boldsymbol{d}) = a_2 \cdot \Omega_{high} - \omega_{i+1}^c \le 0 \quad (11)$$

$$g_3(\boldsymbol{d}) = |\sigma_{max}| - \sigma^* \le 0,$$

$$g_4(\boldsymbol{d}) = |\delta_{max}| - \delta^* \le 0 \quad (12)$$

$$\boldsymbol{d} = \{r_{Ii}, k_j^b ; i = 1, N_e, \ j = 1, N_b\}$$

$$d_i^l \le d_i \le d_i^u, \ i = 1, n$$

(13)

where N_e and N_b are the total numbers of shaft elements and bearings, respectively. The total number of design variables is denoted by $n (= N_e + N_b)$. The definitions of parameters are given in the nomenclature. The ith component of design vector $\boldsymbol{d}$ is denoted by d_i.

4. Proposition of Combined Algorithm

4.1 Immune System

All living organisms are continuously exposed to substances that are capable of causing them harm. Most organisms protect themselves against such substances with physical barriers or chemicals that repel or kill invaders. Animals with backbones have these types of general protective mechanisms, but they also have a more advanced protective system called the immune system[6]. The immune system is a complex network of organs containing cells that recognize foreign substances in the body and destroy them.

Several types of cells compose the immune system as stem cell, precursor cell, lymphocytes, antibodies and antigen. There are basically 2 types of lymphocytes, B cell and T cell. B cells are responsible for generating proper antibodies to an antigen, for tagging it and marking it for removal by other components of the immune system. T cells are responsible of controlling B cells reaction process. Figure 2 shows the production mechanism of antibodies in the immune system.

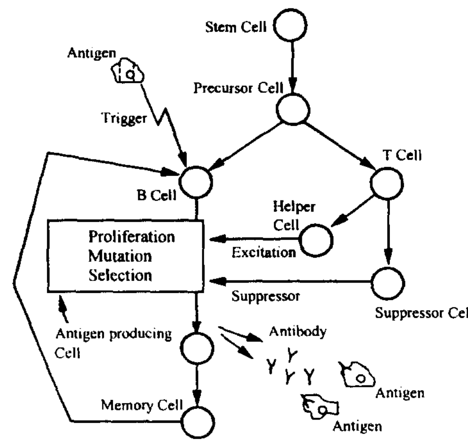

Fig. 2. The production mechanism of antibodies
in the immune system.

4.2 Diversity and Affinity of Immune System

The immune system produces the variety antibody and controls the production of the antibody with the combining of the antigen and the antibody or between two antigens. This coherence is estimated by the affinity. The affinity $ay_{v,w}$ between two antibodies v and w can be also be represented by equation (14).

$$ay_{vw} = 1/(1 + H_{vw}) \quad (14)$$

where $H_{v,w}$ is the hamming distance between antigen v and w. The affinity ax_v between antigen and antibody v is defined by equation (15).

$$ax_v = opt_v \quad (0 \le opt_v \le 1) \quad (15)$$

where opt_v means the evaluated value to show the coherence between antigen and antibody v.

4.3 Combined Algorithm

This paper proposes a combined algorithm (IGA) by introducing the capability of the immune system, which controls the proliferation of clones and suppresses the production of the antibody, to the genetic algorithm, which are search algorithms based on the mechanics of natural selection and natural genetics, in order to prevent the initial convergence of the genetic algorithms and to search for the multi-optimum solution. The antibody in the immune algorithm is the individual in the IGA. Since the genetic algorithm is well documented in many literatures [1,2], the details will be omitted here. Figure 3 shows the flow chart of the IGA. The dotted line indicated in Fig. 3 is the additional part of the immune algorithm.

4.4 Characteristics of the IGA

As defined above, the IGA is that the immune algorithm based on the somatic theory and a net- work hypothesis is introduced to genetic algori- thm. The somatic theory shows that somatic re- combination and mutation contributes to increa- sing the diversity of the individuals. The network hypothesis shows that a mutual recognition net- work among the individuals contributes to control of the proliferation of individuals. Therefore, this algorithm can be expected to have the following characteristics.

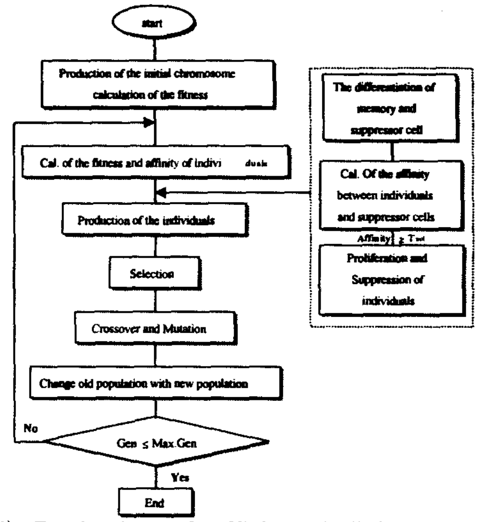

Fig. 3. The flow chart of IGA.

(i) Diversity of the individual : Generating a diversity of the individuals based on the above theories can be expected to obtain the multi- optimum solution to the optimization problem.

(ii) Production of efficient individuals : As the individuals of higher fitness are more remain, the number of efficient individuals are produced. Thus parallel searching in a neighborhood space by a number of efficient individuals can be expected to be quicker and more efficient.

5. Numerical Result and Discussion

The rotor-bearing system[9-11] used in this paper consists of a single spool and three bearings located at stations 3, 6 and 13. Figure 4

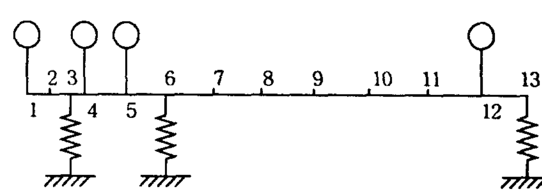

Fig. 4. The schematic of the rotor-bearing system.

shows the schematic of the rotor-bearing system. The details of the rotor configuration are listed in Tables 1 and 2.

Table 1. The configuration data of a rotor-

bearing system.

Station No.	Axial distance to station 1(cm)
1	0.0
2	4.29
3	8.89
4	10.49
5	20.17
6	27.69
7	44.20
8	59.44
9	74.68
10	89.92
11	105.16
12	120.14
13	127.94
$E = 20.69 \times 10^9 N/m^2$, $\rho = 8193.0 kg/m^3$ $r_{oi} = 0.0295m$, $i = 1, 12$	

Table 2. Rigid disk data.

Station No.	Mass (kg)	Polar inertia ($kg \cdot cm^2 \times 10^2$)	Diametral inertia ($kg \cdot cm^2 \times 10^2$)
1	11.38	19.53	9.82
4	7.88	16.70	8.35
5	7.70	17.61	8.80
12	21.70	44.48	22.24

The constraints a_1 and a_2 shown in equation (8) are chosen so that the second critical speed ω_2^c is at least lower than Ω_{low} by the factor of 1.2 while the third critical speed ω_3^c is at least higher than Ω_{high} by the factor of 1.3. The operating speed range is given by Ω_{low}= 830*rad/s* and Ω_{high} = 1770*rad/s*. Therefore, equation (8) can be rewritten as

$$g_1(d) = \omega_2^c - 691.67\, rad/s \leq 0 \quad (16)$$

$$g_2(d) = 2301\, rad/s - \omega_3^c \leq 0 \quad (17)$$

And the constraints σ^* and δ^* shown in equation (9) are chosen as 150*Mpa* and 100μ*m*, respectively. The side constraints of the design variables are given by

$$0.0142\, m \leq r_{Ii} \leq 0.0269\, m, \quad i = 1, 12 \quad (18)$$

$$3.50 \times 10^6 N/m \leq k_i^b \leq 1.75 \times 10^8 N/m, \quad i = 1, 3 \quad (19)$$

The total shaft weight and transmitted force at he third bearing are considered as the objectives to be minimized, simultaneously. In order to identify the merit of the IGA, the optimization result compares to the Shiau and Chang's result [11] using the weighting method (WM). Tables 3 and 4 show the comparison of the optimization result.

Table 3. Initial radius of shaft element and bearing stiffness for initial and multi-objective optimi- zation design

Shaft element	Multi objective : W and F_3^b		
	Initial value	WM	IGA
	Inner radius (cm)		
1	1.882	2.296	2.680
2	1.940	2.369	2.6895
3	1.466	2.690	2.6770
4	1.660	2.618	2.6900
5	2.151	2.690	2.5745
6	2.690	2.690	2.6900
7	2.690	2.690	2.6900
8	2.690	2.690	2.6900
9	2.690	2.690	2.6900
10	2.690	2.690	2.6900
11	1.420	2.441	2.2165
12	1.880	2.287	2.6900
Bearing	Bearing stiffness ($N/m \times 10^8$)		
1	0.035	0.035	0.035
2	1.27	1.28	1.74832
3	0.12	0.035	0.035

It is indicated that the optimum weight and transmitted force of the IGA are smaller than that of WM satisfying all constraints. This implies that the IGA finds more exact solutions than other optimization methods for multi-objective optim- ization using the characteristic of IGA; the diversity of the individual and the production of efficient individuals. Figures 5 and 6 show the Campbell diagram and unbalance response corresponding to the initial and optimum design.

6. Conclusions

This paper proposes the combined optimization

Table 4. Critical speed, shaft weight, transmitted forces, maximum amplitude and bending stress for initial and multi-objective optimization design

	Initial value	WM	IGA
	Critical speed, ω_n^c (rad/s)		
ω_2^c	685.84	514.44	519.22
ω_3^c	2646.14	2365.15	2366.27
	Shaft weight, W (kg)		
W	10.235	6.228	584656
	Transmitted force, F_j^b (N)		
F_1^b	13.562	9.303	8.86
F_2^b	475.599	338.591	327.866
F_3^b	461.366	146.158	141.018
	Maximum bending stress, σ (Mpa)		
MaxBS	45.29	22.688	23.54
	Maximum ampitude, δ (μm)		
MaxAmp	78.9	65.3	64.9

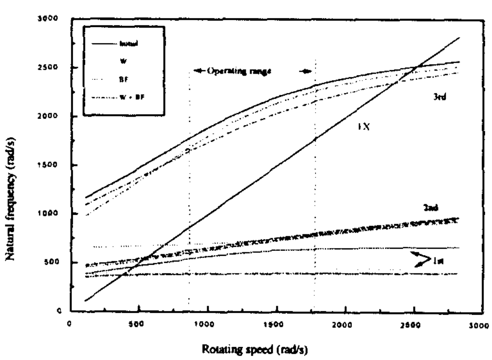

Fig. 5. Campbell diagram of initial and optimum design.

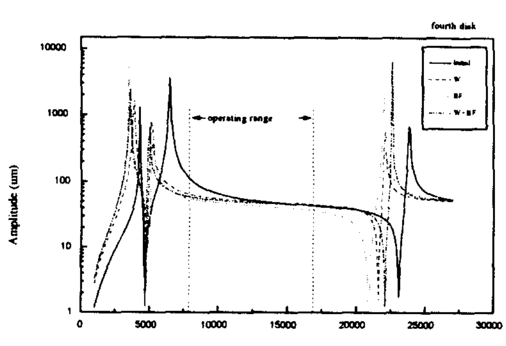

Fig. 6. Unbalance response of initial and optimum design.

algorithm (IGA) for multi-optimization problems by introducing the capability of the immune system that controls the proliferation of clones to the genetic algorithm. In this paper, multi- objective optimization is carried out to demonst- rate the merit of the combined algorithm. The results show that the IGA can reduce both the weight of the shaft and the transmitted forces at the bearing with dynamic behavior constraints. The comparison of the results between the IGA and weighting method (WM) show that the IGA finds out more exact solutions than WM for multi-objective optimization using the characterist- ic of IGA; the diversity of the individual and the production of efficient individuals.

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