A Linear Approach on the Characteristics of Density-Stratified Flow past Complex Terrain

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I. Introduction

The density-stratified flow can be controlled by the Froude number F_r ($\equiv U/Nh$), which is the function of atmospheric stability, maximum height of topography and inflow velocity. According to the previous researches, the flow is linear when the $F_r \gg 1$ while nonlinear when $F_r \ll 1$.

II. The linear theory

For the theoretical description of mountain waves, we consider the steady flow of a vertically unbounded, stratified Boussinesq fluid, over small-amplitude topography given by $z = h(x, y)$. The perturbations to the background wind, pressure and density fields are described by the following linearized equations:

$$U\frac{\partial u'}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \tag{1a}$$

$$U\frac{\partial v'}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial y} \tag{1b}$$

$$U\frac{\partial w'}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{\rho'}{\rho_0}g \tag{1c}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \tag{1d}$$

$$\rho' = \frac{d\bar{\rho}}{dz}\eta \tag{1e}$$

where x, y, z are the downstream, cross stream and vertical coordinates; u', v', w', ρ', p', η are the corresponding perturbation velocity components and the perturbation density, pressure and vertical displacement; and ρ_0, U, $d\bar{\rho}/dz$ are the background mean density, wind speed and vertical density gradient. Using the kinematic condition for steady flow

$$w' = U\frac{\partial \eta}{\partial x} \tag{2}$$

and with U taken as constant, the linear equation set (1) can be reduced to a single equation for $\eta(x, y, z)$ the vertical displacement of a fluid parcel, or a density surface, above its undisturbed level

$$\frac{\partial^2}{\partial x^2}(\nabla^2 \eta) + \frac{N^2}{U^2}\nabla_H^2 \eta = 0 \tag{3}$$

where

$$N^2 \equiv -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz} \tag{4}$$

is the square of the buoyancy frequency, which is assumed to be constant for shallow disturbances. Then, with the aid of a double Fourier transformation, the eq. (3) has the general solutions of the form

$$\eta(x, y, z) = \int\int_{-\infty}^{\infty} \hat{h}(k, l)\, e^{im(k, l)z} e^{i(kx+ly)}\, dkdl \tag{5}$$

where

$$\hat{h}(k, l) = \frac{1}{4\pi^2}\int\int_{-\infty}^{\infty} h(x, y) e^{-i(kx+ly)}\, dxdy$$

$$m^2 = \frac{k^2 + l^2}{k^2}\left(\frac{N^2}{U^2} - k^2\right), \text{ for non-hydrostatic case}$$

$$m = \frac{N}{U}\frac{(k^2 + l^2)^{1/2}}{k}, \text{ for hydrostatic case}$$

III. The results and discussion

In this paper, the density-stratified flow over different types of obstacles is examined using a linear model. The purpose of this paper is to examine the difference of wave propagations between hydrostatic and non-hydrostatic situations. The non-hydrostatic effects become apparent over smaller scale obstacles than large scale ones, and will be play an important role for the transport and dispersion of air particles in the local area.

Figure 1 shows the pattern of the vertical velocity at various levels associated with hydrostatic stratified flow over an isolated mountain. The maximum height of topography is given by $100m$. The region of down motion is widening with height and the well-developed horse-shoes vortex is seen at the level of $3km$. In the non-hydrostatic solution the fluid particles undergo a different oscillation but the shape of horse-shoes vortex is similar to each other.

According to other results, both the wave number of topography ($k=2\pi/L_x$) and Scorer parameter ($\ell^2=N^2/U^2$) determine the pattern of mountain-induced waves. If $k^2>\ell^2$ the wave is evanescent or trapped while if $k^2<\ell^2$ propagating.

References

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[2] Qi Hu, E. R. Reiter and R. A. Pielke, 1988: Analytic Solutions to Long's Model: A Comparison of Nonhydrostatic and Hydrostatic Cases. *Meteorol. Atmos. Phys.*, **39**, 184-196.

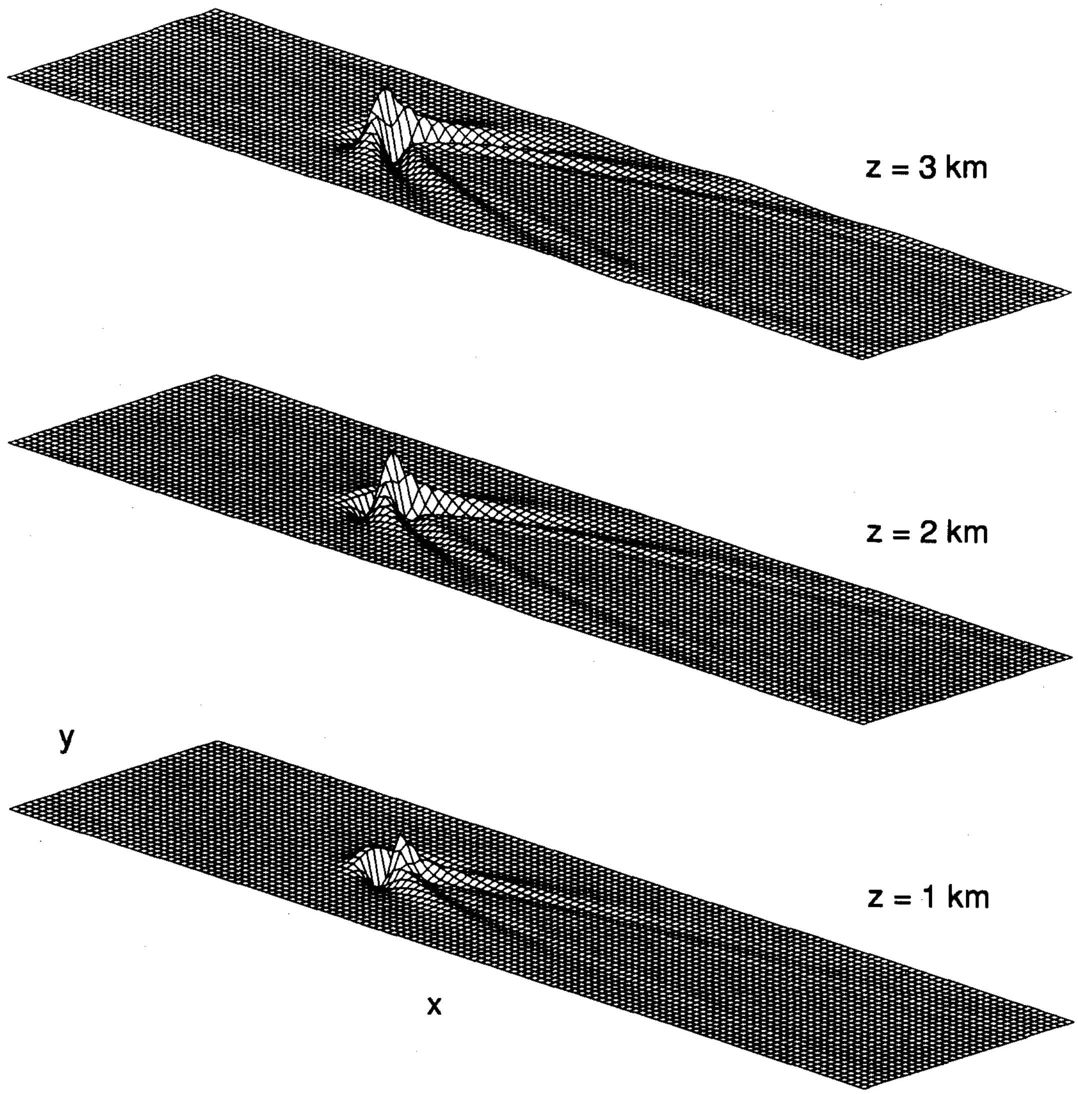

Figure 1. The pattern of the vertical velocity at various levels associated with hydrostatic stratified flow over an isolated mountain.