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Acceleration of the AFEN Method by Two-Node Nonlinear Iteration

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Abstract

A nonlinear iterative scheme developed to reduce the computing time of the AFEN method was tested and applied to two benchmark problems. The new nonlinear method for the AFEN method is based on solving two-node problems and use of two nonlinear correction factors at every interface instead of one factor in the conventional scheme. The use of two correction factors provides higher-order accurate interface fluxes as well as currents which are used as the boundary conditions of the two-node problem. The numerical results show that this new method gives exactly the same solution as that of the original AFEN method and the computing time is significantly reduced in comparison with the original AFEN method.

I. Introduction

The nonlinear scheme[1] has turned out to be very effective in reducing computer memory requirements, computing time, and implementing effort associated with higher-order nodal methods. This scheme solves the modified finite difference method (FDM) current equation given for interface of nodes n and $n+1$ with side-length h :

$$J_g^{n,n+1} = -\frac{2D_g^n D_g^{n+1}}{h(D_g^n + D_g^{n+1})}(\bar{\phi}_g^{n+1} - \bar{\phi}_g^n) - \frac{\tilde{D}_{g,HNM}^{n,n+1}}{h}(\bar{\phi}_g^{n+1} + \bar{\phi}_g^n), \quad (1)$$

where D and $\tilde{D}$ represent the diffusion coefficient and nonlinear correction factor, respectively. Determining the nonlinear correction factor $\tilde{D}$ so that the interface current J should preserve the value of a higher-order nodal method makes the solution of this modified FDM scheme equivalent to that of the higher-order nodal method itself. For the nonlinear FDM iterative scheme with the usual higher-order nodal methods that use the transverse-integration, this is done by solving two-node

problems consisting of neighboring nodes periodically after a specified number of outer iterations of the FDM routine. Using the higher-order nodal method, the two-node problem is solved for the interface current of the two nodes with currently available node-average fluxes and transverse-leakages shapes of both nodes as boundary conditions. The higher-oder correction factor at the interface is updated by equating the resultant higher-order interface current with the modified FDM current given by Eq.(1). Then, the FDM routine is continued utilizing the updated higher-order correction factors. The entire process is repeated until convergence of the effective multiplication factor and the node average fluxes is achieved. Palmtag[2] et al developed a nonlinear scheme for a nodal method that uses flux expansion functions similar to those of the AFEN method. (Actually, their functions are identical to those of the AFEN/PEN hybrid method for the two-group case[5].) But the method consumes long computing time in that it solves nine-node problems.

In this paper, as an acceleration means for the AFEN method[3,4,5,6], the new nonlinear iterative scheme developed in Ref. 7 was tested for computing time. The new nonlinear method is based on solving two-node problems and use of two nonlinear correction factors at every interface instead of one factor in the conventional scheme. The use of two correction factors provides higher-order accurate interface fluxes as well as currents which are used as the boundary conditions of the two-node problem. The numerical results show that this new method gives exactly the same solution as that of the original AFEN method and the computing time is significantly reduced in comparison with the original AFEN method.

II. Theory and Methodology

The new nonlinear scheme adopts two correction factors per interface. The correction factors of node n and node $n+1$, respectively, at the interface "$n, n+1$" are defined as follows :

$$J_g^{n,n+1} = -\frac{2D_g^n}{h}(\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^n) - \frac{2\tilde{D}_g^{n\to n+1}}{h}(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^n), \tag{2a}$$

$$J_g^{n,n+1} = \frac{2D_g^{n+1}}{h}(\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^n) + \frac{2\tilde{D}_g^{n+1\to n}}{h}(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^n), \tag{2b}$$

where $\tilde{\phi}_g^{n,n+1}$ is the interface flux between node n and $n+1$. Employing the two factors makes it possible for this scheme to carry the higher-order accurate interface current and flux simultaneously during the FDM routine. By equating Eq.(2a) and Eq.(2b), the interface current and flux are obtained :

$$J_g^{n,n+1} = -\frac{2(D_g^n D_g^{n+1} - \tilde{D}_g^{n\to n+1}\tilde{D}_g^{n+1\to n})}{h(D_g^n + D_g^{n+1} + \tilde{D}_g^{n\to n+1} + \tilde{D}_g^{n+1\to n})}(\bar{\phi}_g^{n+1} - \bar{\phi}_g^n) \tag{3a}$$
$$-\frac{2(D_g^{n+1}\tilde{D}_g^{n\to n+1} - D_g^n\tilde{D}_g^{n+1\to n})}{h(D_g^n + D_g^{n+1} + \tilde{D}_g^{n\to n+1} + \tilde{D}_g^{n+1\to n})}(\bar{\phi}_g^{n+1} + \bar{\phi}_g^n),$$

$$\tilde{\phi}_g^{n,n+1} = \frac{(D_g^n - \tilde{D}_g^{n\to n+1})\bar{\phi}_g^n + (D_g^{n+1} - \tilde{D}_g^{n+1\to n})\bar{\phi}_g^{n+1}}{(D_g^n + D_g^{n+1} + \tilde{D}_g^{n\to n+1} + \tilde{D}_g^{n+1\to n})}. \tag{3b}$$

Note that the system equation for the node average fluxes derived from the current equation (i.e., Eq.(3a) has the same structure to that of the FDM scheme, that is, a pentadiagonal matrix equation in the two-dimensional geometry. We easily note that if we set the two nonlinear correction factors to zero, Eq.(3a) and Eq.(3b) degenerate into the FDM equations for the interface current and flux, respectively. If the above FDM equation is solved with initially guessed nonlinear correction factors, the node average fluxes, interface currents and interface fluxes can be calculated for use in calculating corner-point fluxes and solving two-node problems. At present, the corner-point fluxes are calculated by the corner-point balance (CPB) method. The simplified form of the CPB equation is given by

$$t_{ij}^L \phi_{i-1j} + t_{ij}^C \phi_{ij} + t_{ij}^R \phi_{i+1j} + t_{ij}^B \phi_{ij-1} + t_{ij}^T \phi_{ij+1} = p_{ij}. \tag{4}$$

All the coefficients of this equation are given in Ref. 3. Eq.(4) is solved once by iteration. Then, the equations for updating nonlinear correction factors are derived. Solving the two-node problem by the AFEN method starts from expanding the intranodal flux distribution of a node into the same non-separable analytic functions as in the original AFEN method except the constant term. The constant term is deleted since Eq.(3) already implies the neutron balance of the node. The flux expansion becomes

$$\begin{aligned}\hat{\xi}_g^n &= A_{g1}^n SNx_g^n x + A_{g2}^n CSx_g^n x + A_{g3}^n SNx_g^n y + A_{g4}^n CSk_g^n y \\ &+ B_{g1}^n SN\frac{\sqrt{2}}{2}x_g^n x SN\frac{\sqrt{2}}{2}x_g^n y + B_{g2}^n SN\frac{\sqrt{2}}{2}x_g^n x CS\frac{\sqrt{2}}{2}x_g^n y \\ &+ B_{g3}^n CS\frac{\sqrt{2}}{2}x_g^n x SN\frac{\sqrt{2}}{2}x_g^n y + B_{g4}^n CS\frac{\sqrt{2}}{2}x_g^n x CS\frac{\sqrt{2}}{2}x_g^n y\end{aligned} \tag{5}$$

In Eq.(5), all coefficients are represented in terms of node average fluxes, interface currents, interface fluxes and corner point fluxes. The two-node equation can be derived by using the continuity condition of the neutron flux across the interface. The final form is given by

$$\begin{aligned}&\{F^{ij}\langle a_1^{ij}\rangle (D^{ij})^{-1} + F^{i-1j}\langle a_1^{i-1j}\rangle (D^{i-1j})^{-1}\}J_{ij}^x \\ &= F^{ij}\langle a_1^{ij} - a_2^{ij} + a_3^{ij}\rangle (D^{ij})^{-1}\frac{J_{ij+1}^y - J_{ij}^y}{2} \\ &- F^{i-1j}\langle a_1^{i-1j} - a_2^{i-1j} + a_3^{i-1j}\rangle (D^{i-1j})^{-1}\frac{J_{i-1j+1}^y - J_{i-1j}^y}{2} \\ &+ U(\phi_{i-1j}, \phi_{i-1j+1}, \phi_{ij}, \phi_{ij+1}, \phi_{i+1j}, \phi_{i+1j+1}, \bar{\phi}_{ij}, \bar{\phi}_{i-1j}).\end{aligned} \tag{6}$$

The equation for the interface flux can be written similarly. After solving the neutron current and flux at the interface of the two-node problem by the AFEN method, the nonlinear correction factors are updated. The equation for updating the nonlinear correction factors are obtained by solving Eq.(2) for the nonlinear correction factors :

$$\tilde{D}_g^{n\to n+1} = \frac{-J_g^{n,n+1}h - 2D_g^n(\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^n)}{2(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^n)}, \tag{7a}$$

$$\tilde{D}_g^{n+1\to n} = \frac{J_g^{n,n+1}h - 2D_g^{n+1}(\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^{n+1})}{2(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^{n+1})}. \tag{7b}$$

Once we update all the correction factors over the whole core, the calculational flow returns to the next FDM outer iteration.

III. Numerical Results

To verify the new nonlinear iterative scheme, two benchmark problems are tested. Benchmark Problem I is a slightly modified two-dimensional IAEA benchmark problem so that the internal boundary condition is changed from mirror symmetry about the node center line into mirror symmetry about the node edge line. This benchmark problem consists of fully homogenized fuel assemblies. The solutions of the new nonlinear iterative AFEN method and the original AFEN method are compared with each other in Fig. 1. From Fig. 1, it is noted that the two solutions are exactly the same if the minor truncation errors are neglected.

17.157 2.838 0.001 0.001	30.952 7.174 0.001 0.001	38.209 8.964 0.001 0.001	32.630 7.556 0.001 0.001	21.915 3.674 0.000 0.000	28.267 6.540 0.000 0.000	30.764 7.237 -0.001 -0.001	22.285 6.130 -0.001 -0.001	3.584 8.428 -0.001 -0.001
	37.973 8.903 0.001 0.001	42.180 9.896 0.001 0.001	39.364 9.228 0.000 0.000	34.002 7.877 0.000 0.000	34.322 8.047 -0.001 -0.001	32.637 7.696 -0.001 -0.001	22.141 6.072 -0.001 -0.001	3.454 8.078 -0.001 -0.001
		44.775 10.505 0.001 0.001	43.062 10.103 0.000 0.000	39.397 9.243 0.000 0.000	37.192 8.748 0.000 0.000	32.911 8.163 0.000 0.000	19.013 5.875 -0.001 -0.001	2.676 6.383 -0.001 -0.001
			39.906 9.355 0.000 0.000	33.935 7.861 0.000 0.000	32.470 7.649 -0.001 -0.001	26.230 7.300 -0.001 -0.001	6.931 14.384 -0.001 -0.001	0.752 3.008 0.000 0.000
				23.200 3.964 0.000 0.000	24.232 5.920 0.000 0.000	17.076 5.259 -0.001 -0.001	2.791 6.896 0.000 -0.001	
					16.668 5.195 -0.001 -0.001	4.804 9.960 -0.001 -0.001	0.633 2.490 -0.001 -0.001	
						0.733 2.925 0.000 0.000		

1. Original AFEN fast flux
2. Original AFEN thermal flux
3. Nonlinear AFEN fast flux error (%)
4. Nonlinear AFEN thermal flux error (%)

Fig. 1. Comparison of the flux distributions for Benchmark Problem I

Table I shows that the total computing time of the nonlinear AFEN method is siginificantly reduced in comparison with that of the original AFEN method. The convergence criteria for multiplication factor and for source are 1.0E-7 and 1.0E-8, respectively.

Benchmark Problem II is devised to test the new nonlinear iterative scheme for large problems. The benchmark problem is the two-dimensional IAEA problem where an assembly is divided into four nodes.

Table I. Comparison of Computing Times[a] (Seconds on HP780)

Benchmark Problem I			Benchmark Problem II		
Original AFEN	Nonlinear AFEN	Speedup	Original AFEN	Nonlinear AFEN	Speedup
6.4	0.5(28[b])	12.8 times	32.1	1.1(27[b])	29.2 times

[a]Without acceleration in outer iteration
[b]Number of updates of nonlinear correction factors

The computing times and number of updates of the nonlinear correction factors are given in Table I. In obtaining the results, the nonlinear correction factors are updated after the FDM routine is updated 12 times. The result shows that the new nonlinear AFEN method is much more effective in large problems than in small problems. In both benchmark problems, the multiplication factors of the original AFEN method were exactly reproduced by the nonlinear AFEN method.

IV. Conclusions

In this paper, a new nonlinear iterative AFEN method is presented and tested for two benchmark problems. The new method is based on solving two-node problems and use of two nonlinear correction factors at every interface instead of one factor in the conventional scheme. The use of two correction factors provides higher-order accurate interface fluxe as well as currents which are used as the boundary conditions of the two-node problem. The numerical results show that the converged solutions of the two methods are exactly the same if the minor truncation errors are neglected. The computing times of the new method are significantly reduced in comparison with those of the original AFEN method. The results indicate that the computing time can be reduced by a factor of ~ 10 for two-dimensional (or higher for three-dimensional) usual nodal problems. (We expect that this speedup factor would be slightly reduced if acceleration in outer iteration is included.) Therefore, the new nonlinear iterative scheme for the AFEN method can be effectively used in practical nuclear design problems.

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