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A GENERALIZED PERTURBATION PROGRAM FOR CANDU REACTOR

Do Heon Kim and Jong Kyung Kim
Hanyang University

Hangbok Choi and Gyuhong Roh
Korea Atomic Energy Research Institute

Won Sik Yang
Chosun University

ABSTRACT

A generalized perturbation program has been developed for the purpose of estimating zonal power variation of a CANDU reactor upon refueling operation. The forward and adjoint calculation modules of RFSP code were used to construct the generalized perturbation program. The numerical algorithm for the generalized adjoint flux calculation was verified by comparing the zone power estimates upon refueling with those of forward calculation. It was, however, noticed that the truncation error from the iteration process of the generalized adjoint flux is not negligible.

INTRODUCTION

The first attempt to apply the generalized perturbation theory (GPT) to CANDU fuel management was made to correct the undesirable flux distributions rather than to minimize fuel cycle cost. The optimization techniques suitable for CANDU reactor fuel management have been studied by Motoda, Wight, Wight and Girouard, Bonin, Dubois and Bonin, Rozon, and Rozon and Hebert.[1] Motoda firstly studied the continuous refueling problem in a slab geometry and one energy group with no restriction on maximum flux. The other studies were engaged in determining an optimal discharge burnup distribution of a reactor under the equilibrium refueling condition, which will maximize the average exit burnup while obeying constraints on maximum power and excess reactivity. Recently, Rozon et al. have investigated the potential of using Reactor Regulating System (RRS) as a response to refueling perturbation in order to select optimal refueling channels.[2]

In this paper, we introduce a procedure for obtaining zone power variation to refueling perturbations using first order GPT equations. In principle, the perturbation program uses RFSP modules for the forward calculation and data transfer and a module *PERTURB was written for the generalized adjoint flux and zone power calculation.

PHYSICS METHOD OF RFSP CODE

The RFSP is a 3-dimensional finite difference diffusion code used for the CANDU reactor design and analysis. It is being used for the lattice property calculation, time-average and instantaneous calculation, refueling simulation, and core transient (kinetics) calculation. It is composed of many modules such as *DATA for model definition, *TIME-AVER for time-average calculation, and *SIMULATE for refueling simulation. Each module executes independently and shares data through the RFSP direct-access file.

The two-group neutron diffusion equation solved by RFSP is as follows:

$$-\nabla \cdot D_1(\vec{r})\nabla\phi_1(\vec{r}) + (\Sigma_{a1}(\vec{r}) + \Sigma_m(\vec{r}))\phi_1(\vec{r}) - \frac{\nu\Sigma_f(\vec{r})}{k_{eff}}\phi_2(\vec{r}) = 0 \quad (1)$$

$$-\nabla \cdot D_2(\vec{r})\nabla\phi_2(\vec{r}) + \Sigma_{a2}(\vec{r})\phi_2(\vec{r}) - \Sigma_m(\vec{r})\phi_1(\vec{r}) = 0 \quad (2)$$

The finite difference scheme of RFSP is a mesh-centered formulation, i.e., the finite difference equation is written in terms of the fluxes at the centers of the nodes. The static diffusion equation can be written as:

$$\boldsymbol{M}\phi = \lambda \boldsymbol{F}\phi \quad (3)$$

and the flux solution is normalized to the total reactor power defined as:

$$P_{tot} = \langle H, \phi \rangle_R = \sum_i H_i \phi_{2i} V_i \quad (4)$$

where $H = V_{cell}\left[\frac{\sum_{all}\phi_g\Sigma_{fg}}{\phi_{thermal}}\right]E_f F_c \times 1.60207\times10^{-5}\times10^{11}\,kW$ for one lattice bundle, E_f is the average energy release (MeV) per fission, F_c is the fraction of power into the coolant, and V_i is the volume correction factor of the node i.

GPT METHOD FOR RFSP

Suppose that the diffusion equation, Eq. (3), has been solved for the reference (unperturbed) system such as:

$$(\boldsymbol{M}_o - \lambda_o \boldsymbol{F}_o)\,\phi_o = 0, \quad (5)$$

the perturbed equation with first order approximation will be written as

$$(\boldsymbol{M}_o - \lambda_o \boldsymbol{F}_o)\Delta\phi = -(\Delta \boldsymbol{M} - \lambda_o \Delta \boldsymbol{F} - \Delta\lambda \boldsymbol{F}_o)\phi_o = S\ . \quad (6)$$

In Eq. (6), the variation of system matrix is known once the refueling channel is selected. In principle, it is possible to get the flux variation by solving a fixed source problem but this requires a large amount of calculation which is not desirable for the selection of optimum refueling channel. If GPT method is used to predict the sensitivity of flux (power) to the refueling operation, the number of calculations can be saved appreciably when estimating the system characteristics of interest.

The CANDU reactor is divided into 14 control zones, and the liquid zone controllers (LZC), located at the center of each zone, are controlled individually to maintain the reference zone power. Therefore, the zone power (or level) can be regarded as an index that represents the deviation of current power distribution from the reference one. For such a case, the system characteristics of interest are the zone power fractions, which are defined for the unperturbed state by:

$$p_{lo} = \frac{\langle H_o, \phi \rangle_l}{\langle H_o, \phi_o \rangle_R}. \tag{7}$$

The goal is to obtain the variation in zone power fractions caused by a single refueling perturbation. For example, refueling of channel j will perturb the zonal power fractions:

$$p_l = p_{lo} + \Delta p_l^{(j)} \tag{8}$$

The variation of zone power fraction can be divided into two components: the direct effect of H and the indirect effect of flux change. The direct component can be obtained by differentiate the zone power fraction with respect to H. The indirect component can be obtained by introducing a generalized adjoint flux (Γ_l^*) to Eq.(6):

$$\langle (\boldsymbol{M}_o^* - \lambda_o \boldsymbol{F}_o^*)\Gamma_l^*, \Delta\phi^{(j)} \rangle = \langle \Gamma_l^*, S_j \rangle \tag{9}$$

Now, the indirect component can be given as

$$\Delta p_l^{(j)}|_{\Delta\phi} = \langle \Gamma_l^*, S_j \rangle \tag{10}$$

under the condition that

$$\langle (\boldsymbol{M}_o^* - \lambda_o \boldsymbol{F}_o^*)\Gamma_l^*, \Delta\phi^{(j)} \rangle = \left\langle \frac{\partial p_l}{\partial \phi}, \Delta\phi^{(j)} \right\rangle \tag{11}$$

When the direct and indirect components are collected, the variation in zone power fraction due to refueling of channel j can be easily obtained:

$$\begin{aligned} \Delta p_l^{(j)} &= \frac{\langle \Delta H^{(j)}, \phi_o \rangle_{V_l}}{\langle H_o, \phi_o \rangle_R} - p_l \frac{\langle \Delta H^{(j)}, \phi_o \rangle_R}{\langle H_o, \phi_o \rangle_R} \\ &- \langle \Gamma_l^*, \left(\Delta \boldsymbol{M}^{(j)} - \lambda_o \Delta \boldsymbol{F}^{(j)}\right)\phi_o \rangle_R + O(\{\Delta\phi\}^2) \end{aligned} \tag{12}$$

where the generalized adjoints are obtained by solving an adjoint source problem for each control zone l as shown in Eq. (13):

$$(\boldsymbol{M}_o^* - \lambda_o F_o^*)\Gamma_l^* = S_l^* \tag{13}$$

and the multigroup spatial source is defined as:

$$S_l^* \equiv \frac{\partial p_l}{\partial \phi}(\vec{r}) = \frac{\{\delta_{V_l}(\vec{r}) - p_l\} H_o(\vec{r})}{\langle H_o, \phi_o \rangle_R} \tag{14}$$

where, $\delta_{V_l} = 1$ only for regions in control zone l.

Eq. (12) contains only the perturbation operators and the unperturbed flux. The generalized adjoint equation, Eq. (13), is more simple than the perturbed equation, Eq. (3), since it is not an eigenvalue equation but a singular source problem. Once the generalized adjoints are calculated, the variation of zone power can be estimated to the second order accuracy by a simple inner product, rather than solving the perturbed field equation for each perturbation.

NUMERICAL TEST

The algorithm of GPT program for CANDU physics code RFSP has been developed. The perturbation program basically adopts RFSP method for input preparation, forward diffusion calculation and data transfer. A numerical test of GPT program for CANDU refueling simulation has been performed for the DUPIC core under a 2-bundle shift refueling scheme. Fig.1 shows forward, adjoint, and generalized adjoint flux in x-direction of the core, where the generalized adjoint flux is for zone 1. Table I shows the variations of zone power and effective multiplication factor for four single refueling and a simultaneous refueling perturbations, and the results are compared to those of RFSP forward calculations. The forward and GPT calculations have shown good agreements when the zone power variation is relatively large. However, when the magnitude of the perturbation is relatively small, it was found that the truncation error during the iteration process is not negligible.

REFERENCE

1. H.W. Bonin, "CANDU Pressurized Heavy Water Reactor Thorium-^{233}U Oxide Fuel Evaluation Based on Optimal Fuel Management," *Nucl. Technol.*, **76**, 390 (1987).
2. D. Rozon, E. Varin, R. Roy, and D. Brissette, "Generalized Perturbation Theory Estimates of Zone Level Response to Refueling Perturbations in a CANDU 600 Reactor," in *Proc. of the ANS Topical Meeting - Advances in Nuclear Fuel Management II*, EPRI TR-107728, p. 15-19, South Carolina (1997).

Table I. Comparison of Zone Power and Effective Multiplication Factor

Refueled Channel		P-7			O-8			M-16		
Δk_{eff}		Forward	GPT	Error*	Forward	GPT	Error*	Forward	GPT	Error*
		1.74E-4	1.57E-4	-9.9	1.55E-4	1.43E-4	-7.5	1.60E-4	1.52E-4	-5.0
$\Delta p_l^{(j)}$		Forward	GPT	Error*	Forward	GPT	Error*	Forward	GPT	Error*
Zone	1	3.1E-4	3.2E-4	3.2	3.0E-4	3.3E-4	8.9	-1.2E-3	-1.1E-3	-3.2
	2	2.6E-3	2.5E-3	-6.7	1.7E-3	1.6E-3	-5.3	-1.0E-3	-9.8E-4	-3.7
	3	-1.3E-3	-1.1E-3	-17.2	-9.7E-4	-8.1E-4	-16.1	-3.9E-4	-4.1E-4	4.6
	4	2.0E-4	8.0E-5	-60.0	7.5E-4	6.4E-4	-14.0	6.5E-4	5.9E-4	-8.9
	5	1.2E-3	1.1E-3	-10.1	9.6E-4	8.7E-4	-9.4	9.1E-5	6.8E-5	-
	6	-1.4E-3	-1.3E-3	-6.6	-1.1E-3	-1.0E-3	-5.2	1.3E-3	1.2E-3	-6.9
	7	-8.3E-4	-8.0E-4	-2.9	-6.3E-4	-6.3E-4	-0.5	1.9E-3	1.8E-3	-6.3
	8	1.4E-4	1.7E-4	14.2	7.6E-5	1.1E-4	-	-1.3E-3	-1.2E-3	-6.1
	9	2.1E-3	2.0E-3	-5.9	1.2E-3	1.1E-3	-4.5	-1.2E-3	-1.1E-3	-6.5
	10	-1.4E-3	-1.2E-3	-17.8	-1.1E-3	-9.3E-4	-16.0	-6.8E-4	-5.8E-4	-14.7
	11	-6.2E-5	-1.9E-4	-	2.6E-4	1.5E-4	-42.9	8.0E-5	6.4E-5	-
	12	8.4E-4	7.5E-4	-10.4	5.4E-4	4.8E-4	-11.3	-3.3E-4	-2.8E-4	-14.0
	13	-1.5E-3	-1.4E-3	-6.5	-1.2E-3	-1.2E-3	-4.5	7.7E-4	7.6E-4	-2.0
	14	-8.9E-4	-8.7E-4	-3.2	-7.4E-4	-7.3E-4	-0.8	1.2E-3	1.2E-3	-2.9

Refueled Channel		G-11			P-7, O-8, M-16, G-11		
Δk_{eff}		Forward	GPT	Error*	Forward	GPT	Error*
		1.43E-4	1.35E-4	-5.8	6.26E-4	5.87E-4	-6.2
$\Delta p_l^{(j)}$		Forward	GPT	Error*	Forward	GPT	Error*
Zone	1	5.3E-4	4.6E-4	-13.7	-9.3E-5	-3.6E-5	-
	2	-5.9E-4	-5.6E-4	-4.9	2.7E-3	2.5E-3	-9.0
	3	2.4E-3	1.9E-3	-20.1	-3.6E-4	-3.5E-4	-0.4
	4	3.0E-4	5.7E-4	91.7	1.9E-3	1.9E-3	1.3
	5	-1.2E-3	-1.1E-3	-8.2	1.1E-3	9.1E-4	-17.1
	6	2.2E-4	2.0E-4	-10.5	-1.0E-3	-9.3E-4	-8.3
	7	-7.7E-4	-7.1E-4	-8.2	-3.1E-4	-3.3E-4	4.4
	8	2.5E-4	2.2E-4	-13.3	-8.1E-4	-7.2E-4	-11.3
	9	-7.3E-4	-6.8E-4	-6.8	1.4E-3	1.3E-3	-7.0
	10	1.7E-3	1.4E-3	-17.8	-1.6E-3	-1.3E-3	-16.4
	11	-5.0E-6	2.1E-4	-	2.9E-4	2.3E-4	-20.3
	12	-1.3E-3	-1.2E-3	-8.9	-1.7E-4	-2.4E-4	42.9
	13	2.9E-5	3.1E-5	-	-1.9E-3	-1.8E-3	6.5
	14	-8.4E-4	-7.7E-4	-8.8	-1.2E-3	-1.2E-3	3.5

* (GPT − Forward) / Forward × 100 (%)

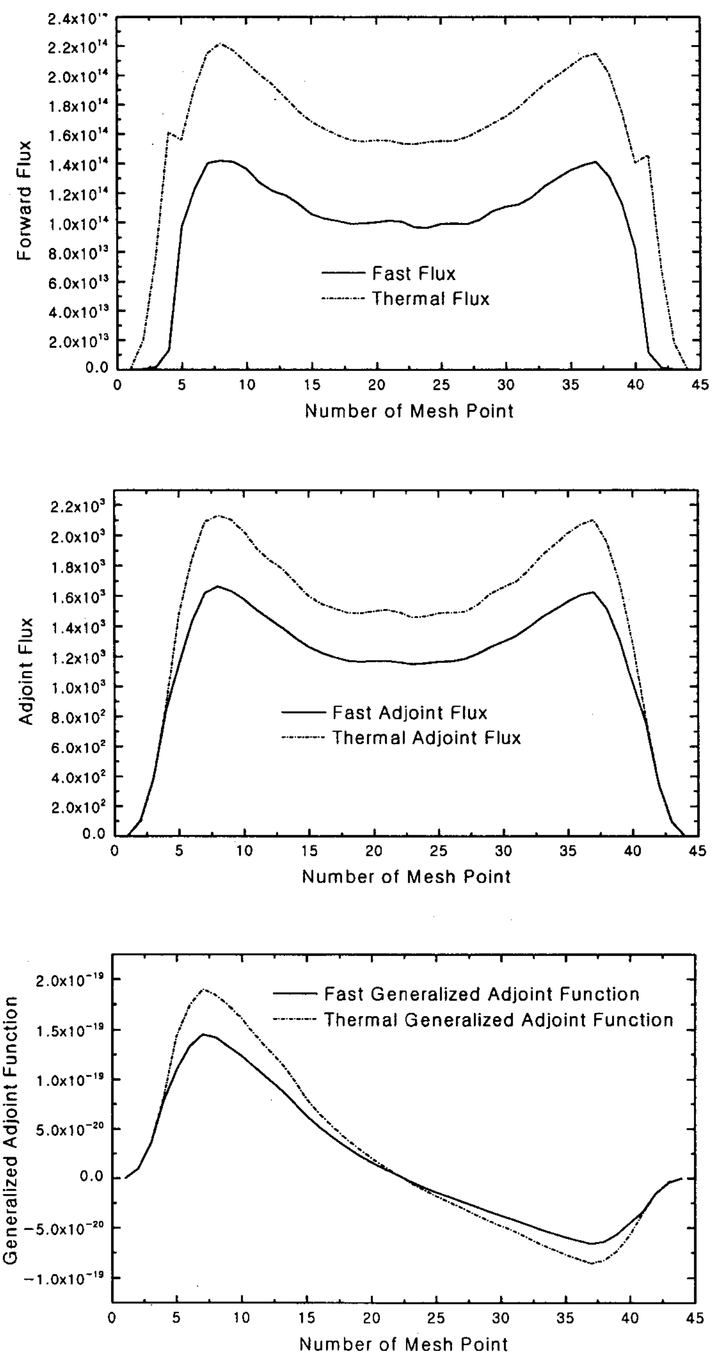

Fig. 1. Forward, Adjoint, and Generalized Adjoint Flux