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Numerical Simulations of Subcritical Reactor Kinetics in Thermal Hydraulic Transient Phases

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Abstract

A subcritical reactor driven by a linear proton accelerator has been considered as a nuclear waste incinerator at Korea Atomic Energy Research Institute (KAERI). Since the multiplication factor of a subcritical reactor is less than unity, to compensate exponentially decreasing fission neutrons, external neutrons from spallation reactions are essentially required for operating the reactor in its steady state. Furthermore, the profile of accelerator beam currents is very important in controlling a subcritical reactor, because the reactor power varies in accordance to the profile of external neutrons. We have developed a code system to find numerical solutions of reactor kinetics equations, which are the simplest dynamic model for controlling reactors. In a due course of our previous numerical study of point kinetics equations for critical reactors, however, we learned that the same code system can be used in studying dynamic behavior of the subcritical reactor. Our major motivation of this paper is to investigate responses of subcritical reactors for small changes in thermal hydraulic parameters. Building a thermal hydraulic model for the subcritical reactor dynamics, we performed numerical simulations for dynamic responses of the reactor based on point kinetics equations with a source term. Linearizing a set of coupled differential equations for reactor responses, we focus our research interest on dynamic responses of the reactor to variations of the thermal hydraulic parameters in transient phases.

I. Introduction

Recently one of our major research projects at Korea Atomic Energy Research Institute (KAERI) is focused on the treatment of high-level radioactive wastes from nuclear power plants. Previously the underground disposal of nuclear wastes has long been considered ever since construction of the first power plant. It, however, has been recognized that this naïve disposal certainly means the inheritance of hazards to future generations for more than tens thousand years as well as nation's economic burden in construction and operation of repository facilities. To relieve public concerns about nuclear waste problem, KAERI has formed a task group to devote R&D in separation and transmutation (S&P) technologies, which are considered as strong supplementary to reduce radiotoxicities and net quantities of spent fuels before sending them to geological burial sites.

A subcritical reactor driven by a proton linear accelerator has been considered as a nuclear waste incinerator at KAERI. Since the multiplication factor of a subcritical reactor is less than unity, external neutrons from spallation reactions are essentially required for operating the reactor in its steady state. Furthermore, the profile of external neutrons is very important in controlling a subcritical reactor since the fission neutrons are exponentially decreasing in this system. It has long been recognized that analysis of reactor dynamics plays an essential role in evaluating reactor stability and designing reactor control system. Recently, we[1,2] have developed a code system for numerically investigating dynamics of nuclear reactors in their transient phases. In a transient phase, the dynamical characteristics of a reactor determine a profile of the reactor power, equivalently, a time rate of population changes inside the reactor.

To study dynamical behavior of subcritical system, we should modify our code system in order to take into consideration of external source terms in the kinetics equations. Our major motivation of this paper is to investigate responses of subcritical reactors for small changes in thermal hydraulic parameters. Building a thermal hydraulic model for the subcritical reactor dynamics, we performed numerical simulations for dynamic responses of the reactor based on point kinetics equations with a source term. Linearizing a set of coupled differential equations for reactor kinetics for a case of a small thermal hydraulic perturbation, we can drop the external source term, which means that there is no difference between critical and subcritical reactors in perturbative treatment of thermal hydraulic effects. We focus our research interest on dynamic responses of the reactor to a small perturbation of the thermal hydraulic parameters in transient phases.

In Section II, we present a brief description of a subcritical reactor for incinerating minor actinides. We also argue certain advantages of a subcritical system over a critical reactor in viewpoints of reactor safeties and controls. Transient models for thermal hydraulic systems in the subcritical reactor are described to prepare for numerical computations. Section III covers a general mathematical modeling of the auxiliary equations required to describe the dynamics of all system components which influence the nuclear behavior of the reactor, that is, the reactor point kinetics equations. Also we describe a linearization procedure to decouple a set of nonlinear differential equations about an operating point in question. Presenting results of numerical simulations, we make our discussion in section IV.

II. Subcritical Reactor for Incinerating Nuclear Wastes

It has been well known[3] that an accelerator driven subcritical reactor for incinerating high-level nuclear wastes has certain advantages in the high-level waste treatment and the nuclear safety management. This system can eliminate the long-lived radioactive wastes in fuel mixtures without requiring a sophisticated reprocessing. Since the spallation neutrons will compensate for the subcriticality, the reactor can be operated without exploiting reactivity margins. Even though the temperature coefficient of certain minor actinides is positive, it does not cause a serious problem in the subcritical reactor, unless it is not very large. However, we have to consider fundamental nuclear safety objectives in designing a subcritical system, such as the control of fission power, adequate cooling systems, reactivity swings, containment of radioactive materials, control of personnel and public exposure, etc.

Since one of strong candidates for incinerating nuclear wastes is a subcritical reactor system, KAERI has established a linear accelerator (1 GeV) project to produce spallation neutrons by injecting a 20 mA CW proton beam into a target. Our previous estimation[4,5] shows that the 1 GeV linear accelerator is capable of producing about 26 neutrons per an incident proton. Thus 20 mA proton beam can yield total 3.25×10^{18} neutrons per second for driving a subcritical reactor. A preliminary study for a subcritical actinide burner[4] shows that the neutron flux of 5.0×10^{15} n/cm^{2}s with average neutron energy of 1.2 MeV is capable for incinerating 2.58% of minor actinides per each fuel cycle of 60 days.

A subcritical reactor can be driven by external neutrons produced from spallation reactions in a target area with high-energy proton beam injected form a linear accelerator. The effective multiplication factor of the reactor core is assumed to be around 0.95. In a viewpoint of controlling subcritical system, the effect of external neutrons can be clearly seen in a transient phase of accelerator beam currents. For example, reactor operators can startup/shutdown the system by turning on/off the beam current injector. As far as the beam current is constant, we can keep the reactor power in its steady state. We assume that the accelerator increases its beam current by 16.7% of its capacity per 10 seconds to drive the reactor in its beginning of the fuel cycle, and that there are certain thermal hydraulic transient phases during a normal operation with a steady beam current. We investigate dynamic responses of subcritical systems for these operating scenarios.

As far as the heat removal and the steam generation from the heat exchanger are concerned, basically we can adopt the thermal hydraulic system designed for LWR power plants. For a purpose of numerical simulations of the subcritical reactor kinetics in thermal hydraulic transient phases, we attach a primary coolant system to the subcritical reactor for removing heats form the fuel pins. The coolants in primary loops transfer their thermal energy to the steam in the heat exchanger. The thermal energy generated from the subcritical reactor is finally converted into the electric energy via a combined system of the turbine and generator. A fraction of the electricity from the generator will be used to supply electric power for operating the linear accelerator.

III. Mathematical Modeling

In contrast to our previous work[1,2] where we just considered the neutronics equations, we are going to study more realistic case by including mechanisms leading to variations of the reactivity dependent on the state of the reactor. If we take reactivity ρ_{ex} as one of the temperature variables, then the point kinetics equation reads

$$\frac{d}{dt}n(t) = \frac{(\rho_{ex}-\beta)}{\Lambda}n(t) + \sum_i \lambda_i C_i(t) + s(t) \tag{1}$$

$$\rho_{ex} = \frac{k_{eff}-1}{k_{eff}} - \alpha_f\left(T_f - T_{fo}\right) - \alpha_c\left(T_c - T_{co}\right) \tag{2}$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i(t) \tag{3}$$

In Eqs. (1~3), $n(t)$, $C(t)$, $s(t)$, k_{eff}, Λ, $\lambda's$, $T's$, $\alpha's$ and $\beta's$ denote the neutron and precursor populations in a reactor, the external neutron source term, the effective multiplication factor, the prompt neutron lifetime, the decay constants of precursors, temperatures, temperature coefficients and the fractions of delayed neutrons, respectively. Also the subscripts i, f, c stand for the values of the i^{th} group precursor, the fuel, the coolant, respectively. The additional subscript o represents the temperatures at the steady state of a reactor. In the above equations we assume that the temperature coefficients be constant.

For thermal hydraulic analysis, the fuel element and coolant heat balance equations are approximately given as follows,

$$M_f c_f \frac{dT_f(t)}{dt} = P(t) - U_f A_f \left(T_f(t) - T_c(t)\right) \tag{4}$$

$$\rho_c V_c c_c \frac{dT_c(t)}{dt} = -W_c c_c \left(T_c(t) - T_h(t)\right) + U_f A_f \left(T_f(t) - T_c(t)\right) \tag{5}$$

where $M's$, $U's$, $A's$, $\rho's$, $V's$, $c's$ are the masses, the heat transmittances, the surface areas, the densities, the volumes, the specific heats for the fuel and coolant, respectively. In Eqs. (4~5), $P(t)$, W_c stand for the reactor power and the mass flow rate of the coolant.

Analogous to the fuel and coolant heat balance equations we introduce a similar equation for heat exchanger as follows,

$$W_g c_g \left(T_g(t) - T_{g,in}\right) = U_g A_g \left(T_h(t) - T_g(t)\right) \tag{6}$$

$$\rho_c V_c c_c \frac{dT_h(t)}{dt} = W_c c_c \left(T_c(t) - T_h(t)\right) - U_g A_g \left(T_h(t) - T_g(t)\right) \tag{7}$$

where the subscripts h, g are representing the values of the heat exchanger and steam, respectively.

Finally, we linearize the nonlinear differential equations based on Taylor series expansion about a reference point in question, i.e., linearization of a product $A = BC$ yields

$$\Delta A = B_o \Delta C + C_o \Delta B \tag{8}$$

If we apply a procedure of Eq. (8) to Eq. (1) for a steady state of a subcritical reactor, the external neutron source term disappears in the linearized equation since it does not depend on thermal hydraulic perturbations. Therefore there is no differences between the critical reactor and the subcritical reactor if the external neutron source is constant, equivalently if the linear accelerator injects stead beam currents into the reactor.

IV. Results and Discussions

In the beginning of fuel cycle, the linear accelerator starts up the subcritical reactor by injecting high-energy proton beams. Since the beam currents are stepwisely increased, the spallation reactions produce synchronously the external neutrons. In this startup phase, the reactor core and coolant are at the preset temperature of 515 K, the steam inlet temperature of heat exchanger is at 400 K.

In Fig. 1 the profile of the reactor power and reactivity clearly show stepwise increase and decrease, respectively, in ramp up phase. The heights of step changes in the power and reactivity are gradually decreasing for both cases, since the feedback effects of temperatures are influencing the reactivity as the reactor increases its power. Note that the small overshoots of the reactor power can also be seen in Fig. 1, and that their sizes are increasing for each step increase of beam currents as the subcriticality is getting bigger and bigger. On the other hand, the reactivity data in Fig. 1 shows that the thermal inertia of hydraulic system makes a gradual saturation of the reactivity, even though the reactor is in temporary steady states right after instantaneous increasing its power at 10 s, 20 s, and so on. Finally, the power of the subcritical system arrives safely at the steady state as the linear accelerator injects constant beam currents into the target.

The temperature profiles of the fuel and coolant and steam are plotted in Fig. 2. The fuel temperature instantly increases soon after starting up the reactor, because fission power heats the fuel elements. However, since the steam inlet temperature of heat exchanger is very low, the coolant loses its thermal energy at the heat exchanger during the first ramp up phase. Consequently it causes the fuel temperature drop rapidly even though the reactor produces marginal fission power. The overall effect of temperature changes causes to decrease subcriticality that can be seen in Fig. 1 in the first two ramps up phases. Therefore we can confirm that the beam currents from the linear accelerator can control the subcritical reactor system.

We assume that the subcritical reactor has been operated in steady state before t=0 without any perturbations in the accelerator and the thermal systems. Introducing a small change in thermal system but keeping the beam currents the same as in steady state, we carry out three case studies for dynamic responses of the subcritical system for a small thermal hydraulic perturbation occurred at t=0.

For a case of 1% increase of the steam mass flow in heat exchanger, the normalized power difference between normal operation and transient phases is plotted in Fig. 3, where the reactor rapidly increases up to 1% of its steady state power. Figure 4 shows that the changes in the fuel, coolant, and steam temperatures. Since the steam mass flow is stepwisely ramped up, it cools the coolant and then the fuel. Note that the coolant and fuel temperatures decrease smoothly, even though the steam temperature drops almost stepwisely. Since the fuel and coolant temperatures are below zero during this phase, their positive contributions to the reactivity cause reactor power increase. This increased reactor power heats the fuel, and increases its temperature above the normal operation. The fuel, coolant, and steam increase their temperatures after hitting their minima. For this case, notice that the subcriticality saturates to a value slightly above the design value of 0.95 as shown in Fig. 3, since the temperature effect due to the subcooled coolant is larger than the heated fuel.

For a case of 1% decrease of the coolant mass flow in heat exchanger, the normalized power difference between normal operation and transient phases is plotted in Fig. 5, where the reactor rapidly increases up to 14.5% of its steady state power. Figure 6 shows that the changes in the fuel, coolant, and steam temperatures. As the coolant mass flow is stepwisely dropped, the coolant loses its temperature in the heat exchanger and then cools the fuel. Note that, due to the thermal inertia of the system, the coolant and fuel temperatures decrease smoothly, even though the coolant mass flow is dropped stepwisely. Since the fuel and coolant temperatures are below zero during this phase, their positive contributions to the reactivity cause reactor power increase. This increased reactor power heats the fuel, and increases its temperature above the normal operation. The fuel, coolant, and steam increase their temperatures after hitting their minima. For this case, notice that the subcriticality saturates to a value slightly above the design value of 0.95 as shown in Fig. 5, since the temperature effect due to the subcooled coolant is larger than the heated fuel.

For a case of 1 K increase of the steam inlet temperature of heat exchanger, the normalized power difference between normal operation and transient phases is plotted in Fig. 7, where the reactor rapidly decreases up to 0.52% of its steady state power. Figure 8 shows that the changes in the fuel, coolant, and steam temperatures. Since the steam inlet temperature is increased stepwisely, it takes less thermal energy from the coolant and then the fuel. Note that the coolant and fuel temperatures increase smoothly, even though the steam inlet temperature is increased stepwisely. Since the fuel and coolant temperatures are above zero during this phase, their negative contributions to the reactivity cause reactor power decrease. This decreased reactor power cools the fuel, and decreases its temperature below the normal operation. The fuel, coolant, and steam decrease their temperatures after hitting their maxima. For this case, notice that the subcriticality saturates to a value slightly below the design value of 0.95 as shown in Fig. 7, since the temperature effect due to the slightly heated coolant is larger than the cooled fuel.

V. Conclusions

In this paper we introduced an accelerator driven subcritical reactor for incinerating nuclear wastes. Building a process dynamics of the reactor and the inherent feedback effects from thermal hydraulic systems, we carried out numerical simulations for dynamic responses of the subcritical system. Linearizing the reactor kinetics equations about the steady state, we pointed out that there are no intrinsic differences between cases of the critical and subcritical systems in treating thermal hydraulic transient phases.

In the beginning of the fuel cycle, our dynamic simulation shows that the accelerator beam currents can synchronously control the reactor power. Since a ramp up of beam currents directly causes increases of the reactor power and the coolant temperature, it makes the reactor power overshoots in ramp up phases. The thermal inertial effect of the thermal hydraulic system turns out to suppress stepwise responses in the reactor power, reactivity, and temperatures for stepwise changes in the thermal hydraulic parameters. The feedback effect with negative temperature coefficients always makes the subcritical system in its stable state. In designing and operating a subcritical system, however, we need to guarantee not to be the reactor in critical condition, since the transitions of the thermal hydraulic system from high to low temperature cause large reactivity swings.

References

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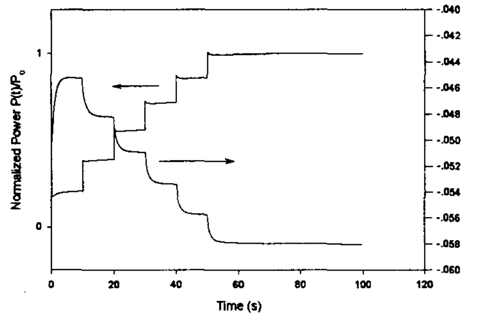

Figure 1. Normalized power and reactivity in ramp up phase.

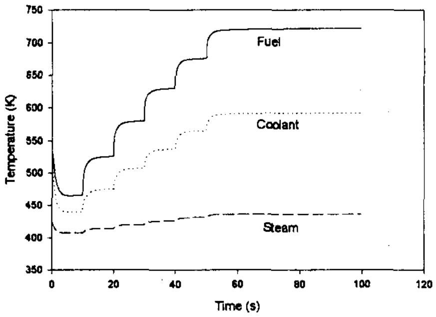

Figure 2. Fuel, coolant, and steam temperatures in ramp up phase.

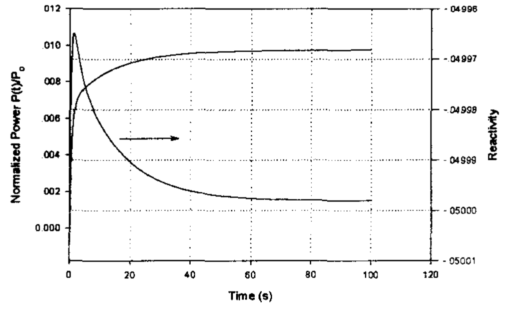

Figure 3. Normalized power difference and reactivity for 1% increase of steam mass flow.

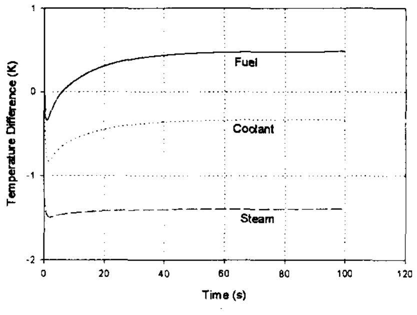

Figure 4. Fuel, coolant, and steam temperature for 1% increase of steam mass flow.

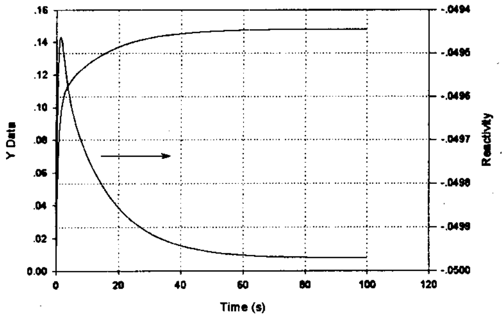

Figure 5. Normalized power difference and reactivity for 1% decrease of coolant mass flow.

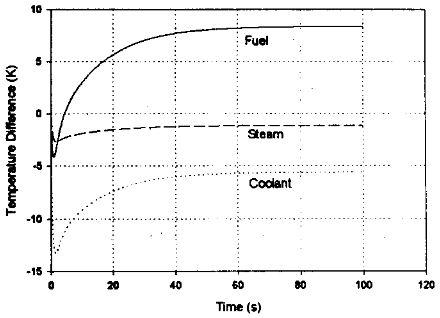

Figure 6. Fuel, coolant, and steam temperature for 1% decrease of coolant mass flow.

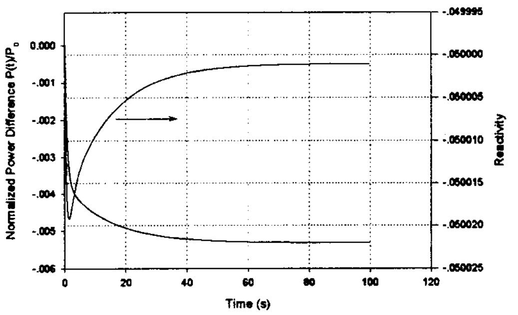

Figure 7. Normalized power difference and reactivity for 1 K increase of steam inlet temperature.

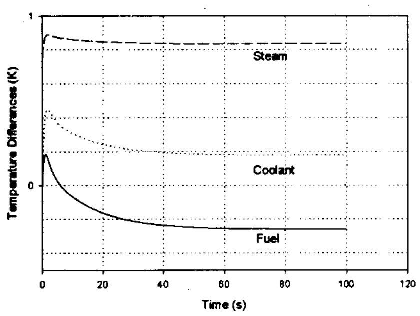

Figure 8. Fuel, coolant, and steam temperature for 1 K increase of steam inlet temperature.