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Tracking Control of RLFJ Robot Manipulator Using Only Position Measurements by Backstepping Method [1]

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Abstract

A tracking controller is presented for RLFJ(rigid link flexible joint) robot manipulators with only position measurements. The controller is developed based on the integrator backstepping design method and on the two observers: the first is simple linear form obsever for the filtered link velocity errors and the other for the actuator velocities. The proposed controller achieves exponential tracking of link positions and velocities while keeping all internal signals bounded. It also guarantees exponential convergence of the estimated signals to their actual ones. Finally, simulation results are included to demonstrate the tracking performance.

1 Introduction

Trajectory tracking control of a robotic manipulator with joint flexibility has received considerable attention in recent years. Since mechanical torques are often applied to the robot links through gearing mechanisms, many researchers have included the effects of joint flexibility (e.g., shaft wind-up, bearing deformation, compressibility of the hydraulic fluid in hydraulic robots, gear train flexibility or elasticity etc.) in the design of advanced robotic controllers. In 1989, Spong[1] proposed the flexible joint robot model and that has been widely used by many researchers in developing their controllers using a variety of techniques such as manifold approach, adaptive/asymptotic adaptive control and so forth. Recently, the concept of '*integrator backstepping*' or '*virtually applied force*' has been used to solve the tracking control problems of the flexible joint manipulators.
One of the main problems of the above controllers is that they require full states: link positions, link velocities, actuator positions, and actuator velocities. For this reason, usually, robotic manipulators are equipped with one position and one velocity sensor for each joint. The available position sensors, such as encoders, give us very accurate measurements of the joint displacements. Whereas the velocity measurements, such as tachometers, are often contaminated by noise and due to their sizes and production costs, in many of today's robot applications, they are seldom used. To address these issues, a number of researchers have proposed various techniques. For example, Tomei[4] proposed an observer design technique that estimates actuator positions and velocities from the measurements of link positions and velocities. With regard to partial state feedback control algorithms, Tomei[5] and Kelly *et al.*[6] proposed controllers for set-point regulation. Recently, based on the model-based observer approach, Lim *et al.*[7] and Chang *et al.*[8] developed the tracking controllers with only link position and actuator position measurements and established local exponential tracking of link positions and velocities.
This paper presents a tracking controller for flexible joint robot manipulators with only position measurements. The tracking controller is developed in steps by using the observer techniques and the integrator backstepping design method.
This paper is organized as follows. In section 2, the model of an n-link flexible joint robot manipulator is described and some of its important properties are given. In section 3, the tracking controller is developed and its exponential convergence is proved. Simulation results of the proved controller is given in section 4 and the conclusion is given in section 5.

2 Model Description and Control Objective

Consider an n-link flexible joint robot manipulator, whose dynamics can be described in Spong[1] and Spong and Vidyasagar[2] as:

$$D(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) + K(q_1 - q_2) = 0 \quad (1)$$

$$J\ddot{q}_2 + B\dot{q}_2 + K(q_2 - q_1) = u, \quad (2)$$

where $q_1 \in R^n$ and $q_2 \in R^n$ represent respectively the vectors of link positions and actuator positions, $K \in R^{n\times n}$ is the diagonal matrix representing joint stiffnesses, $D(q_1) \in R^{n\times n}$ is the link inertia matrix, $C(q_1,\dot{q}_1)\dot{q}_1 \in R^n$ represents the Coriolis and centrifugal terms, $g(q_1) \in R^n$ represents the gravitational terms, $u \in R^n$ is the vector of actuator input torques, $J \in R^{n\times n}$ is the diagonal matrix representing actuator inertia and $B \in R^{n\times n}$ is the diagonal matrix representing actuator damping. Note that the link dynamics (1) and the motor dynamics (2) are coupled only by the *elastic torque term* $K(q_2 - q_1)$. As introduced in Spong[1], there are two sources of interaction between the degrees-of-freedom: the torque transmitted through the spring and the inertial coupling represented by the off-diagonal terms of the inertia matrix and the associated Coriolis forces. In the dynamic model (1) and (2), the inertial coupling is ignored and only the torque transmitted through the joints serves to couple the actuators to the links. In practice the inertial coupling between actuators and links is considerably weaker and can most often be ignored in the control system design (see [1] and [2]). The equations (1) and (2) are nonlinear but they have several fundamental properties that can be exploited to facilitate the control system design. These properties are:

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Property P1: *The link inertia matrix $D(q_1)$ is symmetric, positive definite, and both $D(q_1)$ and $D^{-1}(q_1)$ are uniformly bounded:*

$$\|D(q_1)\|_2 \le D_M \quad \text{and} \quad \|D^{-1}(q_1)\|_2 \le D_m$$

where $\|\cdot\|_2$ denotes the matrix induced two-norm and D_M, D_m are known positive constants.

Property P2: *$C(q_1, x)$ is bounded with respect to any vector x:*

$$\|C(q_1, x)\|_2 \le C_M \|x\|$$

where C_M is a known positive constant and $\|x\|$ denotes the standard Euclidean norm of the vector x.

Property P3: *If $C(q_1, \dot{q}_1)$ is suitably chosen using the Christofel symbols, the matrix $N(q_1, \dot{q}_1) = \dot{D}(q_1) - 2C(q_1, \dot{q}_1)$ is skew symmetric and for this choice, $C(q_1, \dot{q}_1)$ satisfies*

$$\begin{aligned} C(q_1, x)y &= C(q_1, y)x \\ C(q_1, z + \alpha x)y &= C(q_1, z)y + \alpha C(q_1, x)y, \end{aligned}$$

where x, y, and z are any vectors in R^n and α is any scalar value.

In developing the controllers, we assume that the desired trajectory vector $q_{1d}(t)$ of link positions is differentiable up to fourth order and $q_{1d}(t)$ and its derivatives are upper limited as follows:

Assumption A1: *There exist five positive constants d_i, $i = 0, 1, \cdots, 4$ that satisfy the inequalities:*

$$\|q_{1d}(t)\| \le d_0, \quad \|q_{1d}^{(i)}(t)\| \le d_i, \quad i = 1, \cdots, 4$$

where $q_{1d}^{(i)}(t) = d^{(i)} q_{1d}/dt^i$.

Assumption A2: *There exist positive constants P_M such that*

$$\|q_1\| \le P_M$$

In practical applications, q_1 is bounded due to the mechanical structure of the flexible joint robot manipulator. Given only position measurements, our control objective is to achieve exponential tracking of link positions while keeping all internal signals bounded. This indicates that neither the link velocities nor the actuator velocities are available for measurement. In order to quantify the tracking performance, we define the tracking error $e_1 \in R^n$ of link positions as $e_1 = q_1 - q_{1d}$, and the filtered tracking error $r_1 \in R^n$ of link velocities as $r_1 = \dot{e}_1 + \lambda_1 e_1$, where λ_1 is a positive constant. The objective of controller design is then to drive e_1 and r_1 to zero exponentially fast.
For subsequent development of the controllers, we rewrite the dynamic system (1) and (2) in state space form :

$$D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) + Kq_1 = Kx_1 \tag{3}$$

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = -J^{-1}Kx_1 - J^{-1}Bx_2 + J^{-1}u + J^{-1}Kq_1, \tag{5}$$

where $x_1 = q_2$ and $x_2 = \dot{q}_2$.
In the following, we use a hat over a variable (i.e. $\widehat{(\cdot)}$) to denote the estimated value of the variable $(\cdot)$ and a tilde over a variable (i.e., $\widetilde{(\cdot)}$) to denote the estimation error, which is defined as the difference between the actual value and the estimated value of the variable.

3 Controller Design and Stability Analysis

The proposed tracking controller is developed based on the integrator backstepping design method, which views the dynamic system (3), (4) and (5) as the cascade of two parts: the link dynamics (3) and the actuator dynamics (4) and (5). According to this method, the link dynamics (3) is considered to be controlled by the actuator position x_1 through the flexible joints and the actuator dynamics is controlled by the actuator input torque u. Since x_2 is not available for measurement in our problem setting, the backstepping design procedure consists of three steps: (i) the development of the virtual control law α_1 for x_1 that regulates the tracking errors of link positions and velocities; (ii) the design of the actuator velocity observer that determines $\widehat{x}_2$ and regulates $\widetilde{x}_2 = x_2 - \widehat{x}_2$; (iii) the derivation of the actual control law u that regulates the error $z_1 = x_1 - \alpha_1$. We now focus on step (i), the development of α_1 and rewrite (3) as

$$\begin{aligned} D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) + Kq_1 \\ = Kx_1 = K\alpha_1 + Kz_1. \end{aligned} \tag{6}$$

If the link velocity measurements are available, then a natural choice for α_1 would be the Slotine and Li's control law ([3]):

$$\begin{aligned} &\alpha_1(q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, r_1) \\ &= q_1 + K^{-1}\left[D(q_1)a_1 + C(q_1, \dot{q}_1)v_1 + g(q_1) - k_d r_1\right], \end{aligned} \tag{7}$$

where $a_1 = \ddot{q}_{1d} - \lambda_1 \dot{e}_1 = \ddot{q}_{1d} - \lambda_1(r_1 - \lambda_1 e_1)$, $v_1 = \dot{q}_{1d} - \lambda_1 e_1$, and k_d is a positive constant. Substituting (7) into (6) and noting that $r_1 = \dot{q}_1 - v_1$ and $\dot{r}_1 = \ddot{q}_1 - a_1$, we obtain the tracking error dynamics of link positions

$$D(q_1)\dot{r}_1 + C(q_1, \dot{q}_1)r_1 + k_d r_1 = Kz_1. \tag{8}$$

Assuming that the errors z_1 is zero, the asymptotic stability of the error dynamics (8) then follows immediately by choosing the Lyapunov function

$$V = \frac{1}{2} r_1^T D(q_1) r_1$$

and showing that its derivative $\dot{V} \le -k_d r_1^2$. In fact, $\dot{q}_1$ and hence r_1 is not measurable and the virtual control law (7) cannot be used as it is. Therefore, we propose to modify (7) by replacing $\dot{q}_1$ with $\dot{q}_{1d}$ and r_1 with $\widehat{r}_1$, the estimates of r_1. Then, a physically realizable control law α_1 for x_1 to follow is determined as

$$\begin{aligned} \alpha_1(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, \widehat{r}_1) = q_1 + K^{-1}[D(q_1)(\ddot{q}_{1d} \\ -\lambda_1 \widehat{r}_1 + \lambda_1^2 e_1) + C(q_1, \widehat{\dot{q}}_1)v_1 + g(q_1) - k_d \widehat{r}_1], \end{aligned} \tag{9}$$

where $\widehat{\dot{q}}_1 = \widehat{r}_1 + v_1$ and the estimates $\widehat{r}_1$ of r_1 is computed from the observer:

$$\widehat{r}_1 = p_1 + l_1 e_1, \tag{10}$$

$$\dot{p}_1 = -l_1 \widehat{r}_1 + l_1 \lambda_1 e_1, \tag{11}$$

where $l_1 > 0$ is the design constant to be determined during the stability analysis. p_1 here is the auxiliary variable that enables observer design without link velocity measurements.
At this point, we will mention two characteristics of this paper. The first is that we used $\widehat{\dot{q}}_1$, the estimate of $\dot{q}_1$, in

(9) instead of using $\dot{q}_{1d}$. The second is that we used simple linear form observers in (10) and (11) for the unknown link velocity. Since the link dynamics (3) is highly nonlinear, many other papers have suggested nonlinear form observers. Though the observers above have simple linear form, it shows better tracking performance in steady state contrary to other papers. The details will be shown in simulations. In addition, the observers similar to (10) and (11) can be found in [7], but the design constant in [7] consists of many sub-design constants which depend on the desired trajectory and the structure of the robot whereas the design constants here are simple.
Applying (9) into (6) using (7), (8) and (10) yields

$$D(q_1)\dot{r}_1 + C(q_1,\dot{q}_1)r_1 + k_d r_1 = D(q_1)(\lambda_1 r_1 - \lambda_1 \hat{r}_1) \\ -C(q_1,\dot{q}_1)v_1 + C(q_1,\hat{\dot{q}}_1)v_1 + k_d(r_1 - \hat{r}_1) + Kz_1. \quad (12)$$

Here, using the relations $v_1 = \dot{q}_{1d} - \lambda_1 e_1$ and $\hat{\dot{q}}_1 = \hat{r}_1 + v_1$ and Property P3, (12) can be rewritten as the link error dynamics

$$D(q_1)\dot{r}_1 + C(q_1,\dot{q}_1)r_1 + k_d r_1 = \lambda_1 D(q_1)\tilde{r}_1 - C(q_1,\dot{q}_{1d})\tilde{r}_1 \\ +\lambda_1 C(q_1,q_1)\tilde{r}_1 - \lambda_1 C(q_1,q_{1d})\tilde{r}_1 + k_d\tilde{r}_1 + Kz_1, \quad (13)$$

where the term Kz_1 will be driven to zero by another control law α_2 to be developed below. Then, with the substitution of (11), the dynamics of $\tilde{r}_1 = r_1 - \hat{r}_1$ becomes

$$\begin{aligned}\dot{\tilde{r}}_1 = & \ \dot{r}_1 - \dot{\hat{r}}_1 \\ = & -(l_1 - \lambda_1)\tilde{r}_1 - D^{-1}(q_1)C(q_1,\dot{q}_1)r_1 \\ & -k_d D^{-1}(q_1)(r_1 - \tilde{r}_1) - D^{-1}(q_1)C(q_1,\dot{q}_{1d})\tilde{r}_1 \\ & +\lambda_1 D^{-1}(q_1)C(q_1,q_1)\tilde{r}_1 - \lambda_1 D^{-1}(q_1)C(q_1,q_{1d})\tilde{r}_1. \quad (14)\end{aligned}$$

Let's choose at this point a Lyapunov function candidate

$$V_1 = \frac{1}{2}\gamma e_1^T e_1 + \frac{1}{2}r_1^T D(q_1) r_1 + \frac{1}{2}\tilde{r}_1^T\tilde{r}_1, \quad (15)$$

where γ is some positive constant and compute its derivative $\dot{V}_1$ along the trajectories (13) and (14) with a definition V_M such that

$$\|\dot{q}_1\| \le d_1 + \|r_1\| + \lambda\|e_1\| := V_M. \quad (16)$$

Then,

$$\begin{aligned}\dot{V}_1 = & \ \gamma e_1^T\dot{e}_1 + \frac{1}{2}r_1^T\dot{D}(q_1)r_1 + r_1^T D(q_1)\dot{r}_1 + \tilde{r}_1^T\dot{\tilde{r}}_1 \\ \le & -\gamma\lambda_1\|e_1\|^2 + \gamma\|e_1\|\|r_1\| - k_d\|r_1\|^2 + \lambda_1 D_M\|r_1\|\|\tilde{r}_1\| \\ & +C_M d_1\|r_1\|^2 + \lambda_1 C_M V_M\|e_1\|\|r_1\| + k_d\|r_1\|\|\tilde{r}_1\| \\ & -(l_1-\lambda_1)\|\tilde{r}_1\|^2 + D_m C_M V_M\|r_1\|\|\tilde{r}_1\| \\ & +k_d D_m\|r_1\|\|\tilde{r}_1\| + k_d D_m\|\tilde{r}_1\|^2 + D_m C_M d_1\|\tilde{r}_1\|^2 \\ & +\lambda_1 D_m C_M P_M\|\tilde{r}_1\|^2 + \lambda_1 D_m C_M d_0\|\tilde{r}_1\|^2 + r_1^T K z_1 \\ \le & -\left(\gamma\lambda_1 - \frac{1}{2}\gamma - \frac{1}{2}\lambda_1 C_M V_M\right)\|e_1\|^2 \\ & -\left(k_d - C_M d_1 - \frac{1}{2}\gamma - \frac{1}{2}\lambda_1 C_M V_M - \frac{1}{2}\lambda_1 D_M\right. \\ & \left. -\frac{1}{2}k_d - \frac{1}{2}D_m C_M V_M\right)\|r_1\|^2 \\ & -(l_1 - \lambda_1 - k_d D_m - D_m C_M d_1 - \lambda_1 D_m C_M P_M \\ & -\lambda_1 D_m C_M d_0 - \frac{1}{2}\lambda_1 D_M - \frac{1}{2}k_d - \frac{1}{2}D_m C_M V_M \\ & -\frac{1}{2}k_d D_m)\|\tilde{r}_1\|^2 + r_1^T K z_1.\end{aligned}$$

Here, we define σ as $\sigma := \gamma - \frac{1}{2}C_M V_M$ for a γ satisfying that $\gamma > \frac{1}{2}C_M V_M$. We can now define positive constants κ_1, κ_2 and κ_3 as

$$\begin{aligned}\kappa_1 &= \lambda_1\sigma - \frac{1}{2}\gamma, \\ \kappa_2 &= \frac{1}{2}k_d - C_M d_1 - \frac{1}{2} - \frac{1}{2}\lambda_1 C_M V_M - \frac{1}{2}\lambda_1 D_M \\ & \quad -\frac{1}{2}D_m C_M V_M \ \ \textit{and} \\ \kappa_3 &= l_1 - \lambda_1 - k_d D_m - D_m C_M d_1 - \lambda_1 D_m C_M P_M \\ & \quad -\lambda_1 D_m C_M d_0 - \frac{1}{2}\lambda_1 D_M - \frac{1}{2}k_d - \frac{1}{2}D_m C_M V_M \\ & \quad -\frac{1}{2}k_d D_m,\end{aligned}$$

then we have

$$\dot{V}_1 \le -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\tilde{r}_1\|^2 + r_1^T K z_1. \quad (17)$$

Next we develop another virtual control law α_2 for $\hat{x}_2$ in order to drive Kz_1 in (6)to zero. First note that $\alpha_1(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, \hat{r}_1)$ is equivalent to $\alpha_1(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, p_1)$ since $\hat{r}_1 = p_1 + l_1 e_1 = p_1 + l_1(q_1 - q_{1d})$. Hence, the first step to find α_2 is to differentiate z_1 as follows:

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 - \dot{\alpha}_1(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, p_1) \\ &= x_2 - \frac{\partial\alpha_1}{\partial q_1}\dot{q}_1 - \frac{\partial\alpha_1}{\partial q_{1d}}\dot{q}_{1d} - \frac{\partial\alpha_1}{\partial\dot{q}_{1d}}\ddot{q}_{1d} - \frac{\partial\alpha_1}{\partial\ddot{q}_{1d}}q_{1d}^{(3)} \\ & \quad -\frac{\partial\alpha_1}{\partial p_1}\dot{p}_1. \quad (18)\end{aligned}$$

Since $\dot{q}_1 = r_1 + \dot{q}_{1d} - \lambda_1 e_1 = r_1 + \dot{q}_{1d} - \lambda_1(q_1 - q_{1d})$, it follows from (18) that

$$\dot{z}_1 = x_2 - \frac{\partial\alpha_1}{\partial q_1}r_1 - \rho_1(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{1d}^{(3)}, p_1), \quad (19)$$

where

$$\begin{aligned}\rho_1 &= \frac{\partial\alpha_1}{\partial q_1}(\dot{q}_{1d} - \lambda_1(q_1 - q_{1d})) + \frac{\partial\alpha_1}{\partial q_{1d}}\dot{q}_{1d} + \frac{\partial\alpha_1}{\partial\dot{q}_{1d}}\ddot{q}_{1d} \\ & \quad +\frac{\partial\alpha_1}{\partial\ddot{q}_{1d}}q_{1d}^{(3)} + \frac{\partial\alpha_1}{\partial p_1}\dot{p}_1.\end{aligned}$$

Here, we rewrite the equation (19) as

$$\dot{z}_1 = \hat{x}_2 + \tilde{x}_2 - \frac{\partial\alpha_1}{\partial q_1}r_1 - \rho_1(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{1d}^{(3)}, p_1). \quad (20)$$

A good choice for α_2 would then be the control law:

$$\alpha_2(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{1d}^{(3)}, x_1, p_1) = \\ -\hat{r}_1 - k_1 z_1 + \frac{\partial\alpha_1}{\partial q_1}\hat{r}_1 + \rho_1, \quad (21)$$

where $k_1 > 0$ is a design constant. By defining $z_2 = \hat{x}_2 - \alpha_2$, we have from (20) and (21) that

$$\dot{z}_1 = -\hat{r}_1 - k_1 z_1 + z_2 + \tilde{x}_2 - \frac{\partial\alpha_1}{\partial q_1}\tilde{r}_1. \quad (22)$$

Next we move on to step (ii), the design of an actuator velocity observer to regulate $\widetilde{x}_2$ in (20). Because x_2 is not available for measurement, we introduce yet another observer that estimates x_2 from x_1 and drives $\widetilde{x}_2$ to zero exponentially fast. From the actuator dynamics (5), we then propose to formulate the observer as follows :

$$\begin{aligned}\widehat{x}_2 &= p_2 + l_2 x_1 && (23)\\ \dot{p}_2 &= J^{-1}K(q_1 - x_1) - (J^{-1}B + l_2)\widehat{x}_2 + J^{-1}u + v && (24)\end{aligned}$$

where $l_2 > 0$ is the observer gain and v is the auxiliary signals to be designed during the stability analysis. Then, the dynamics of $\widetilde{x}_2 = x_2 - \widehat{x}_2$ becomes

$$\dot{\widetilde{x}}_2 = -l_2\widetilde{x}_2 - J^{-1}B\widetilde{x}_2 - v. \qquad (25)$$

Once again, if we choose a Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_1^T K z_1 + \frac{1}{2}\widetilde{x}_2^T\widetilde{x}_2$$

and compute its derivative $\dot{V}_2$, we have

$$\begin{aligned}\dot{V}_2 \leq\ & -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\widetilde{r}_1\|^2\\ & + r_1^T K z_1 - z_1^T K\widetilde{r}_1 - k_1\lambda_{max}^K\|z_1\|^2 + z_1^T K z_2\\ & - z_1^T K\frac{\partial\alpha_1}{\partial q_1}\widetilde{r}_1 + z_1^T K\widetilde{x}_2 + \widetilde{x}_2^T\dot{\widetilde{x}}_2\\ \leq\ & -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\widetilde{r}_1\|^2 - k_1\lambda_{max}^K\|z_1\|^2\\ & - \left(l_2 - \lambda_{max}^{J^{-1}B}\right)\|\widetilde{x}_2\|^2 + \lambda_{max}^K\|z_1\|\|\widetilde{r}_1\|\\ & + \lambda_{max}^K\left\|\frac{\partial\alpha_1}{\partial q_1}\right\|\|z_1\|\|\widetilde{r}_1\| - \widetilde{x}_2^T(v - Kz_1) + z_1^T K z_2,\end{aligned}$$

where λ_{max}^K and $\lambda_{max}^{J^{-1}B}$ denote the maximum eigenvalues of the matrix K and the matrix $J^{-1}B$, respectively. Then, we obtain

$$\begin{aligned}\dot{V}_2 \leq\ & -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\widetilde{r}_1\|^2 - k_1\lambda_{max}^K\|z_1\|^2\\ & - \left(l_2 - \lambda_{max}^{J^{-1}B}\right)\|\widetilde{x}_2\|^2 + \lambda_{max}^K\left(1 + \left\|\frac{\partial\alpha_1}{\partial q_1}\right\|\right)\\ & \|z_1\|\|\widetilde{r}_1\| - \widetilde{x}_2^T(v - Kz_1) + z_1^T K z_2.\end{aligned}$$

Note here that e_1 is bounded due to Assumption and (16), hence $\frac{\partial\alpha_1}{\partial q_1}$ is also bounded. Letting $\left\|\frac{\partial\alpha_1}{\partial q_1}\right\| \leq \beta_1$ for some $\beta_1 > 0$, we have

$$\begin{aligned}\dot{V}_2 \leq\ & -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\widetilde{r}_1\|^2 - k_1\lambda_{max}^K\|z_1\|^2\\ & - \left(l_2 - \lambda_{max}^{J^{-1}B}\right)\|\widetilde{x}_2\|^2 + \lambda_{max}^K(1+\beta_1)\|\widetilde{r}_1\|\|z_1\|\\ & - \widetilde{x}_2^T(v - Kz_1) + z_1^T K z_2. \qquad (26)\end{aligned}$$

Note here that $\widehat{x}_2$ is not the true control variable for the flexible joint robot system (1) and (2) either. Therefore, we have to go one step further and move on to step (iii) to solve for the true control variable u. Since u is related in (5) to $\hat{x}_2$, we differentiate z_2 to derive an equation that relates x_2 with u. Differentiating z_2, we obtain

$$\begin{aligned}\dot{z}_2 &= \dot{\hat{x}}_2 - \dot{\alpha}_2\\ &= \dot{p}_2 + l_2\dot{x}_1 - \dot{\alpha}_2\\ &= J^{-1}u - J^{-1}K(x_1 - q_1) - J^{-1}B\widehat{x}_2 + v + l_2\widetilde{x}_2\\ &\quad - \frac{\partial\alpha_2}{\partial q_1}\dot{q}_1 - \frac{\partial\alpha_2}{\partial q_{1d}}\dot{q}_{1d} - \frac{\partial\alpha_2}{\partial\dot{q}_{1d}}\ddot{q}_{1d} - \frac{\partial\alpha_2}{\partial\ddot{q}_{1d}}q_{1d}^{(3)}\\ &\quad - \frac{\partial\alpha_2}{\partial q_{1d}^{(3)}}q_{1d}^{(4)} - \frac{\partial\alpha_2}{\partial x_1}x_2 - \frac{\partial\alpha_2}{\partial p_1}\dot{p}_1\\ &= J^{-1}u - \frac{\partial\alpha_2}{\partial q_1}r_1 + l_2 x_2 - \frac{\partial\alpha_2}{\partial x_1}x_2\\ &\quad - \rho_2(q_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{1d}^{(3)}, q_{1d}^{(4)}, x_1, \widehat{x}_2, p_1),\end{aligned}$$

where

$$\begin{aligned}\rho_2 =\ & J^{-1}K(x_1 - q_1) + J^{-1}B\widehat{x}_2 - v\\ & + \frac{\partial\alpha_2}{\partial q_1}(\dot{q}_{1d} - \lambda_1 e_1) + \frac{\partial\alpha_2}{\partial q_{1d}}\dot{q}_{1d} + \frac{\partial\alpha_2}{\partial\dot{q}_{1d}}\ddot{q}_{1d}\\ & + \frac{\partial\alpha_2}{\partial\ddot{q}_{1d}}q_{1d}^{(3)} + \frac{\partial\alpha_2}{\partial q_{1d}^{(3)}}q_{1d}^{(4)} + \frac{\partial\alpha_2}{\partial p_1}\dot{p}_1.\end{aligned}$$

By choosing yet another Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2}z_2^T K z_2$$

and computing its derivative $\dot{V}_3$, we have

$$\begin{aligned}\dot{V}_3 =\ & \dot{V}_2 + z_2^T K\dot{z}_2\\ \leq\ & -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\widetilde{r}_1\|^2 - k_1\lambda_{max}^K\|z_1\|^2\\ & - \left(l_2 - \lambda_{max}^{J^{-1}B}\right)\|\widetilde{x}_2\|^2 + \lambda_{max}^K(1+\beta_1)\|\widetilde{r}_1\|\|z_1\|\\ & - \widetilde{x}_2^T(v - Kz_1) + z_1^T K z_2 + z_2^T K\left[J^{-1}u - \frac{\partial\alpha_2}{\partial q_1}r_1\right.\\ & \left. + l_2\widetilde{x}_2 - \frac{\partial\alpha_2}{\partial x_1}x_2 - \rho_2\right]. \qquad (27)\end{aligned}$$

If we choose the control law as

$$u = J\left[-z_1 - k_2 z_2 + \frac{\partial\alpha_2}{\partial q_1}\widehat{r}_1 + \frac{\partial\alpha_2}{\partial x_1}\widehat{x}_2 + \rho_2\right], \qquad (28)$$

where $k_2 > 0$ is a design constant and substitute it into (27), we have

$$\begin{aligned}\dot{V}_3 \leq\ & -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \kappa_3\|\widetilde{r}_1\|^2 - k_1\lambda_{max}^K\|z_1\|^2\\ & - k_2\lambda_{max}^K\|z_2\|^2 - \left(l_2 - \lambda_{max}^{J^{-1}B}\right)\|\widetilde{x}_2\|^2\\ & + \lambda_{max}^K(1+\beta_1)\|\widetilde{r}_1\|\|z_1\| + \lambda_{max}^K\left\|\frac{\partial\alpha_2}{\partial q_1}\right\|\|\widetilde{r}_1\|\|z_2\|\\ & - \widetilde{x}_2^T K\left(v - z_1 + \frac{\partial\alpha_2}{\partial x_1}^T z_2 - l_2 z_2\right). \qquad (29)\end{aligned}$$

In order to eliminate the last term in (29), let's choose v as

$$v = z_1 - \frac{\partial\alpha_2}{\partial x_1}^T z_2 + l_2 z_2.$$

Note here that $\frac{\partial\alpha_2}{\partial q_1}$ is bounded and let $\left\|\frac{\partial\alpha_2}{\partial q_1}\right\| \leq \mu_1$ for some $\mu_1 > 0$. Then we have

$$\dot{V}_3 \leq -\kappa_1\|e_1\|^2 - \kappa_2\|r_1\|^2 - \left(\kappa_3 - \frac{1}{2}\lambda_{max}^K(1+\beta_1)\right.$$

$$-\frac{1}{2}\lambda_{max}^{K}\mu_1\Big) \|\widetilde{r}_1\|^2 - \lambda_{max}^{K}\left(k_1 - \frac{1+\beta_1}{2}\right)\|z_1\|^2$$
$$-\lambda_{max}^{K}\left(k_2 - \frac{\mu_1}{2}\right)\|z_2\|^2 - \left(l_2 - \lambda_{max}^{J^{-1}B}\right)\|\widetilde{x}_2\|^2$$

Choosing the constant gains λ_1, k_d, l_1, k_1, k_2, and l_2 as follows:

$$\begin{aligned}
\lambda_1 &\geq \frac{1}{\sigma}(\eta + \frac{\gamma}{2}),\\
k_d &\geq \eta D_M + 2C_M d_1 + 1 + \lambda_1 C_M V_M + \lambda_1 D_M\\
&\quad + D_m C_M V_M,\\
l_1 &\geq \eta + \lambda_1 + k_d D_m + D_m C_M d_1 + \lambda_1 D_m C_M P_M\\
&\quad + \lambda_1 D_m C_M d_0 + \frac{1}{2}\lambda_1 D_M + \frac{1}{2}k_d + \frac{1}{2}k_d D_m\\
&\quad + \frac{1}{2}\lambda_{max}^{K}(1 + \beta_1 + \mu_1),\\
k_1 &\geq \eta + \frac{1+\beta_1}{2},\\
k_2 &\geq \eta + \frac{\mu_1}{2},\\
l_2 &\geq \eta + \lambda_{max}^{J^{-1}B},
\end{aligned}$$

where η is a positive constant. From the definition of V_M in (16), defining a region of attraction $\mathcal{D}_R$ as

$$\mathcal{D}_R = \{(e_1(0), r_1(0))| d_1 + \|r_1(0)\| + \lambda\|e_1(0)\| \leq V_M\} \quad (30)$$

where $e_1(0)$ and $r_1(0)$ are the initial conditions of $e_1(t)$ and $r_1(t)$ respectively. If the tracking errors satisfy the initial condition (30), then we conclude that

$$\begin{aligned}
\dot{V}_3(t) &\leq -\eta\|e_1\|^2 - \eta D_M\|r_1\|^2 - \eta\|\widetilde{r}_1\|^2 - \eta\lambda_{max}^{K}\|z_1\|^2\\
&\quad -\eta\lambda_{max}^{K}\|z_2\|^2 - \eta\|\widetilde{x}_2\|^2\\
&\leq -2\eta V_3(t),
\end{aligned}$$

which verifies that $V_3(t) \leq V_3(0)e^{-2\eta t}$ for all $t \geq 0$. Therefore, e_1, r_1, $\widetilde{r}_1$, z_1, z_2 and $\widetilde{x}_2$ all converge to zero exponentially fast with exponential rate η. Moreover, the size of region $\mathcal{D}_R$ can be made arbitrarily large by adjusting the controller gains.

4 Simulation

4.1 One Link FJ Robot

Consider a single link flexible joint manipulator:

$$\begin{aligned}
D\ddot{q}_1 + Mgl\sin(q_1) + K(q_1 - q_2) &= 0\\
J\ddot{q}_2 + B\dot{q}_2 + K(q_2 - q_1) &= u,
\end{aligned}$$

where $D = 10.0(kg \cdot m^2)$, $Mgl = 5.0(Nm)$, $K = 100.0(Nm/rad)$, $J = 1.0(kg \cdot m^2)$ and $B = 1.0(Nm \cdot sec/rad)$. The initial position and velocity are set respectively to $q_1(0) = q_2(0) = 0.5$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$ and the desired link trajectory is set to $q_{1d}(t) = 1.0\sin(t)$. And the observer constants are $l_1 = 50.0$, $k_d = 20.0$, and $\lambda_1 = 10.0$ and the controller gains are set respectively to $k_1 = 40.0$ and $k_2 = 40.0$.

Simulation results are shown from Fig 1 to 3. They show exponential convergence of desired trajectory. Additionally, Fig 4 shows the comparison of link position tracking error performance. Solid line represents $e_1 = q_1 - q_{1d}$ when linear link velocity observer is used which we proposed in this paper and dotted line when nonlinear link velocity observer is used. The nonlinear link velocity observer used here is combination form of (11) and $-k_d D^{-1}(q_1)\widehat{r}_1 - D^{-1}(q_1)C(q_1, \dot{q}_{1d})\widehat{r}_1$ term. Simulation parameters and design constants are the same on both cases with the values used before except that $k_d = 10.0$. As is shown in Fig 4, when linear link velocity observer is used, the steady-state error is reduced remarkably but an oscillation phenomenon is appeared in the transient time.

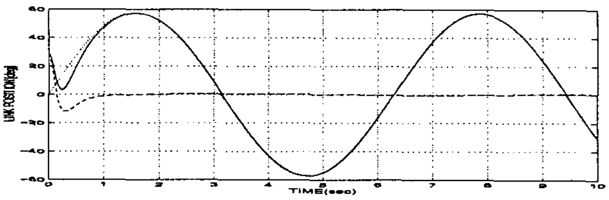

Figure 1: Tracking performance of link position (dotted: q_{1d}; solid: q_1; dashed: $q_1 - q_{1d}$)

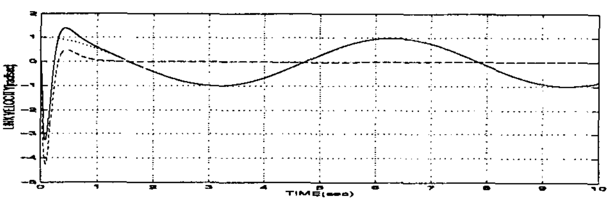

Figure 2: Tracking performance of link velocity (dotted: $\dot{q}_{1d}$; solid: $\dot{q}_1$; dashed: $\dot{q}_1 - \dot{q}_{1d}$)

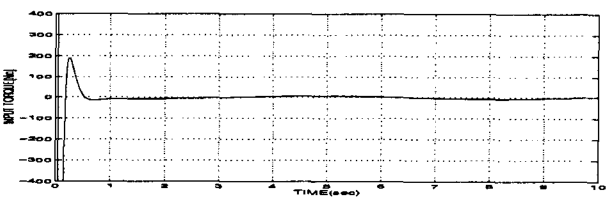

Figure 3: Applied actuator torque

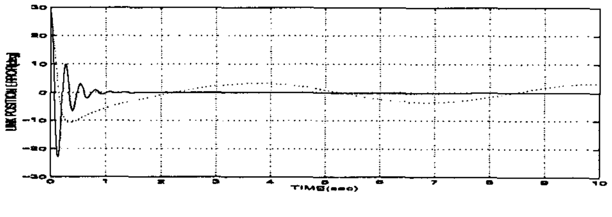

Figure 4: Tracking error of link position when linear/nonlinear link velocity observer is used where $D = 10.0(kg \cdot m^2)$ (dotted: e_1 when nonlinear observer used; solid: e_1 when linear observer is used)

4.2 Two Link FJ Robot

Consider a two link planar robot with flexible joint manipulator, operating in the gravity field. The dynamics are given in (1), (2) and the parameters are as follows:

$$D(q_1) = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \text{kg}\cdot\text{m}^2,$$

$$C(q_1, \dot{q}_1) = \begin{bmatrix} -p_3\dot{q}_{12}s_2 & -p_3(\dot{q}_{11} + \dot{q}_{12})s_2 \\ p_3\dot{q}_{11}s_2 & 0 \end{bmatrix} \text{Nm}\cdot\text{sec/rad},$$

$$g(q_1) = \begin{bmatrix} p_4c_{12} + p_5c_1 \\ p_4c_{12} \end{bmatrix} \text{kg}\cdot\text{m}^2,$$

$K = diag[10.0 \;\; 5.0]$ Nm/rad, $J = diag[0.5 \;\; 0.2]$ kg·m^2 and $B = diag[4.0 \;\; 0.5]$ Nm·sec/rad where $p_1 = 3.31$, $p_2 = 0.116$, $p_3 = 0.16$, $p_4 = 2.274$, $p_5 = 45.366$, c_1, c_2, c_{12} denotes respectively $\cos(q_{11})$, $\cos(q_{12})$, and $\cos(q_{11} + q_{12})$. The initial positions and desired link trajectories are same as one link robot simulation. The initial velocities are set $\dot{q}_1(0) = \dot{q}_2(0) = 1.0$. And the observer constants are $l_1 = 120.0$, $k_d = 15.0$, and $\lambda_1 = 9.0$ and the controller gains are set respectively to $k_1 = 360.0$ and $k_2 = 180.0$. Simulation results are shown from Fig 5 to 9.

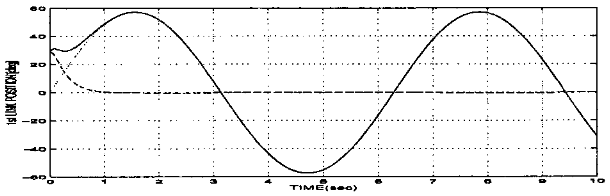

Figure 5: Tracking performance of 1st link position (dotted: q_{11d}; solid: q_{11}; dashed: $q_{11} - q_{11d}$)

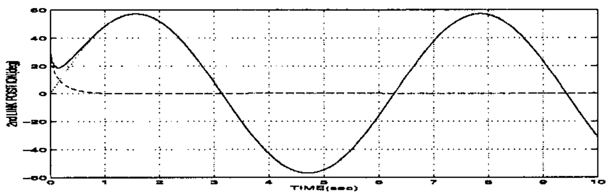

Figure 6: Tracking performance of 2nd link position (dotted: q_{12d}; solid: q_{12}; dashed: $q_{12} - q_{12d}$)

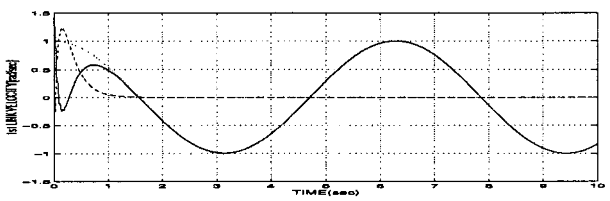

Figure 7: Tracking performance of 1st link velocity (dotted: $\dot{q}_{11d}$; solid: $\dot{q}_{11}$; dashed: $\dot{q}_{11} - \dot{q}_{11d}$)

5 Conclusion

A dynamic tracking controller has been proposed for RLFJ robot manipulators, which tracks the link positions and velocities from position measurements. It is designed based on the integrator backstepping techniques and on two observers: the first is simple linear form obsever for

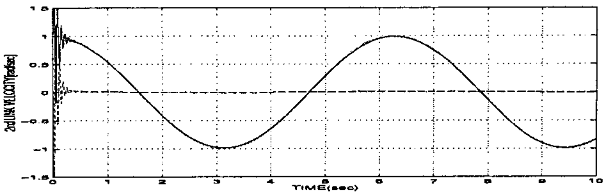

Figure 8: Tracking performance of 2nd link velocity (dotted: $\dot{q}_{12d}$; solid: $\dot{q}_{12}$; dashed: $\dot{q}_{12} - \dot{q}_{12d}$)

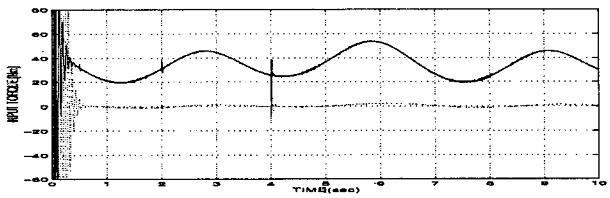

Figure 9: Applied actuator torque (above: 1st actuator torque u_1; below: 2nd actuator torque u_2)

the filtered link velocity errors and the other for the actuator velocities. The proposed controller guarantees that all internal signals remain bounded and drives the link position tracking error to zero exponentially fast if the initial errors are bounded in the region whose size can be made arbitrarily large by adjusting the control gains. It also drives all of the estimated signals to their actual ones exponentially fast. Simulation results of the controller were presented as a successful performance validation of the proposed theoretical development.

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