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Guidance and Control System Design for the Descent Phase of a Vertical Landing Vehicle

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Abstract

This study deals with guidance and control laws for an optimal reentry trajectory of a vertical landing reusable launch vehicle (RLV) in the future. First, a guidance law is designed to create the reference trajectory which minimizes propellant consumption. Then, a nonlinear feedback controller based on a linear quadratic regulator is designed to make the vehicle follow the predetermined reference trajectory. The proposed method is simulated for the first stage of the H-II scale rocket.

Nomenclature

$A(t)$ = system matrix
$B(t)$ = control matrix
$\mathbf{a}_T(t)$ = thrust acceleration vector
C = effective exhaust velosity
C_A = vehicle axis aerodynamic coefficient
C_D = drag coefficient
C_L = lift coefficient
C_N = vehicle-normal axis aerodynamic coefficient
$\mathbf{d}_i$ = defect at the center of the ith segment
$\mathbf{f}(\mathbf{x}, \mathbf{u})$ = dynamics
g = gravity acceleration
$K(t)$ = feedback gain matrix
$m(t)$ = mass of vehicle
$P(t)$ = solution of the algebraic Riccati equation
Q = weighting matrix on state
$\mathbf{R}$ = weighting matrix on control
R = radius of the Earth
$r(t)$ = distance from the Earth's center
S = reference area of the vehicle
$\mathbf{S}$ = equality constraint vector
t = elapsed time
$t_1(t_{d1})$ = inertial flight time (dummy variable)
$t_2(t_{d2})$ = powered flight time (dummy variable)
$T(t)$ = magnitude of thrust
$u(t)$ = horizontal velocity
$\mathbf{u}(t)$ = control vector
$w(t)$ = vertical velocity
$x(t)$ = downrange
$\mathbf{x}(t)$ = state vector
X = horizontal component of external force
$z(t)$ = altitude
Z = vertical component of external force
$\alpha(t)$ = angle of attack
μ = gravitational constant
$\rho(z)$ = atmospheric density
$\rho_s = \rho(0)$ = atmospheric density at sea level
$\theta(t)$ = pitch angle
$\psi(\mathbf{x}, \mathbf{u}, t)$ = boundary constraint vector

subscripts
n : reference value i : node number
0 : initial value f : final value
a : concerned with aerodynamic force

1. Introduction

The aim of this study is to develop guidance and control laws for recovering a launch vehicle such as the first stage of the H-II rocket by vertical landing without the aid of aerodynamic control surfaces. For such a reusable launch vehicle (RLV) to return to a target landing site, the design of a reference trajectory and a nonlinear feedback controller for the descent phase is proposed [1, 2].

The design method comprises of two stages. First, an optimal trajectory to minimize fuel consumption is designed through direct collocation with nonlinear programming (DCNLP) for simplified dynamics [3], in which centrifugal force is neglected and aerodynamic characteristics are extremely simplified to escape the complexity of an optimization calculus [2].

However, due to the neglected and simplified terms, the actual trajectory is perturbed from the designed reference trajectory. Dispersion of entry conditions such as initial position and velocity also influences the trajectory offsets. Therefore, a nonlinear feedback controller based on a linear quadratic regulator (LQR) is designed to restrict the actual trajectory to the reference trajectory [4, 5]. In the design of the LQR, the nonlinear dynamics of the vehicle is linearized about the reference trajectory, assuming a small perturbation [6, 1].

The proposed method was applied to simulate the descent phase of the first stage of the H-II class rocket using simulation software MATLAB and Simulink. The computer simulation showed that the controlled trajectory well follows the reference trajectory over a fairly wide range of initial state dispersion.

2. Equations of Motion of a RLV

The longitudinal translational and the mass equations are given by

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{u} \\ \dot{w} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} u \\ w \\ T\cos\theta/m \\ T\sin\theta/m - \mu/r^2 \\ -T/C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -uw/r \\ u^2/r \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ X_a/m \\ Z_a/m \\ 0 \end{bmatrix} \quad (1)$$

$$r = z + R \quad (2)$$

The aerodynamic force is given by

$$\begin{bmatrix} X_a \\ Z_a \end{bmatrix} = \begin{bmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} C_A \\ C_N \end{bmatrix} \frac{\rho(u^2+w^2)S}{2} \quad (3)$$

where ρ is the exponential atmospheric density.

$$\rho = \rho_s \exp(-z/H), \rho_s = 1.752\ [kg/m^3],\ H = 6.7\ [km] \quad (4)$$

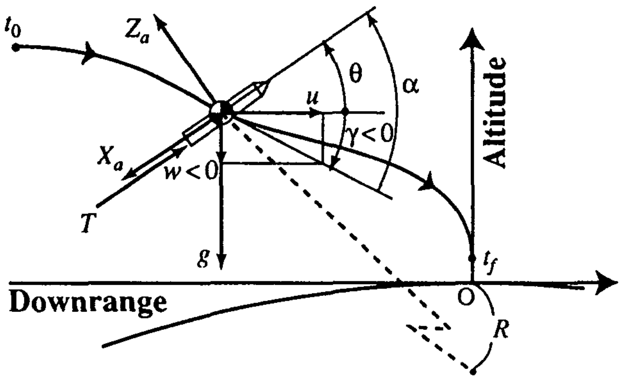

Fig. 1 Definition of variables.

3. Optimal Trajectory Design

A landing guidance law for a minimal trajectory is required to minimize propellant consumption. The problem is described as the following optimal control problem.

Performance index

$$J = \int_{t_0}^{t_f} (-\dot{m})\cdot dt = \frac{1}{C}\int_{t_0}^{t_f} T\cdot dt \quad (5)$$

Constraint

$$\dot{x}_n = f_n(x_n, u_n) \quad (6)$$

$$f_n = \begin{bmatrix} u \\ w \\ \frac{T}{m}\cos\theta \\ \frac{T}{m}\sin\theta - g \\ -T/c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{\rho u}{2m}\sqrt{u^2+w^2}SC_D \\ -\frac{\rho w}{2m}\sqrt{u^2+w^2}SC_D \\ 0 \end{bmatrix} \quad (7)$$

$$x = [x, z, u, w, m]^T,\ u_n = [T, \theta]^T \quad (8)$$

Constraint on control

$$0 \le T \le T_{max} \quad (9)$$

Boundary conditions

$$\psi_0(x_0, u_0, t_0) = 0 \quad (10)$$

$$\psi_f(x_f, u_f, t_f) = 0 \quad (11)$$

Here f_n in Eq. (6) indicates a nominal dynamics where the centrifugal term is neglected, and the aerodynamic term is simply approximated as a sphere.

$$C_D = 0.2, \qquad C_L = 0.$$

According to the Pontryagin's maximum principle, the optimal input takes a value of either 0 in the period t_1, or a known maximum value T_{max} in the period t_2, without taking an intermediate value. Thus, it is found that the powered flight period t_2 occupies the last part of the entire descent phase. Therefore, Eq. (5) can be transformed into

$$J = \frac{T_{max}}{C}\cdot t_2\ , \qquad t_f - t_0 = t_1 + t_2 \quad (12)$$

4. Nonlinear Programming Problem [3]

First, in order to apply NLP, the trajectories in state and control spaces are divided into segments. Then, state vectors and control vectors at each node are assembled into the NLP state vector:

$$X' \equiv [x_0^T, u_0^T, \cdots, x_i^T, u_i^T, \cdots, x_n^T, u_n^T, t_1, t_2, t_{d1}, t_{d2}] \quad (13)$$

Here Eq. (12) is involved in Eq. (13). The last two dummy variables t_{d1} and t_{d2} are introduced to convert the apparent inequalities $t_1 > 0$, $t_2 > 0$ to additional equivalent equality constraints.

$$S \equiv [t_1 - t_{d1}^2, t_2 - t_{d2}^2]^T = \mathbf{0} \tag{14}$$

The defects and problem constraints are collected into the NLP constraint vector C:

$$C^T \equiv [d_1^T, \cdots, d_n^T, S^T, \psi_0^T, \psi_f^T] = \mathbf{0} \tag{15}$$

Here, the defect vectors are calculated as

$$\begin{aligned} d_i &= f_i(x_{ci}, u_{ci}) - \dot{x}_{ci} \\ x_{ci} &= \frac{1}{2}(x_{li} + x_{ri}) + \frac{\Delta t}{8}\{f(x_{li}) - f(x_{ri})\} \\ u_{ci} &= \frac{1}{2}(u_{li} + u_{ri}) \\ \dot{x}_{ci} &= \frac{3}{2\Delta t}(x_{li} + x_{ri}) - \frac{1}{4}\{f(x_{li}) - f(x_{ri})\} \end{aligned} \tag{16}$$

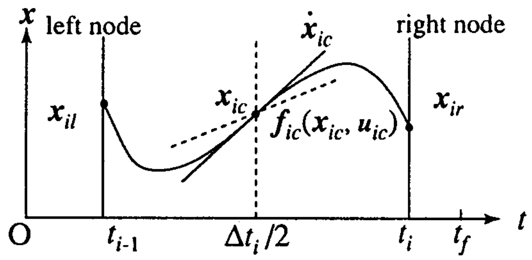

Fig. 2 Direct collocation with nonlinear programming

5. Tracking to the Reference Trajectory [5]

The guidance scheme illustrated above makes use of rough aerodynamic characteristics. Therefore, to track the actual trajectory to the reference trajectory, an additional feedback compensator is needed. For the feedback controller design, a perturbation equation from the reference trajectory can be written in linear form as

$$\Delta\dot{x} = A(t)\Delta x + B_t \Delta a_T \tag{17}$$

$$\Delta x = [\Delta x, \Delta z, \Delta u, \Delta w, \Delta m]^T = x - x_n \tag{18}$$

$$\Delta a_T = [\Delta a_{Tx}, \Delta a_{Tz}]^T = a_T - a_{Tn} \tag{19}$$

The components of $A(t)$ and B_t are obtained by differentiating the nominal dynamics f_n by x and u along the reference trajectory.

$$A(t) = \left.\frac{\partial f_n}{\partial x}\right|_r = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{\rho u V S C_D}{2mH} & -\dfrac{\rho(u^2+V^2)SC_D}{2mV} & -\dfrac{\rho u w S C_D}{2mV} \\ 0 & \dfrac{2m}{(R+z)^3} - \dfrac{\rho w V S C_D}{2mH} & -\dfrac{\rho u w S C_D}{2mV} & -\dfrac{\rho(w^2+V^2)SC_D}{2mV} \end{bmatrix} \tag{20}$$

$$B_t = \frac{\partial f}{\partial a_T} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{21}$$

In Eq. (17), the modelling error $\Delta f = f(x, u) - f_n(x, u)$ is neglected.

$$a_{Tn} = \begin{bmatrix} a_{Tnx} \\ a_{Tnz} \end{bmatrix} = \begin{bmatrix} \cos\theta_n \\ \sin\theta_n \end{bmatrix}\frac{T}{m} \tag{22}$$

Equation (17) is linear with respect to control (acceleration) Δa_T so that LQR works effectively. The nonlinearity between u and a_T is separated from the feedback controller. Thus, the actual control variables T, θ are generated by combining a nonlinear feedforward portion and a linear feedback portion as follows.

Nonlinear feedforward portion :

$$u = u_n + \Delta u = \begin{bmatrix} T \\ \theta \end{bmatrix} = \begin{bmatrix} m\sqrt{a_{Tx}^2 + a_{Tz}^2} \\ \tan^{-1}(a_{Tz}/a_{Tx}) \end{bmatrix} \tag{23}$$

Linear feedback portion :

$$a_T = \begin{bmatrix} a_{Tx} \\ a_{Tz} \end{bmatrix} = \begin{bmatrix} \cos\theta_n \\ \sin\theta_n \end{bmatrix}\frac{T}{m} + \begin{bmatrix} \Delta a_{Tx} \\ \Delta a_{Tz} \end{bmatrix} \tag{24}$$

Assuming the function A(t) is slowly varying, the time-variant system in Eq. (17) could be regarded as a time-invariant system. Thus, at each discrete time t_k, a linear optimal control problem is formulated as shown in the next section.

6. Linear Quadratic Regulator (LQR)

An optimal feedback control law which minimizes a quadratic performance index

$$J = \int_0^\infty (\Delta x^T Q\, \Delta x + \Delta u^T R\, \Delta u)dt \tag{25}$$

is obtained as

$$\Delta a_T = -K(t_k)\,\Delta x \tag{26}$$

$$K(t_k) = R^{-1}B_t^T P(t_k) = R^{-1}[P_{12}^T\ P_{22}] \tag{27}$$

Here $P_{12}(t_k)$ and $P_{22}(t_k)$ are the submatrices of the positive definite solution of the algebraic Riccati equation

$$[PA + A^T P - PBR^{-1}B^T P + Q]_{t_k} = O \tag{28}$$

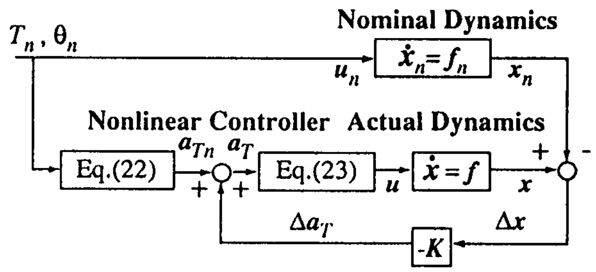

Fig. 3 Block diagram of guidance and tracking control

7. Simulation results

In the optimal trajectory design, the initial and terminal conditions, (10) and (11), were specified below.

$$\begin{bmatrix} x_i \\ z_i \\ u_i \\ w_i \\ m_i \end{bmatrix} = \begin{bmatrix} free \\ 176[km] \\ 5.6[km/s] \\ 0[km/s] \\ free \end{bmatrix}, \quad \begin{bmatrix} x_f \\ z_f \\ u_f \\ w_f \\ m_f \end{bmatrix} = \begin{bmatrix} 0[km] \\ 5[km] \\ 0[km/s] \\ -0.005[km/s] \\ 20{,}000[kg] \end{bmatrix} \tag{29}$$

$$\begin{bmatrix} T(0) \\ \theta(0) \end{bmatrix} = \begin{bmatrix} free \\ \pi \end{bmatrix}, \quad \begin{bmatrix} T(t_f) \\ \theta(t_f) \end{bmatrix} = \begin{bmatrix} free \\ \pi/2 \end{bmatrix} \tag{30}$$

Since the initial mass of the vehicle is to be minimized by the numerical optimization procedure, it was left free. The final mass was specified above including the residual fuel for the final vertical landing phase.

The simulation program was written in Matlab and Simulink. The simulation begins at the completion of the inversion maneuver.

Aerodynamic forces were considered in this simulation. Due to the lack of the aerodynamic data concerning the vehicle, the modified wind tunnel test data, C_A and C_N, at Mach 0.3 for the DC-X vehicle were roughly approximated as

$$\begin{bmatrix} C_A \\ C_N \end{bmatrix}_r = \begin{bmatrix} 0.755\cos\alpha \\ 1.2\sin\alpha \end{bmatrix} \tag{31}$$

and used in the simulation during the entire descent phase. The other parameters are as follows.

$$S = 80\ [m^2]$$

$$T_{max} = 86t = 842.8\ [kN]$$

$$I_{sp} = 445\ [sec]$$

$$C = I_{sp} \cdot g = 4.3639\ [km/sec]$$

Figure 4 (a) shows the designed reference trajectory and controlled trajectory. It was confirmed that the vehicle could follow the reference trajectory well.

Figures 4(b) - (d) each illustrate the time histories of variables of interest.

Figure 5 shows the results for the assumption of 10[km] of initial position dispersion. Figure 5 (a) shows that the resultant trajectory could sufficiently follow the reference trajectory even from dispersed initial points. Due to atmospheric uncertainties, however, a large correction of thrust was required to be generated by the tracking controller, to follow the reference trajectory.

In the case of Fig. 6, where the initial velocity is assumed to be dispersed by 1[km/sec] in each direction, similar phenomena is found as for Fig. 5.

8. Summary and Conclusions

In this paper a guidance and control scheme for a RLV VL without aerodynamic control surfaces, by combining DCNLP and trajectory tracking scheme with nonlinear feedforward and linear feedback compensators is presented.

Computer simulation results were performed for the H-II first stage rocket. The results for the initial condition dispersions showed that the tracking performance of the presented controller was considerably strong.

9. References

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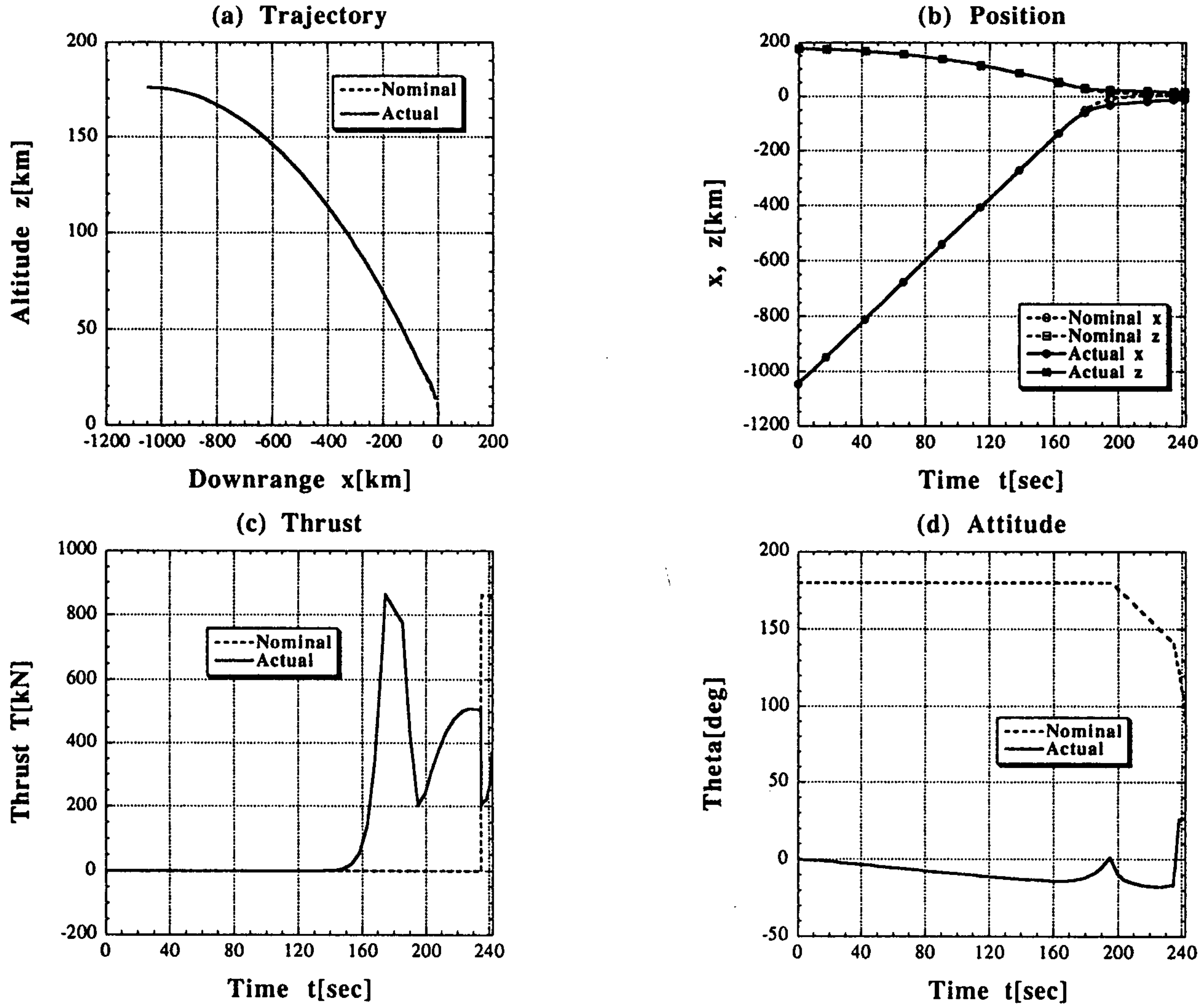
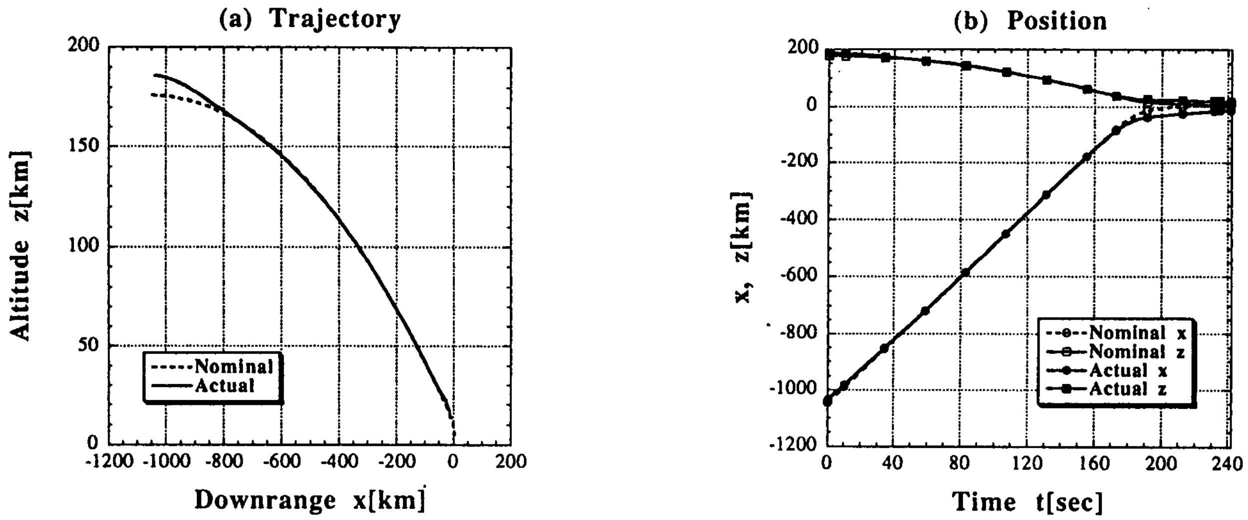


Fig. 4 Trajectory and time histories

$Q = \mathrm{diag}(10^{-3}, 10^{-3}, 10^{-4}, 10^{-4})$, $R = I_2$, $del\_x_0 = del\_z_0 = del\_u_0 = del\_w_0 = 0$



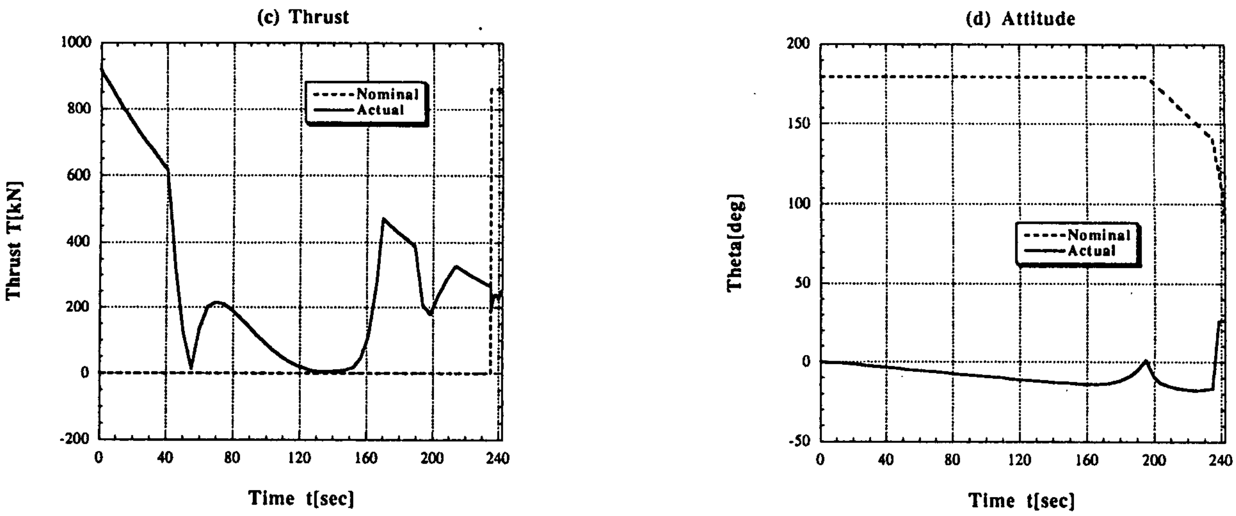


Fig. 5 Trajectory and time histories

$Q = \mathrm{diag}(10^{-5}, 10^{-5}, 10^{-4}, 10^{-4})$, $R = I_2$, $del\_x_0 = 10[km]$, $del\_z_0 = 10[km]$

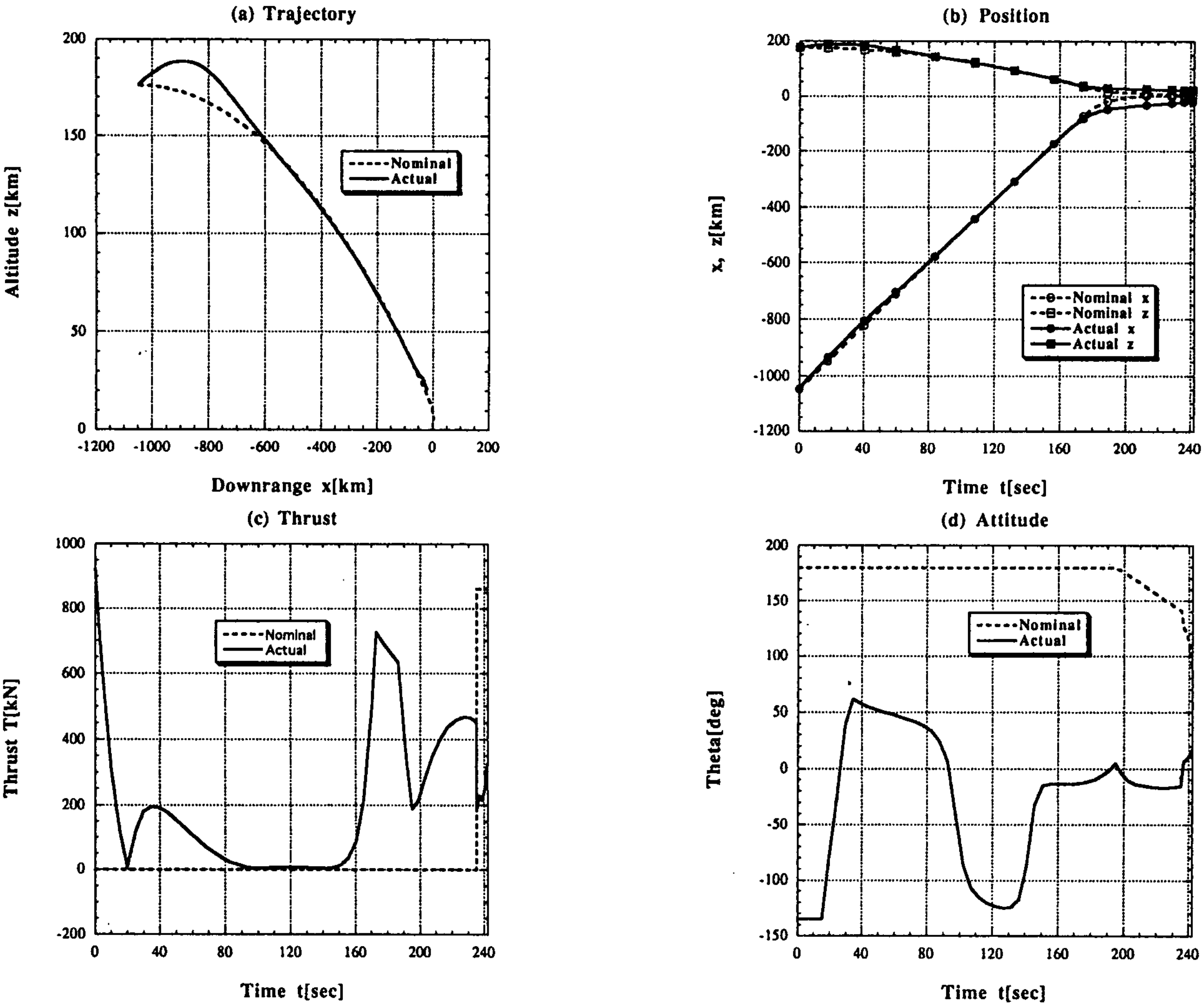


Fig. 6 Trajectory and time histories

$Q = \mathrm{diag}(10^{-5}, 10^{-5}, 10^{-4}, 10^{-4})$, $R = I_2$, $del\_u_0 = 1[km/sec]$, $del\_w_0 = 1[km/sec]$