Proceedings of the 13th KACC, October 1998

Time Domain Identification of an Interval System and Some Extremal Properties

Youngtae Woo, Taeshin Cho, Sunwook Choi, Youngchol Kim

Department of Electronic Engineering, Chungbuk National University
Chongju 361-763, Korea
Tel : +82-431-261-2475
Fax : +82-431-272-2475

Email : yckim@cbucc.chungbuk.ac.kr

Abstract

This paper presents time domain identification of an interval system. We conjectured that Markov parameters (Pulse Responses) from Kharitonov plants would envelope those of the whole interval system. The examination on interrelations between Markov parameters from Kharitonov plants of an interval system and those of the whole interval system determines the validity of the conjecture and is used to give some extremal properties of Markov parameters. The results of this paper are shown in simulations on MBC500 Magnetic Bearing System and a given interval system.

Key words : Markov parameters, Kharitonov plants, Interval system, extremal properties

1. Introduction

It is natural to try to model a family of uncertain systems using the interval framework, since obtaining a very accurate mathematical description of a system is usually impossible and very costly. A recent trend in the area of system identification is an attempt to model the system uncertainties to fit the available and design tools of robust control The interval transfer function is interpreted as a family of transfer function whose coefficients are bounded by some known intervals and centered at the nominal values. But, in many cases this is unnatural in the sense that physical parameter perturbations do not correspond to transfer function coefficients [9]. A major motivation for this paper is the challenge posed by the interval system modeling. In this paper, we have three main objectives in view of time domain identification of an interval system and extremal properties : *1)* to present the efficiency of ERA(Eigensystem Realization Algorithm) as one of the identification algorithms(A-1, section 4). After confirming the result from simulations, we apply ERA to real data obtained from MBC500 Magnetic Bearing System(A-2, section 4), *2)* to examine whether the bound of Markov parameters derived in Kharitonov plants involve the envelope of any parameters in interval or not(B, section 4). This effort leads to the results that the bound between the 16 different trajectories of Markov parameters based on Kharitonov plants can't involve arbitrary coefficients in interval system, *3)* to investigate the relationship between the characteristics of Markov parameters in time history and the step responses of Kharitonov plants(C, section 4). In section 2, the theoretical background used for identifying a model is introduced. In section 3, problem formulation is described. And Simulation results are shown in section 4. Finally, Conclusions are described in section 5.

This work was supported in part by MOST of Korea under grant 97-I-01-03-A-101.

2. Theoretical background

Consider the discrete linear time invariant system described by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{1}$$

where $x(k)$ is an $n \times 1$ vector, $y(k)$ an $m \times 1$ vector, and $u(k)$ is an $r \times 1$ vector with A, B, C and D being system matrices with appropriate dimensions. Assuming zero initial conditions, the set for a sequence of $k = 0, 1, \cdots, l-1$ can be written as

$$\begin{aligned} x(l-1) &= \sum_{i=1}^{l-1} A^{i-1} Bu(l-1-i), \\ y(l-1) &= \sum_{i=1}^{l-1} CA^{i-1} Bu(l-1-i) + Du(l-1) \end{aligned} \tag{2}$$

Eq. (2) can be assembled into Markov parameters(a sequence of pulse-response) matrix Y_k with dimension m by r as follows:

$$Y_0 = D,\ Y_1 = CB,\ Y_2 = CAB,\ \cdots,\ Y_k = CA^{k-1}B . \quad (3)$$

Eq. (3) can be grouped in a matrix from to yield

$$y_L = Y_L U_L \quad (4)$$

where

$$y_L = [y_0\ y_1\ y_2\ \cdots y_{l-1}]$$
$$Y_L = [D\ CB\ CAB\ \cdots\ CA^{l-2}B]$$
$$U_L = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(l-1) \\ & u(0) & u(1) & \cdots & u(l-2) \\ & & u(0) & \cdots & u(l-3) \\ & & & \ddots & \vdots \\ & & & & u(0) \end{bmatrix} \quad (5)$$

Consider the case where A is asymptotically stable so that for some sufficiently large p, $A^k \approx 0$ for all time steps $k \ge p$. Eq. (4) can then be approximated by (6)

$$y_P = Y_P U_P$$

where

$$y_P = [y(0)\ y(1)\ y(2)\ \cdots\ y(p)\ \cdots\ y(l-1)]$$
$$Y_P = [D\ CB\ CAB\ \cdots\ CA^{p-2}B]$$
$$U_P = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(p) & \cdots & u(l-1) \\ & u(0) & u(1) & \cdots & u(p-1) & \cdots & u(l-2) \\ & & u(0) & \cdots & u(p-2) & \cdots & u(l-3) \\ & & & \ddots & \vdots & \cdots & \vdots \\ & & & & u(0) & \cdots & u(l-p-1) \end{bmatrix} \quad (7)$$

In this procedure, ERA (Eigensystem Realization algorithm) based on Markov parameters can then be effectively applied to the problem of system identification. It includes the original minimal realization approach of Ho and Kalman[7], and the following steps.

Step1) Form a block data matrix which is obtained by deleting some rows and columns of the generalized Hankel matrix composed of the Markov parameters as follows :

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \cdots & Y_{k+\alpha+\beta-2} \end{bmatrix} \quad (8)$$

Then, construct a block Hankel matrix $H(0)$ by arranging the Makov parameters into blocks with given α, β.

Step2) Decompose $H(0)$ using singular value decomposition.

$$H(0) = R\Sigma S^T = [R_2 : R_0]\begin{bmatrix} \Sigma_2 & 0 \\ 0 & \Sigma_0 \end{bmatrix}[S_2 : S_0]^T \quad (9)$$

Step3) Determine the order of the system by examining the singular values of the Hankel matrix $H(0)$.

Step4) Construct a minimum order realization $[\hat{A}, \hat{B}, \hat{C}]$ using a shifted block Hankel matrix $H(1)$.

$$\begin{aligned} \hat{A} &= \Sigma_2^{-1/2} R_2^T H(1) S_2 \Sigma_2^{-1/2} \\ \hat{B} &= \Sigma_n^{1/2} S_n^T E_r \\ \hat{C} &= E_m^T R_n \Sigma_n^{1/2} \end{aligned} \quad (10)$$

It is defined that $E_m^T = [I_m\ O_m\ \cdots O_m]$ where m is the number of outputs, and $E_r^T = [I_r\ O_r\ \cdots O_r]$ where r is the number of inputs, O_i as a null matrix of order i, I_i as an identity matrix of order i.

Step5) Convert the estimated discrete state-space model to the continuous transfer function

3. Problem Formulation

Consider an interval system,

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{a_m s^m + a_m s^{m-1} + \cdots + a_1 s + a_0} \quad (11)$$

where

$$a_i \in [a_i^-\ a_i^+]\ i = 1,2,\cdots,m\ ,\quad b_i \in [b_i^-\ b_i^-]\ j = 1,2,\cdots,n .$$

For the linear interval system where the uncertain parameters lie in intervals and appear linearly in the numerator and denominator coefficients of the transfer functions , it is known that, by the Generalized Kharitonov theorem, a set of plants which are the combinations of the Kharitonov polynomials in interval system characterize the frequency domain behavior of linear interval system [9]. Therefore, each Kharitonov polynomial of numerator and denominator is

$$\begin{aligned} K_D^1 &= a_m^- s^m + a_{m-1}^- s^{m-1} + a_{m-2}^+ s^{m-2} + a_{m-3}^+ s^{m-3} + \cdots \\ K_D^2 &= a_m^+ s^m + a_{m-1}^+ s^{m-1} + a_{m-2}^- s^{m-2} + a_{m-3}^- s^{m-3} + \cdots \\ K_D^3 &= a_m^+ s^m + a_{m-1}^- s^{m-1} + a_{m-2}^- s^{m-2} + a_{m-3}^+ s^{m-3} + \cdots \\ K_D^4 &= a_m^- s^m + a_{m-1}^+ s^{m-1} + a_{m-2}^+ s^{m-2} + a_{m-3}^- s^{m-3} + \cdots \end{aligned} \quad (12)$$

$$\begin{aligned} K_N^1 &= b_n^- s^n + b_{n-1}^- s^{n-1} + b_{n-2}^+ s^{n-2} + b_{n-3}^+ s^{n-3} + \cdots \\ K_N^2 &= b_n^+ s^n + b_{n-1}^+ s^{n-1} + b_{n-2}^- s^{n-2} + b_{n-3}^- s^{n-3} + \cdots \\ K_N^3 &= b_n^+ s^n + b_{n-1}^- s^{n-1} + b_{n-2}^- s^{n-2} + b_{n-3}^+ s^{n-3} + \cdots \\ K_N^4 &= b_n^- s^n + b_{n-1}^+ s^{n-1} + b_{n-2}^+ s^{n-2} + b_{n-3}^- s^{n-3} + \cdots \end{aligned} \quad (13)$$

From eq. (12) and eq. (13), 16 Kharitonov plants can be derived as follows;

$$G_v(s) = \frac{K_N^i}{K_D^j} \qquad i = 1,2,3,4\ j = 1,2,3,4\ v = 1,2,\cdots,16 \quad (14)$$

From eq(14), we can conjecture that Markov parameters estimated by ERA from the 16 plants of interval system would envelope Markov parameters of the whole interval system. It would be of use to find extremal properties between the characteristics of Markov parameters and the step responses of interval system. As a counterexample to examine the validity of the conjecture, an interval system can be considered. In this procedure, it needs an evaluation about the efficiency of ERA and examination of the bound of Markov parameters derived from Kharitonov plants. In the following section, the results are shown through several examples.

4. Simulation Results

A-1) Example 1: The efficiency of ERA

To evaluate the efficiency of the ERA estimation method, we Consider a given linear time invariant model whose coefficients are fixed,

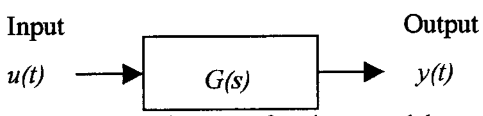

Fig. 1 Block diagram of a given model

The given linear time invariant model in fig. 1 is,

$$G(s)=\frac{17.7223s^2+25s+14.8112}{s^4+7s^3+17.7223s^2+25s+14.8112} \qquad (15)$$

As in Fig.2, the excited input signal data and the output signal data, the response of the given system, are used. The specification of these signals is as follows;

- $u(t)$: 1023 × 1 *Input vector (PRBS)*
- $y(t)$: 1023 × 1 *Output vector*
- *Sampling time : 0.05 sec*
- *Frequency bandwidth of input signal :* 10^{-1} Hz ~ 10^{2} Hz
- *Excitation time : 51.1 sec*

By eq. (6) and (7), Markov parameters derived from the input/output data are shown in fig. 3. The number of Markov parameters used in this simulation is 301.
The coefficients of model estimated by eq. (8), (9), (10) from Markov parameters are compared with those of the given model. The order of the estimated model is the same of the given model.

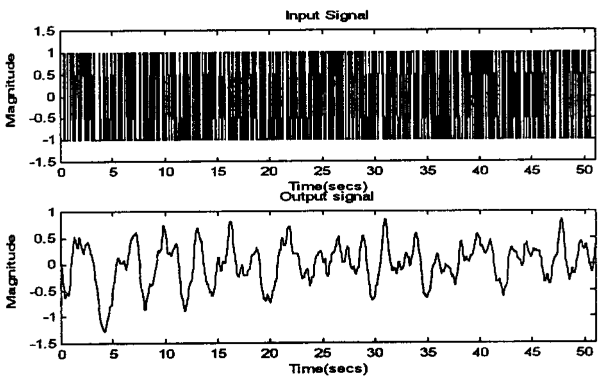

Fig.2 Input / Output signal

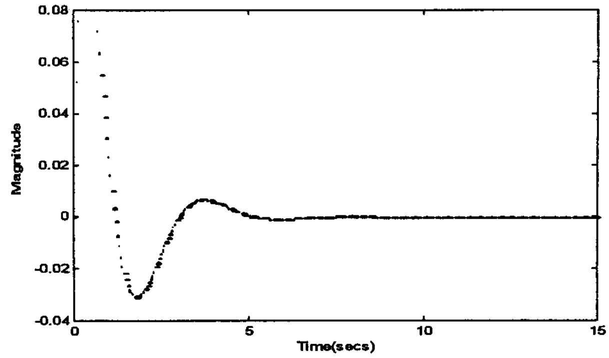

Fig.3 Markov parameters

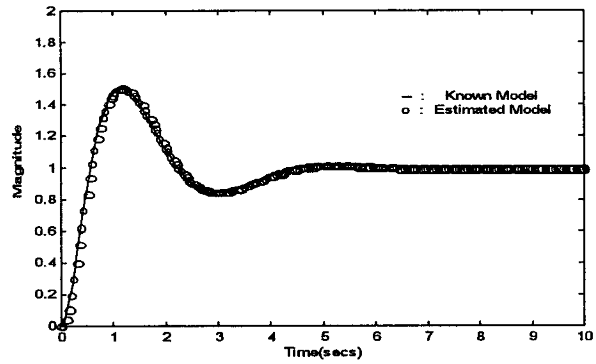

Fig.4 Comparison of step response between the Given Model and the Estimated Model

Table. 1 Comparison of coefficients between the given model and the estimated model

	Given Model	Estimated Model
a_4	1	1
a_3	7	7.0302
a_2	17.7223	17.5875
a_1	25	25.5119
a_0	14.8112	15.5577
b_2	17.7223	16.6482
b_1	25	25.3336
b_0	14.8112	15.4539

Fig. 4 and Table.1 show that ERA estimates the given model with a high accuracy.

A-2) Example 2: The application of ERA to Magnetic Bearing System

As a practical case of identifying an unknown system, we experimented with Magnetic Bearing System which operates stably. In this simulation, a closed-loop system is considered. And the structure of this system is represented in fig. 5.

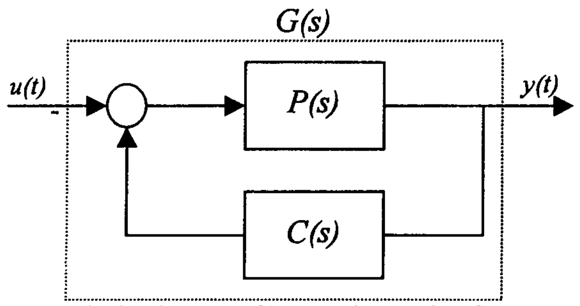

Fig.5 Block Diagram of Magnetic Bearing System

In fig. 6, the input signal and output signal used in this simulation are shown. The specification of these signals in this experiment is as follows;

- $u(t)$: 50002×1 *Input vector (PRBS)*
- $y(t)$: 50002×1 *Output vector*
- *Sampling time : 0.0001 sec*
- *Frequency bandwidth of input signal :* 10Hz ~10kHz
- *Excitation time : 5 sec*

And, 2000 Markov parameters derived from eq. (6) and (7) are shown in fig. 7. After examining the singular values of the Hankel matrix $H(0)$, the minimal order of this system 12, is determined by eq. (8), (9) and (10). The estimated transfer function of this system is,

$$G_B(s) = \frac{N(s)}{D(s)} \qquad (16)$$

where,

$$N(s) = -0.002134s^{12} + 60.2001s^{11} - 379940.6442s^{10} + 896799249.9701s^9$$
$$- 6066962651178.26s^8 - 932837689816997s^7 - 1.958302e{+}18s^6$$
$$- 1.859089e{+}020s^5 - 2.044439e{+}023s^4 - 1.046559e{+}025s^3$$
$$- 8.428408e{+}027s^2 - 1.788379e{+}029s - 1.187480e{+}032$$
$$D(s) = s^{12} + 305.6821s^{11} + 25290786.417s^{10} + 6921924171.4647s^9$$
$$+ 12966412199201.3s^8 + 2.173869124e{+}015s^7 + 2.1728e{+}018s^7$$
$$+ 2.300373e{+}020s^5 + 1.598700e{+}023s^4 + 9.791511e{+}024s^3 +$$
$$+ 5.313943e{+}027s^2 + 1.430588e{+}029s + 6.492615e{+}031$$

The step response of estimated transfer function is shown in fig. 8. Finally, to check the validity of the estimated transfer function, the original output data and the estimated model's output data are compared in fig. 9.

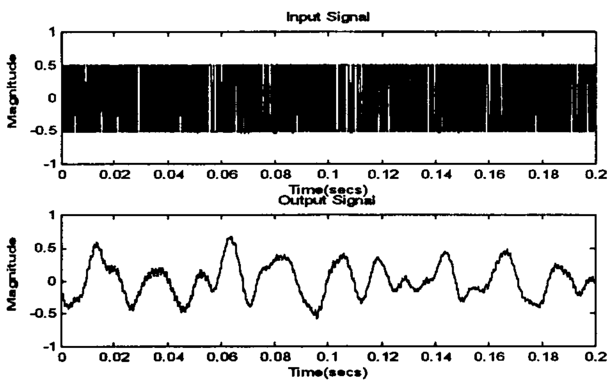

fig.6 Input / Output signal

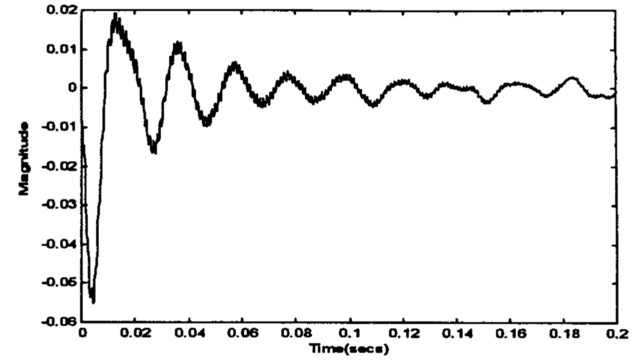

Fig.7 Markov parameters

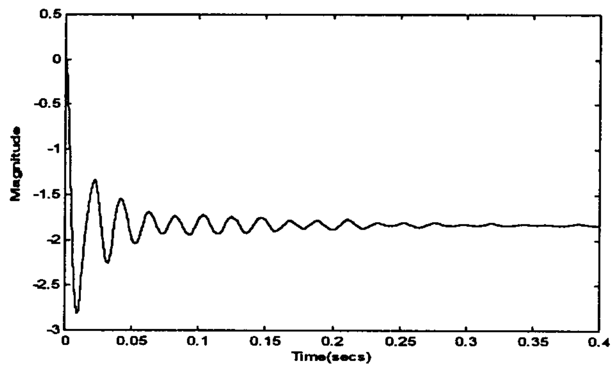

Fig.8 Step response of the estimated model

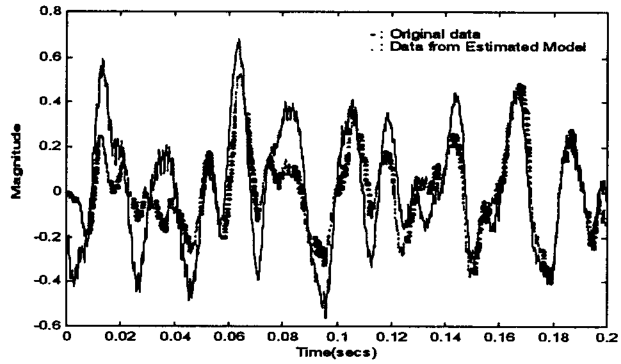

Fig.9 Model Validation Check

B) Example 3: The bound of Markov parameters

As a main example, an interval system is considered. The structure of $G(s)$ in fig.1 is assumed to be an interval plant. The given interval plant is,

$$G(s) = \frac{[17.3223\ 17.823]s^2 + [25\ 25.5]s + 14.8112}{s^4 + [7\ 7.5]s^3 + [17.3223\ 17.8223]s^2 + [25\ 25.5]s + 14.8112} \qquad (15)$$

Kharitonov plants of a given interval system are selected by eq. (12), (13), (14). The same PRBS used in example 1 is excited to the 16 plants. The specification of these signals is :

- $u(t)$: 1023×1 *Input vector (PRBS)*
- $y(t)$: 1023×1 *Output vector*
- *Sampling Time : 0.05 sec*
- *Frequency Bandwidth of input signal :* 10^{-1} Hz ~ 10^{2} Hz
- *Excitation time : 51.1 sec*

From the time history data of input/output signal, 16 different kinds of Markov parameters are derived. Those parameters are shown in fig. 11. To check whether the Markov parameters from the 16 plants envelope all the Markov parameters derived from the interval system, several plants in the interval system are arbitrarily selected. The Markov parameters from 16 Kharitonov plants with those from arbitrarily selected plants are compared in fig. 12. Fig.13, the zoomed plot of fig.12, shows that the Markov parameters from 16 Kharitonov plants do not envelope even the Markov parameters from arbitrarily selected plants.

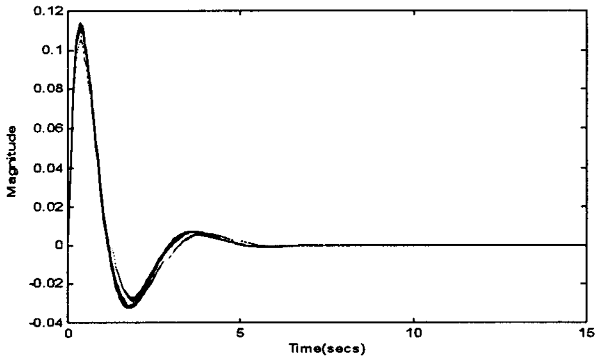

Fig.11 Markov parameters from 16 Kharitonov plants of the interval system

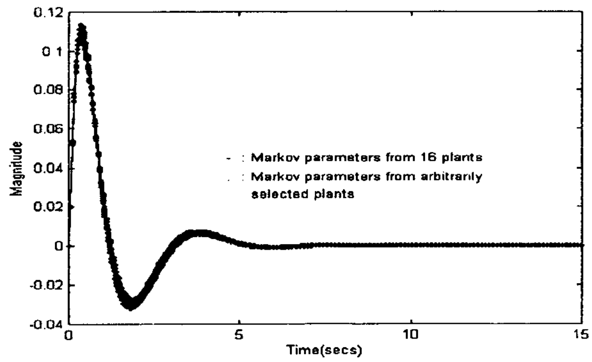

Fig.12 Comparison of Markov parameters from 16 Kharitonov plants with those from arbitrarily selected plants.

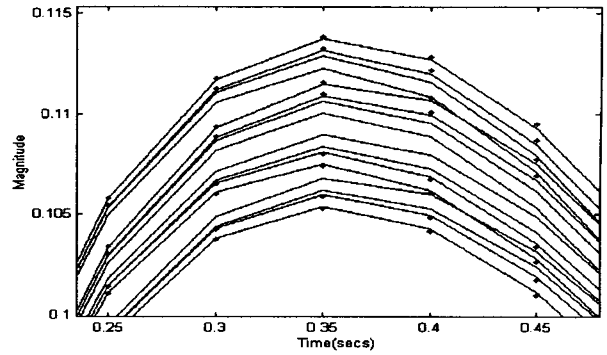

Fig.13 Zoomed Markov parameters of Fig.12

C) Example 4: Maximal overshoot

In this example, the relationship between a model whose Markov parameters have the maximal absolute values at each time step and the maximum overshoot in the step responses of Kharitonov plant is investigated. This result is shown in fig.14 and 15.

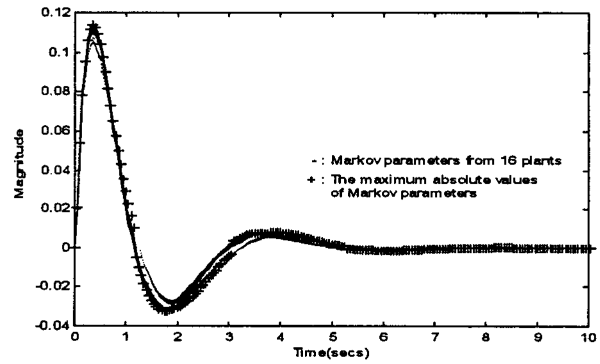

Fig.14 Comparison of Markov parameters

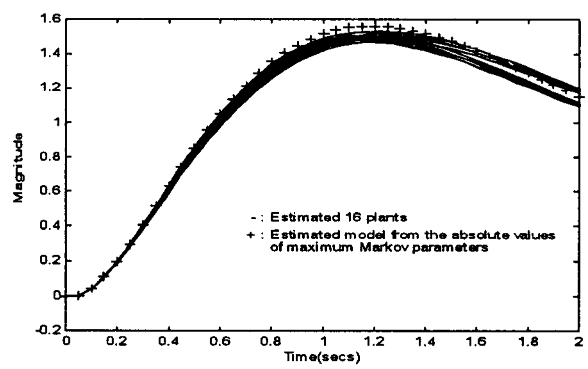

Fig.15 Comparison of Step Responses

5. Conclusion

In this paper, we have presented some results obtained through counterexamples for time domain identification of an interval system. By the results, the conjecture that Markov parameters (Pulse Responses) from Kharitonov plants would envelope those of the whole interval system was proved to be not valid. Also, the result that a plant whose Markov parameters have the maximal absolute value at each time step gives the maximum overshoot in step responses of an interval system is shown as one of extremal properties of the interval system. Finding the bound of coefficients of an interval system remained to be developed in the future.

References

[1] Juang, J.N., *Applied System Identification*. Prentice-Hall,Englewood Cliffs, New Jersey, 1994.

[2] Ljung, L., *System Identification theory for the user*. Prentice-Hall, Englewood Cliffs, New Jersey, 1987.

[3] Bayard, D. S., "An Algorithm for State Space Frequency Domain Identification without Windowing Distortions," *Proceedings of the Control and Decision Conference,* December 1992.

[4] Chen, C.T., *Linear System Theory and Design*, Holt, Rienharts and Winston, Inc., New York, New Yowk, 1984.

[5] Gantmacher, F. R., *The Theory of Matrices*, Chelsea, New York, New York, 1959.

[6] Juang, J.N. and Pappa, R. S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction," *Journal of Guidance, Control, and Dynamics*, vol. 8, No. 5, pp. 620-627, 1985,.

[7] B.L Ho and R.E, Kalman,"Efficient construction of linear state variable models from input/output functions," *Regelungstechnik*, vol. 14, pp. 545-548,1996.

[8] Kailath, T., *Linear Systems*, Prentice-Hall, Englewood Cliffs, New Jersey, 1980.

[9] Bhattacharyya, S.P, Chapellat., H and Keel, L. H, *Robust Control : The Parametric Approach*, Prentice Hall, 1995.