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QFT Tunning of Multivariable Mu Controllers

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ABSTRACT

We argue that the combination of optimal control synthesis and QFT tuning enables design of controllers with levels of performance that surpasses what can be achieved using only a single technique. Using a constructive example, we demonstrate how the strength of each technique is utilized to arrive at a particularly desired controller in terms of tradeoffs between performance and controller complexity.

KEYWORDS: *μ-analysis and synthesis, QFT, robust stability, robust performance*

I. INTRODUCTION

In the past decade, norm-based optimal control techniques have taken center stage for solving simple as well as complex control problems in linear, time-invariant feedback systems. They allow for plant descriptions that include various classes of norm-bounded uncertainties in unstructured models. And handle single-loop and multi-loop problems alike. The performance specifications are defined in terms of H_∞ norms of closed-loop transfer functions and optimal control generates the solution if it exists. In contrast, control engineers often use experience and insight into a particular problem as the design guidelines and prefer use of manual loop shaping as the means of generating the controller. This approach has the advantage in that the designer can work directly with frequency responses. In QFT[3], the quality of the design strongly depends on the skills of the control engineer, with respect to manual loop shaping. But it also requires a great deal of experience to be used for complex problems such as multivariable and non-square systems, and plants with a large number of resonances. Naturally, the availability of an initial design would be of great help to the QFT designer. Based on our experience with both H_∞ and QFT design approaches, it appears that by combing both into a single design process, the control engineer could enjoy the benefits offered by each approach. Considering the weaknesses and strengths of optimal control and QFT, it seems worthwhile to explore the possibilities of combining the two approaches into a single, sequential, design procedure

II. THE DESIGN PROBLEM

This section describes a design example taken from the μ-analysis and synthesis toolbox [1]. It involves a 2x2-pitch axis controller of an experimental highly maneuverable airplane, HIMAT. The block diagram of the HIMAT control problem is shown in Fig. 1.

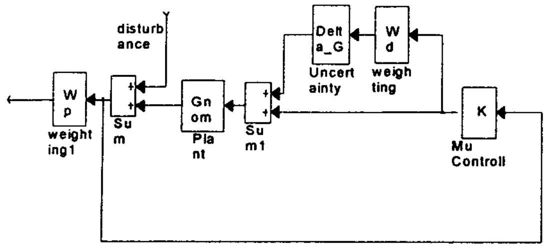

Figure 1: Closed-loop interconnection structure of HIMAT

The interested reader should refer to [1] for additional insight into the problem and the design via μ technique.

III. QFT TUNING

The QFT design is done as follows. A diagonal QFT controller, F, is inserted before the μ controller K as shown in Fig. 2.

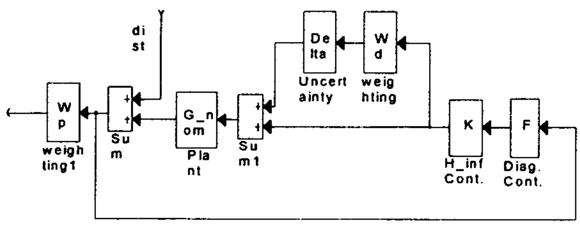

Figure 2: New closed-loop inter-connection structure of HIMAT showing QFT controller F

The robust stability constraint becomes

$$\left\| W_{del} KFG_{nom} (I + KFG_{nom})^{-1} \right\|_{\infty} < 1$$

where $F = \mathrm{diag}[f_1, f_2]$.

The robust performance constraint is that

$$\left\| W_P (I + GKF)^{-1} \right\|_{\infty} < 1$$

is satisfied for each $G \in G$. The technical issue we must deal with now is that the QFT design framework for MIMO systems is inherently different from the norm - based approach. In QFT, performance specifications are placed on each SISO element in the matrix function of interest. Clearly, it is virtually impossible for QFT to deal directly with norm-based specifications. However, we believe that for the purpose of controller tuning, it is possible to modify the weights from a norm-based formulation into the QFT's framework and still maintain the basic performance requirements.

We first modify the full block uncertainty Δ_G as follows. Consider a block diagonal structure

$$\begin{bmatrix} \Lambda_1(s) & 0 \\ 0 & \Lambda_2(s) \end{bmatrix}$$

and approximate the frequency responses of Λ_i using an N-point representation of their boundaries

$$\tilde{\Lambda}_i(j\omega) = \left\{ \begin{matrix} \cos(n\pi / N) + j\sin(n\pi / N) : \\ n = 1, \dots N \end{matrix} \right\} \subseteq \Lambda_i(j\omega)$$

resulting in an approximated frequency response set of the uncertainty

$$\tilde{\Delta}_G(j\omega) = \begin{bmatrix} \tilde{\Lambda}_1(j\omega) & 0 \\ 0 & \tilde{\Lambda}_2(j\omega) \end{bmatrix}$$

The approximate plant family becomes

$$\tilde{G} = \left\{ G = G_{nom}(I + \tilde{\Delta}_G W_{del}) : \tilde{\Delta}_G \text{ stable} \right\}$$

The nominal plant remains unchanged and this approximate plant family consists of (N+1)x(N+1) members. Let us now define a new plant P consisting of the original plant G cascaded with the μ-controller K, P = GK. The new plant family is:

$$\tilde{P} = \left\{ P = GK : G \in \tilde{\Delta}_G \right\}$$

With the QFT controller F (Fig. 9), the robust output sensitivity specification becomes

$$\left\| W_P S \right\|_{\infty} < 1, \quad \text{for each } P \in \tilde{P}$$

where $S = (I+PF)^{-1}$. For each $P \in \tilde{P}$ we compute the sensitivity with nominal QFT control (i.e., F = I),

$$S_m = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{\begin{bmatrix} 1+p_{22} & -p_{12} \\ -p_{21} & 1+p_{11} \end{bmatrix}}{1+p_{11}+p_{22}+\det[P]}.$$

$$P_i = \{p_{ij}\}_m \in \tilde{P}, \quad m = 1 \dots (N+1)^2$$

and use it as the baseline frequency response for QFT tuning. That is, since the μ controller K already satisfies our performance specification, the magnitude of the μ sensitivity for each plant in the approximate family

$W_m = |S_m|, \ m = 1, \dots, (N+1)^2$

is used to define the following QFT's robust performance problem. With the QFT controller $F = \mathrm{diag}[f_1, f_2]$ included, the sensitivity transfer function becomes

$$S_m = \begin{bmatrix} S_{11_m} & S_{12_m} \\ S_{21_m} & S_{22_m} \end{bmatrix} = \frac{\begin{bmatrix} 1+p_{22_m}f_2 & -p_{12_m}f_2 \\ -p_{21_m}f_1 & 1+p_{11_m}f_1 \end{bmatrix}}{1+p_{11_m}f_1+p_{22_m}f_2+\det[P_m]f_1f_2}$$

$, m = 1, \dots, (N+1)^2$

and for each plant in the family $\tilde{P}$ we write the sensitivity specification as

$|s_{ij_m}| \le w_{ij_m}, \ i,j = 1,2, \ m = 1, \dots, (N+1)^2$

While robust stability of the design is automatically guaranteed in optimal control, in QFT one must include this constraint explicitly. Assuming no unstable pole/zero cancellations in the loop, the feedback system is robust stable if the nominal system is stable and

$|I + P_m(j\omega)F(j\omega)| > 0, \quad m = 1, \dots, (N+1)^2$

the QFT robust stability margin specification takes on the form

$|s_{ii_m}| \le \max\left|\frac{1}{w_P}\right|, \ i = 1,2, \ m = 1, \dots, (N+1)^2$

where

$$s_{ii_m} = \frac{1}{1+p^e_{ii_m}f_i}$$

and where

$$p^c_{11_m} = \frac{p_{11_m} - \det|P_m|f_2}{1+p_{22_m}f_2} \quad p^c_{22_m} = \frac{p_{22_m} - \det|P_m|f_1}{1+p_{11_m}f_1}$$

and we have used $\max|1/w_P| = 2$.

The QFT Toolbox [2] is used to generate the corresponding bounds at a set of frequencies. These bounds are then intersected to yield worst case bounds. To achieve nominal stability we actually loopshape $p^c_{110}f_1$ with the nominal plant p^c_{110} corresponding to G_0. A screen capture of a typical interactive loop shaping environment is shown. Specifically, two QFT bounds, the original (f_1 = 1) and the tuned nominal loops are shown in Fig3. The effect of a 3rd order f_1 on the loop response is highlighted at ω = 500 (see arrow).

After f_1 is designed, we proceed to tune f_2. Again, to achieve nominal stability we actually loopshape $p^c_{220}f_2$ with the nominal plant p^c_{220} corresponding to G_0. A screen capture of a typical interactive loopshaping environment is shown Specifically, two QFT bounds, the original (f_1 = 1) and the tuned nominal loops are shown in Fig 4. The effect of a 5th order f_2 on the loop response is highlighted at ω = 300 (see arrow).

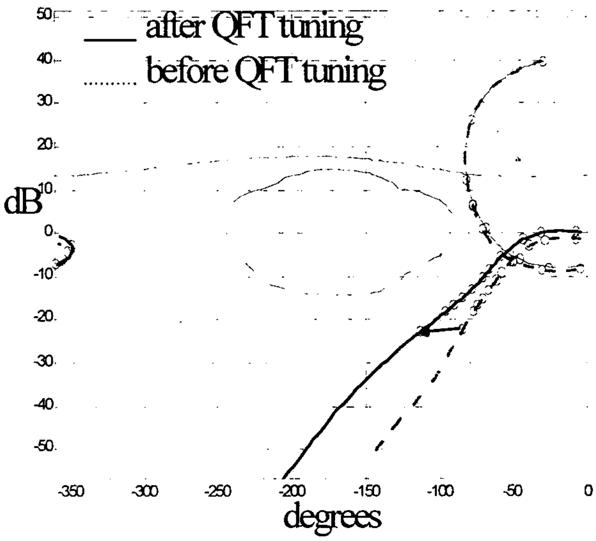

Figure 3: QFT bounds, tuned and un-tuned nominal loops in the first loop

Since we have been tuning the approximate plant only, at each step we analyze the structured singular values using the μ toolbox. As it turns out, using a few QFT iterations, we were able to find the direction μ changes for small changes in the open-loop response. This is especially useful when we tune the response over a "small" frequency band. That insight is exactly what makes QFT tuning so powerful. It is important to note that the QFT performance bounds are not exact relative to the original, norm-based specifications. And so, it is feasible that the nominal loop does not satisfy its bounds, yet the structured singular value is below 1. This insight is learned during the QFT tuning/μ analysis cycle.

In Figs 3-4, one can observe what QFT can offer in terms of reducing the high-frequency gain. While satisfying the low-frequency robust performance bounds (the line across the Nichols chart), and avoiding the robust stability margins

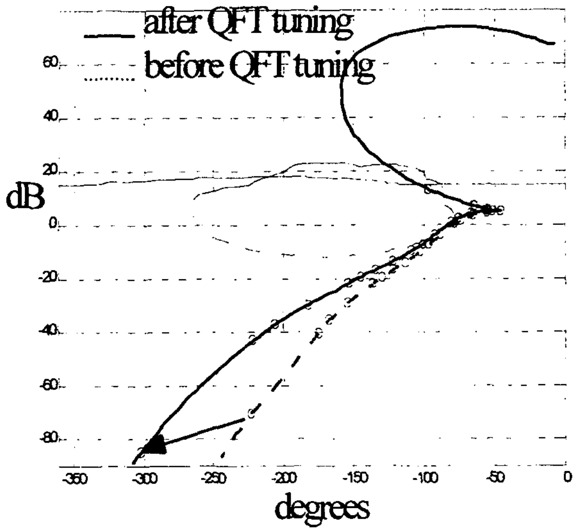

Figure 4: QFT bounds and final design for the second loop

bound (the closed curve in Nichols chart), one can attempt to reduce high frequency gain by adding/tuning any number of "far-off" poles. The designer can tune the values of such poles by interactively dragging the loop response to the left/down at a specific frequency. The feasible limit for such shifts is exactly the QFT bound. This is a rather straight forward process yet it does require experience.

We compare the resulting reduction in the controller high frequency gains between the two design approaches. Figure 5 depicts such values for the 12th-order combined QFT/μ design (without sensor noise), and the 12th-order μ design with sensor noise. As seen, the μ/QFT design satisfies the robust performance with the lowest bandwidth.

IV. CONCLUSIONS

In this paper we have shown using a generic, multivariable, robust performance problem, that the combination of μ-synthesis and QFT tuning led to a controller whose

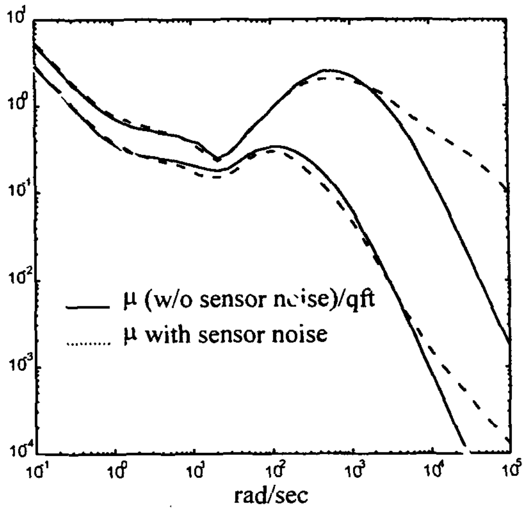

Figure 5: Comparing controller singular values of the two design approaches

performance levels may not be achievable if only a single technique was used. This design approach enjoys the strength of μ-synthesis in dealing with complex multivariable problems (such as non-square and/or highly coupled plant) and QFT's ability to deal directly with plant frequency response plant and easily tune control response over narrow frequency bands. Our findings strongly suggest that the historical academic competition between the two design philosophies should end.

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