Proceedings of the 13th
KACC, October 1998

A hierarchical fuzzy controller using structured Takagi-Sugeno type fuzzy inference engine

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Abstract

In this paper, a new hierarchical fuzzy inference system (HFIS) using structured Takagi-Sugeno type fuzzy inference units(FIUs) is proposed. The proposed HFIS not only solves the rule explosion problem in conventional HFIS, but also overcomes the readability problem caused by the structure where outputs of previous level FIUs are used as input variables directly. Gradient descent algorithm is used for adaptation of fuzzy rules. The ball and beam control is performed in computer simulation to illustrate the performance of the proposed controller.

1. Introduction

One of the recent topics in the fuzzy control area is the reduction of the number of fuzzy rules. In general, single output fuzzy system having n input variables with m membership functions to each inputs needs m^n fuzzy rules. The number of rules grow exponentially to the number of inputs. This is the rule explosion problem and is a important issue in real application where many input variables are concerned. Too many rules results in big memory size, much calculation time, and degradation of performance to unacceptable levels. So, it is necessary to reduce the number of fuzzy rules to use fuzzy systems appropriately.

Hierarchical fuzzy inference system(HFIS)[1,2] has been proposed to solve that problem. Fuzzy inference units(FIUs) in those structure have a few input variables, say, 2 or 3 input variables, and are connected to other FIUs hierarchically to be a input of upper level FIU. Generally this structure needs less number of fuzzy rules than that of conventional fuzzy inference system where all the inputs are gathered to only one FIU. Furthermore, rule adaptation algorithms using GA[3,4], gradient descent[5] etc. have been proposed also to adapt fuzzy rules automatically in HFIS.

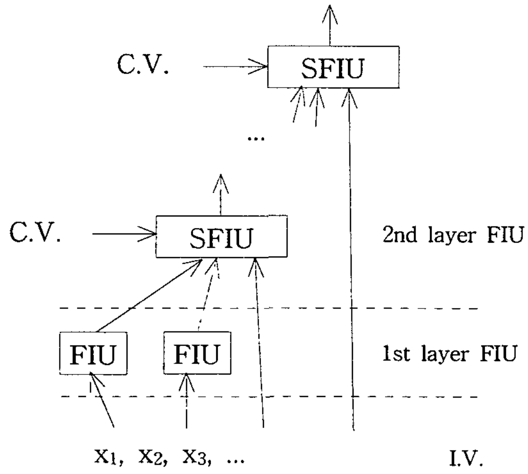
However, the HFIS scheme has the disadvantage of poor readability. In other word, as the number of layers grows, man can less and less aware of the meaning of the fuzzy system. Even if automatic fuzzy rule generation and adaptation have been successfully performed, man can not gather much information from that structure. It goes without saying that man can not represent his knowledge easily in the HFIS manner. It is because the outputs of the previous level fuzzy controller is used as input variables directly and controller output does not provide enough meaning to man like as other input variables. The outputs of previous level controller may have a meaning of ,at most, plus, minus, big, and small.

In this paper, a new hierarchical fuzzy inference system using structured Takagi-Sugeno type FIUs is proposed in Section 2. The proposed HFIS overcomes the readability problem maintaining reduced number of rules of conventional HFIS. And gradient descent algorithm with rule-confliction handling is shown to be a learning method of proposed HFIS in Section 3. In Section 4, ball and beam control is performed in computer simulation to illustrate the performance of the proposed HFIS.

2. S-HFIS

A hierarchical fuzzy inference system using structured Takagi-Sugeno type fuzzy inference(S-HFIS) consists of conventional FIUs in the first layer, and structured Takagi-Sugeno type fuzzy inference units(SFIUs) from the second layer. This structure is depicted in Fig. 1.

This is a structure where from 2nd layer, FIUs in the conventional HFIS are replaced with structured Takagi-Sugeno type FIUs.



C.V. : condition variable

I.V. : input variable

FIU : fuzzy inference unit

SFIU : structured Takagi-Sugeno FIU

Fig. 1 Structure of S-HFIS

At the first layer, usual fuzzy inference system like as Mamdani's or Takagi-Sugeno's one is used because the input variables of this layer have enough physical meaning. From the second layer, SFIUs are used as inference units and they have the following:

(a) Rule base:

$$Rule^{i}: IF\ C_1\ is\ \Omega^{i}_{c_1}\ \text{and}\ C_2\ is\ \Omega^{i}_{c_2}\ \ldots\text{and}\ C_L\ is\ \Omega^{i}_{c_L}$$
$$THEN\ y^{i}\ is\ \sum_p (a^{i}_{p} x_p) + \sum_q (b^{i}_{p} Yprev_q) + r^{i}$$
$$l \in \{1 \ldots L\}$$
$$p \in \{1 \ldots P\} \qquad (1)$$
$$q \in \{1 \ldots Q\}$$

L: number of condition variables

P: number of input variables

Q: number of outputs of previous level controller

(b) gaussian membership function

$$\omega_{c^{n}_{l}}(x) = \exp\left(-\frac{(m_{c^{n}_{l}} - x)^2}{2\sigma^2_{c^{n}_{l}}}\right) \qquad (2)$$
$$n \in \{1 \ldots N\}$$

N: number of membership functions in each linguistic varialbles

(c) product inference

$$\mu^{i} = \Pi^{L}_{l=1} \omega_{c^{i}_{l}} \qquad (3)$$

(d) single-tone fuzzifier

(e) center average defuzzifier

$$y = \frac{\sum_{i=1}^{I} \mu^{i} Y^{i}}{\sum_{i=1}^{I} \mu^{i}} \qquad (4)$$
$$i \in \{1 \ldots I\}$$

I: number of fuzzy rules

In this rule base structure, consequent part is a function of input variables and outputs of previous level controllers. But in the antecedent part, variables must not be ones of consequent part as is done in conventional HFIS. Condition variables and input variables are divided into different group. The condition variables must be ones having enough physical meaning to man. This feature gives us benefits of enhanced readability of fuzzy rule base. Fig. 2 shows the block diagram of SFIU.

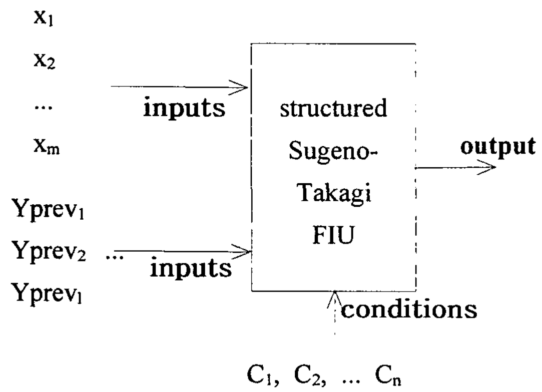


Fig. 2 structured Takagi-Sugeno FIU

Explanatory examples

Here are some meaningful examples of HFIS using structured Takagi-Sugeno FIU.

Example 1:

IF (C_1, C_2, ..., C_L) is some condition,

THEN Y = $\Sigma_q(\lambda_q Yprev_q)$

Taking average or weighted average maybe one of the most feasible way if some expert would express his knowledge in HFIS manner. But the condition variables should be meaningful ones for the rule base being readable. If Yprev has some physical meaning, it can be used as condition variable. Fuzzy rule base selection among local fuzzy rule base and mixtures of them by fuzzy inference falls into this category.

Example 2
IF (Yprev, x_1, x_2, ...) is some condition,
THEN Y= a_1 Yprev + b_1 x_1 + b_2 x_2 ...

This is a conventional HFIS using Takagi-Sugeno type fuzzy inference. In this example, input variables 'x's are used as condition variable 'C's. But, Yprev rarely has clear meaning to man.

Rule adaptation

Automatic rule adaption is done by gradient descent algorithm.

Parameter adaptation: Adjustable parameters in the antecedent part of S-HFIS are center(m) and sigma(σ) of input variables and condition variables. In the consequent part of fuzzy rule, adjustable parameters are Y^i itself when using 0th order Takagi-Sugeno FIU or Mamdani type FIU as first layer. In the other layers, they are coefficients(a, b, r) of Y^i. Conventional gradient descent algorithm be adopted to adapt all parameter.

Rule confliction handling: Two rule is said to be conflicting if there are two rules with same antecedent conditions but having different consequent results. In HFIS, It is often that variables of antecedent part are divided into other FIUs and can not describe the whole system by separated FIU only. In such a case, they have conflicting rules because they do not consider whole input space, but focus on only the concerned input variables. That case should be solved in the higher layer FIU because there are more variables concerned. In the case of rule confliction, the adaptation procedure is allowed to only higher level FIU. If there are rule confliction also, same procedure is applied until all of the variables are included.

3. Ball and beam control example

The ball and beam system is shown in Fig. 3.[6] The purpose of the control is that output will converge to zero from arbitrary initial conditions in a certain region.

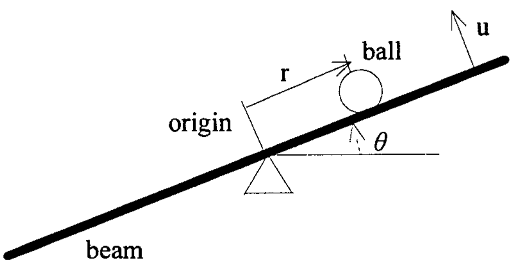


Fig. 3 The ball and beam system

And this system can be represented by the state space model from Hauser, Sastry, and Kokotovic as follows.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1 x_4^2 - G\sin x_3) \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \tag{5}$$
$$y = x_1$$
$$B = 1.2$$
$$G = 9.81$$

,where

$$[x_1, x_2, x_3, x_4] = [r, \dot{r}, \theta, \dot{\theta}] \tag{6}$$

Controller design
S-HFIS shown in Fig. 4 is a controller structure for ball and beam system. This structure is not optimal, but a choice to demonstrate S-HFIS scheme.

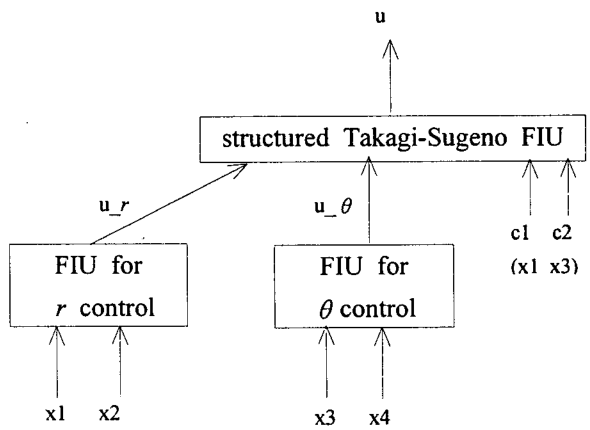


Fig. 4 A controller structure for ball and beam system using structured HFIS

The rule base of 2nd layer FIU is of the form:

$$\begin{aligned} &Rule_2^{ij}: IF\ C_1\ is\ \Omega_{c_1}^{i}\ \text{and}\ C_2\ is\ \Omega_{c_2}^{j} \\ &\qquad THEN\ y^{ij}\ is\ a_2^{ij}u_\theta + b_2^{ij}u_y + r_2^{ij} \\ &\Omega_{c_1} \in \{NE_{c_1}, PO_{c_1}\} \equiv \{\Omega_{c_1}^1, \Omega_{c_1}^2\} \\ &\Omega_{c_2} \in \{NE_{c_2}, PO_{c_2}\} \equiv \{\Omega_{c_2}^1, \Omega_{c_2}^2\} \end{aligned} \tag{7}$$

So, we have 4 fuzzy rules with 4 membership functions of the form:

$$\begin{aligned} &\omega_{c_l^n}(x) = \exp\left(-\frac{(m_{c_l^n} - x)^2}{2\sigma_{c_l^n}^2}\right) \\ &l \in \{1...L\} \\ &n \in \{1...N\} \\ &L\text{: number of condition variables} \\ &N\text{: number of membership functions} \\ &\quad \text{in each linguistic varialbles} \end{aligned} \tag{8}$$

Adaptation parameters are a,b,r in the consequent part and m, σ in the antecedent part. We have 20 (4×3 + 4×2) parameters in the 2nd layer FIU.

In the first layer, tow conventional Mamdani type or 0th order Takagi-Sugeno FIUs are used to control r and θ. In this example, we use Takagi-Sugeno FIU and the FIU to control r is stated as

$$Rule_1^{ij}\text{: } IF\ x_1\ is\ \Omega_{x_1}^{i}\ \text{and}\ x_2\ is\ \Omega_{x_2}^{i}\quad THEN\ u^{ij}\ is\ a_1^{ij}x_1+b_1^{ij}x_2+r_1^{ij}$$

$$\begin{aligned}
\Omega_{x_1} &\in \{NE_{x_1}, PO_{x_1}\} \equiv \{\Omega_{x_1}^1, \Omega_{x_1}^2\}\\
\Omega_{x_2} &\in \{NE_{x_2}, PO_{x_2}\} \equiv \{\Omega_{x_2}^1, \Omega_{x_2}^2\}\\
\omega_{x_p^n}(x) &= \exp\left(-\frac{(m_{x_p^n}-x)^2}{2\sigma_{x_p^n}^2}\right)\\
p &\in \{1...P\}\\
n &\in \{1...N\}
\end{aligned} \tag{9}$$

P: *number of input variables*
N: *number of membership functions in each linguistic varialbles*

FIU to control θ is of the same form. And 20 adaptation parameters are exist for each FIU also.

Rule adaptation:
Let cost function to be minimized is

$$J = \frac{1}{2}e^2 \tag{10}$$

where

$$e = y_d - y \tag{11}$$

and

$$y = \frac{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}y^{ij}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}} \tag{12}$$

Rule adaptation by gradient descent algorithm is

$$p(k+1) = p(k) - \eta\frac{\Delta J}{\Delta p(k)} \tag{13}$$

as well known. In the 2nd layer, the derivatives are calculated as

$$\frac{\Delta J}{\Delta a_2^{ij}} = \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta y^{ij}}\frac{\Delta y^{ij}}{\Delta a_2^{ij}} = -e\frac{\omega_{c_1^i}\omega_{c_2^j}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}}u_r \tag{14}$$

$$\frac{\Delta J}{\Delta b_2^{ij}} = \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta y^{ij}}\frac{\Delta y^{ij}}{\Delta b_2^{ij}} = -e\frac{\omega_{c_1^i}\omega_{c_2^j}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}}u_\theta \tag{15}$$

$$\frac{\Delta J}{\Delta r_2^{ij}} = \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta y^{ij}}\frac{\Delta y^{ij}}{\Delta r_2^{ij}} = -e\frac{\omega_{c_1^i}\omega_{c_2^j}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}} \tag{16}$$

and

$$\begin{aligned}
\frac{\Delta J}{\Delta m_{c_1^i}} &= \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta \omega_{c_1^i}}\frac{\Delta \omega_{c_1^i}}{\Delta m_{c_1^i}}\\
&= -e\frac{\sum_j y^{ij}\omega_{c_2^j} - y\sum_j \omega_{c_2^j}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}}\omega_{c_1^i}\frac{2(c_1-m_{c_1^i})}{\sigma_{c_1^i}^2}
\end{aligned} \tag{17}$$

$$\begin{aligned}
\frac{\Delta J}{\Delta \sigma_{c_1^i}} &= \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta \omega_{c_1^i}}\frac{\Delta \omega_{c_1^i}}{\Delta \sigma_{c_1^i}}\\
&= -e\frac{\sum_j y^{ij}\omega_{c_2^j} - y\sum_j \omega_{c_2^j}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}}\omega_{c_1^i}\frac{2(c_1-m_{c_1^i})}{\sigma_{c_1^i}^3}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\Delta J}{\Delta m_{c_2^j}} &= \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta \omega_{c_2^j}}\frac{\Delta \omega_{c_2^j}}{\Delta m_{c_2^j}}\\
&= -e\frac{\sum_i y^{ij}\omega_{c_1^i} - y\sum_i \omega_{c_1^i}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}}\omega_{c_2^j}\frac{2(c_2-m_{c_2^j})}{\sigma_{c_2^j}^2}
\end{aligned} \tag{19}$$

$$\begin{aligned}
\frac{\Delta J}{\Delta \sigma_{c_2^j}} &= \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta \omega_{c_2^j}}\frac{\Delta \omega_{c_2^j}}{\Delta \sigma_{c_2^j}}\\
&= -e\frac{\sum_i y^{ij}\omega_{c_1^i} - y\sum_i \omega_{c_1^i}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}}\omega_{c_2^j}\frac{2(c_2-m_{c_2^j})}{\sigma_{c_2^j}^3}
\end{aligned} \tag{20}$$

respectively.
In the 1st layer, derivatives in FIU for r are calculated as

$$\begin{aligned}
\frac{\Delta J}{\Delta p_1} &= \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta u_r}\frac{\Delta u_r}{\Delta p_1} = -err_1\frac{\Delta u_r}{\Delta p_1}\\
p_1 &\in \{a_1^{ij}, b_1^{ij}, r_1^{ij}, m_{x_1^i}, \sigma_{x_1^i}\}
\end{aligned} \tag{21}$$

where

$$err_1 = \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta u_r} = -e\frac{\omega_{c_1^i}\omega_{c_2^j}a_2^{ij}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}} \tag{22}$$

Derivatives in FIU for θ can be written in the same manner with

$$err_2 = \frac{\Delta J}{\Delta y}\frac{\Delta y}{\Delta u_\theta} = -e\frac{\omega_{c_1^i}\omega_{c_2^j}b_2^{ij}}{\sum_i\sum_j \omega_{c_1^i}\omega_{c_2^j}} \tag{23}$$

as the case of r control by straightforward calculation.

Target controller for learning: The control raw $u(x)$ from the input output linearization algorithm of Hauser, Sastry, and Kokotovic is determined as follows and this is the learning target for S-HFISs.

$$\begin{aligned}
\upsilon(x) &= -\alpha_3\Phi_4(x) - \alpha_2\Phi_3(x) - \alpha_1\Phi_2(x) - \alpha_0\Phi_1(x)\\
\Phi_1(x) &= x_1\\
\Phi_2(x) &= x_2\\
\Phi_3(x) &= -BG\sin(x_3)\\
\Phi_4(x) &= -BGx_4\cos(x_3)
\end{aligned} \tag{24}$$

The alpha is chosen so that

$$s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \tag{25}$$

is a Hurwitz polynomial and we use alpha as follows.

$$a_0=1,\ a_1=4,\ a_2=6,\ a_3=4 \tag{26}$$

Then *u(x)* is determined as

$$\begin{aligned} u(x) &= (v(x)-b(x))/a(x) \\ a(x) &= -BG\cos(x_3) \\ b(x) &= BGx_4^2\sin(x_3) \end{aligned} \tag{27}$$

Fig.5 shows the simulation result from four initial conditions, **x(0)** = $[2.4, -0.1, 0.6, 0.1]^T$, $[1.6, 0.05, -0.6\ -0.05]^T$, $[-1.6, -0.05, 0.6, 0.05]^T$, $[-2.4, 0.1, -0.6, -0.1]^T$. The value of B and G is 0.7143 and 9.81 respectively.

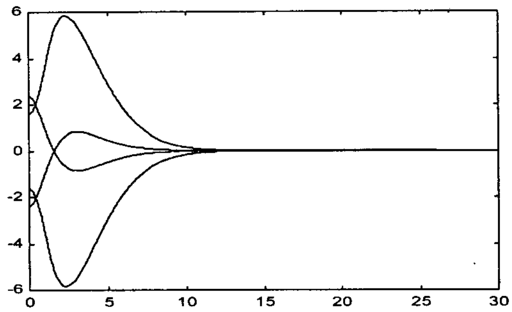


Fig. 5 Outputs r(t) of the closed loop ball and beam system using the input output linearization algorithm of Hauser, Sastry, and Kokotovic and four initial conditions

Simulation

The ranges of variables are [-5, 5],[-2, 2],[-π/4, π/4],[-0.8, 0.8],[-5, 5],[-π/4, π/4] to x_1,x_2,x_3,x_4,C_1,C_2 each to each. And initial value of sigma is set to its range. For example, 10 is assigned to σ of x_1. Fuzzy rule bases are all set to random number between -1 and 1. All the adaptation gain(η) is set to 0.1. Sampling time is 0.01 sec. Target samples consists of 4×3000 input output pairs from four different initial values.

Simulation result: After 200 iterations of learning procedure, parameter adaptation for second layer FIU is shown in TABLE I and TABLE II.

TABLE I

Rule base of 2nd layer FIU after 200 epochs learning

	C_1	C_2	u
$rule^{11}$	NE	NE	$-1.51u_r - 3.21u_\theta + 0.88$
$rule^{12}$	NE	PO	$-1.38u_r - 2.04u_\theta - 1.11$
$rule^{21}$	PO	NE	$1.51u_r - 2.99u_\theta + 1.64$
$rule^{22}$	PO	PO	$1.51u_r - 2.62u_\theta - 0.71$

TABLE II

Values of *m* and σ in the 2nd layer precedence variables after 200 epochs learning

		m	σ
C_1	NE	-4.99	9.98
	PO	4.99	10.00
C_2	NE	-0.65	1.04
	PO	0.78	2.47

From the plot of RMS error shown in Fig. 6, we notice that the learning is done in quite short epochs. The outputs of the closed loop ball and beam system using S-HFIS after 200 epochs learning is shown in Fig. 7.

Fig. 8 shows the result when 1 layer Takagi-Sugeno FIU is used as controller with same conditions. And In Fig. 9, plots of RMS error is depicted. Upper one is result of 1 layer FIU, and lower one is of S-HFIS. As shown in Fig. 9, S-HFIS is superior to the 1 layer FIU in learning maintaining readability.

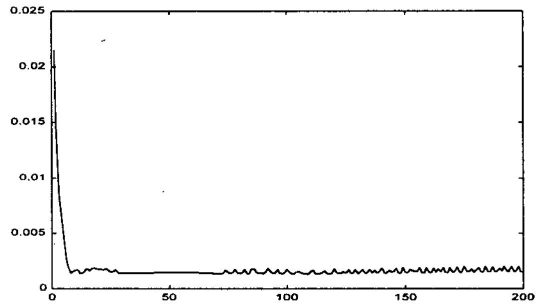


Fig. 6 RMS error during 200 epochs learning

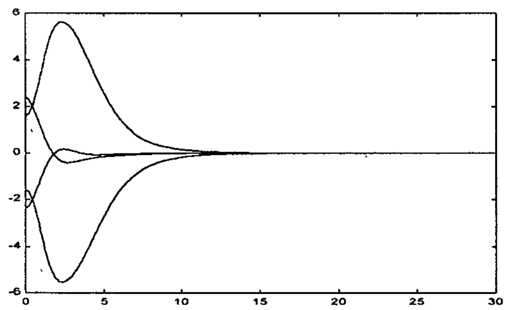


Fig. 7 Outputs r(t) of the closed loop ball and beam system using the proposed HFIS and four initial conditions after 200 epochs learning

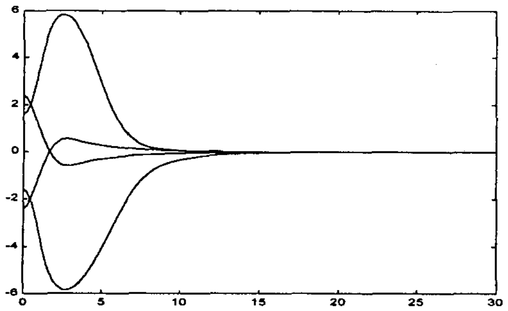


Fig. 8 Outputs r(t) of the closed loop ball and beam system using the 1 layer Takagi-Sugeno FIS and four initial conditions after 200 epochs learning

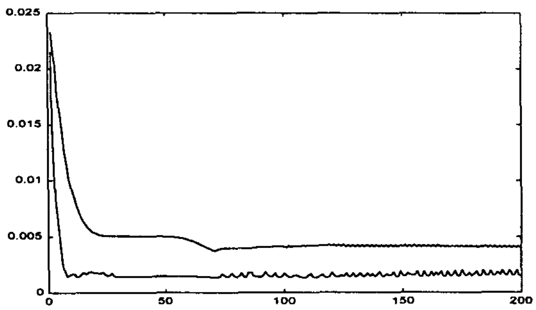


Fig. 9 Comparison of RMS error during 200 epochs learning

4. Conclusions

A new HFIS using structured Takagi-Sugeno type fuzzy inference units is proposed. We show that the proposed HFIS not only solve the rule explosion problem, but also overcomes the readability problem in conventional HFIS. And in the simulation it is shown that gradient descent algorithm is successfully used to adapt both antecedent and consequent parameters of S-HFIS.

Simulations with more complicated control system with more than 2 layer hierarchy is now in progress and GA learning scheme for S-HFIS is being tested.

5. References

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