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Design of Fuzzy Model Based Controller for Uncertain Nonlinear Systems

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Abstract - This paper addresses analysis and design of a fuzzy model-based-controller for the control of uncertain SISO nonlinear systems. In the design procedure, we represent the nonlinear system by using a Takagi-Sugeno fuzzy model and construct a global fuzzy logic controller via parallel distributed compensation and sliding mode control. Unlike other parallel distributed controllers, this globally stable fuzzy controller is designed without finding a common positive definite matrix for a set of Lyapunov equations, and has good tracking performance. The stability analysis is conducted not for the fuzzy model but for the real underlying nonlinear system. Furthermore, the proposed method can be applied to partially known uncertain nonlinear systems. A numerical simulation is performed for the control of an inverted pendulum, to show the effectiveness and feasibility of the proposed fuzzy control method.

1. Introduction

A systematic analysis and design procedure for fuzzy control systems is difficult since they are essentially nonlinear. In this paper, stability analysis and design of the Takagai-Sugeno (TS) fuzzy model-based-control system for uncertain SISO nonlinear systems are presented. On the basis of the TS fuzzy model, some fuzzy-model-based controls have been investigated in the literature [1-4]. Sometimes, they are called parallel distributed compensation (PDC). These kinds of design approaches suffer from a few limitations: 1) A common positive definite matrix must be found to satisfy a set of Lyapunov equations, which is difficult especially when the number of fuzzy rules required to give a good plant model is large. 2) The performance of the closed-loop system is difficult to predict. 3) The stability is guaranteed only for the simplified TS fuzzy models although they have been successfully applied to the original, underlying nonlinear systems. 4) The tracking problem of nonlinear systems is not easy to discuss.

In [5-6] we presented a new kind of TS fuzzy model-based-controller for known SISO nonlinear systems. In this paper, we extend the result of the above approach to the control of uncertain SISO nonlinear systems. A simulation is included for the control of the inverted pendulum system, to show the effectiveness and feasibility of the proposed fuzzy control method.

2. TS Fuzzy Model

Consider a class of uncertain and complex SISO nonlinear dynamic systems :

$$x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u \tag{1}$$

where the scalar x is the output state variable of interest, the scalar u is the system control input, and $\mathbf{x} = [x \ \ \dot{x} \ \ \ldots \ \ x^{(n-1)}]^T$ is the state vector. In equation (1), $f(\mathbf{x})$ is an unknown nonlinear continuous function of $\mathbf{x}$, and $g(\mathbf{x})$ is an unknown nonlinear continuous and invertible function of $\mathbf{x}$. This SISO nonlinear system can be approximated by the TS fuzzy model, proposed in [1], which combines the fuzzy inference rule and the local linear state model [2-4]. The ith rule of the TS fuzzy model, representing the complex SISO system (1), is the following:

Plant Rule i: IF $x(t)$ is F_1^i and ... and $x^{(n-1)}(t)$ is F_n^i
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{b}_i u(t)$ (2)
$(i = 1, 2, \ldots, r)$

where Rule i denotes the ith fuzzy inference rule, F_j^i ($j = 1, 2, \ldots, n$) are fuzzy sets, $\mathbf{x}(t) \in R^n$ is the state vector, $u(t) \in R^1$ is the input control, $\mathbf{A}_i \in R^{n \times n}$, $\mathbf{B}_i \in R^{n \times 1}$, r is the number of fuzzy IF-THEN rules.

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, the dynamic fuzzy model (2) can be expressed as the following global model:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \frac{\sum_{i=1}^{r} w_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))} \\ &= \sum_{i=1}^{r} \mu_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)) \\ &= \mathbf{A}(\mu(\mathbf{x}(t)))\mathbf{x}(t) + \mathbf{B}(\mu(\mathbf{x}(t)))u(t)\end{aligned} \tag{3}$$

where

$$w_i(\mathbf{x}(t)) = \prod_{j=1}^{n} F_j^i(x^{(j-1)}(t))$$

$$\mu_i(\mathbf{x}(t)) = \frac{w_i(\mathbf{x}(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))}$$

$$\mu(\mathbf{x}(t)) = (\mu_1(\mathbf{x}(t)), \mu_2(\mathbf{x}(t)), \cdots, \mu_n(\mathbf{x}(t)))$$

and $F_j^i(x^{(j-1)}(t))$ is the grade of membership of $x^{(j-1)}(t)$ in F_j^i. It is assumed, as usual, that

$$w_i(\mathbf{x}(t)) \geq 0,\ (i = 1, 2, \ldots, r),\ \sum_{i=1}^{r} w_i(\mathbf{x}(t)) > 0$$

for all t. Therefore,

$$\mu_i(\mathbf{x}(t)) \geq 0,\ (i = 1, 2, \ldots, r),\ \sum_{i=1}^{r} \mu_i(\mathbf{x}(t)) = 1$$

for all t. For simplicity of notation, let $w_i = w_i(\mathbf{x}(t))$, $\mu_i = \mu_i(\mathbf{x}(t))$, and $\mu = \mu(\mathbf{x}(t))$.

Definition 1: Model (3) is called the global state-space model of the fuzzy system (2). If the pairs $(\mathbf{A}_i, \mathbf{B}_i)$, $i = 1, 2, \ldots, r$ are controllable, the fuzzy system (2) is called locally controllable.

3. Robust TS Fuzzy Model-Based-Controller

First, Let us define controller rule as (4).

Controller Rule i: If x_1 is F_{i1} and ... and x_n is F_{in}
THEN $u = -\mathbf{K}_i\mathbf{x} + r$ (4)
$(i = 1, 2, \ldots, r)$

where L_i and r are scalar values. The scalar input r will be determined later. Equation (4) can be rewritten as

$$u = \frac{\sum_{i=1}^{r} w_i(-\mathbf{K}_i\mathbf{x} + r)}{\sum_{i=1}^{r} w_i} = -\sum_{i=1}^{r} v_i\mathbf{K}_i\mathbf{x} + r \tag{5}$$

where $v_i = w_i / \sum_{i=1}^{r} w_i$. The closed-loop system is obtained from the feedback interconnection of the nonlinear system (1) and the controller (5), and can be described by the following equation:

$$x^{(n-1)} = F(\mathbf{x}) + g(\mathbf{x})r \tag{6}$$

where $F(\mathbf{x}) = f(\mathbf{x}) - g(\mathbf{x})\sum_{i=1}^{r} v_i\mathbf{K}_i\mathbf{x}$

In order to proceed, we have to make the following assumption.

Assumption 1. There exists functions $f^U(\mathbf{x})$, $g^U(\mathbf{x})$, and $g_L(\mathbf{x})$ such that $|f(\mathbf{x})| \leq f^U(\mathbf{x})$ and $0 \leq g_L(\mathbf{x}) \leq g(\mathbf{x}) \leq g^U(\mathbf{x})$.

Based on $f^U(\mathbf{x})$, $g^U(\mathbf{x})$, and $g_L(\mathbf{x})$, and observing (6), the upper bound function of $F(\mathbf{x})$ can be easily obtained as

$$\begin{aligned} |F(\mathbf{x})| &= |f(\mathbf{x}) - g(\mathbf{x})\sum_{i=1}^{r} v_i\mathbf{K}_i\mathbf{x}| \\ &\leq f^U + g^U |\sum_{i=1}^{r} v_i\mathbf{K}_i\mathbf{x}| = F^U(\mathbf{x}) \end{aligned} \tag{7}$$

Let $\tilde{x} = x - x_d$ be the tracking error in the variable x, and let

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x} \cdots \tilde{x}^{(n-1)}]^T \tag{8}$$

In order to incorporate sliding mode control theory into the fuzzy-model-based control architecture, we first define a time-varying surface $S(t)$ in the state-space $\mathbf{R}^n$ by the scalar equation $s(\mathbf{x};t) = 0$, with

$$s(\mathbf{x};t) = (\frac{d}{dt} + \lambda)^{n-1}\tilde{x} = \tilde{x}^{(n-1)} + a_1\tilde{x}^{(n-2)} + \cdots + a_{n-1}\tilde{x} \tag{9}$$

where λ is a strictly positive constant.

Given an initial condition, the problem of tracking n dimensional vector $\mathbf{x}_d$ can be reduced to that of keeping the scalar quantity s at zero. More precisely, nth order tracking problem in x can be replaced by a 1st order stabilization problem in s [7].

The simplified, 1st order problem of keeping the scalar s at zero can now be achieved by choosing the control law such that

$$\frac{d}{dt}s^T s \leq -\eta |s| \qquad \text{outside of } S(t) \tag{10}$$

Differentiating $s(\mathbf{x};t)$ with respect to time, we obtain

$$\begin{aligned} \dot{s} &= \overline{F}(\mathbf{x}) + g(\mathbf{x})r, \\ \overline{F}(\mathbf{x}) &= F(\mathbf{x}) - \mathbf{x}_d^{(n)} + a_1\tilde{\mathbf{x}}^{(n-1)} + \ldots.. + a_{n-1}\dot{\tilde{\mathbf{x}}} \end{aligned} \tag{11}$$

Since $F(\mathbf{x})$ and $g(\mathbf{x})$ are unknown; only their bounds can be used to construct u. In this case, the control law r is chosen to be

$$r = -g_L^{-1}\{Q\,\mathrm{sgn}(s) - Ks\} \tag{12}$$

where $K > 0$ and

$$Q = [F^U + |\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1)} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}|]$$

Substituting (12) into (11), we have

$$\begin{aligned} \dot{s} &= \\ &\overline{F} - \{gg_L^{-1}[F^U + |\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}|]\}\mathrm{sgn}(s) - gg_L^{-1}Ks \end{aligned} \tag{13}$$

$$\begin{aligned} s^T\dot{s} &= s^T\overline{F} \\ &- s^T\{gg_L^{-1}[F^U + |\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}|]\mathrm{sgn}(s)\} - s^T gg_L^{-1}Ks \\ &\leq -s^T gg_L^{-1}Ks \\ &+ |s^T||\overline{F}| - |s^T|\{gg_L^{-1}[F^U + |\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}|]\} \\ &\leq -s^T gg_L^{-1}Ks \leq 0 \end{aligned} \tag{14}$$

Therefore, the closed- loop fuzzy system (6) is asymptotically stable. The results are summarized in the following theorem.

Theorem 1: If the dynamic fuzzy model described in (1) is locally controllable, then the closed-loop fuzzy system described in (6), with control law (12), is asymptotically stable.

Note that the controllability condition in Definition 1 is only required for the design of local compensators in each rule. Although the proposed control scheme is able to guarantee the stability of the overall system, it suffers from chattering problem because of the switching function in the control law. The chattering problem is inherent in the sliding mode control but can be eliminated by replacing the switching function by a saturation function. In this method, the boundary layer around the sliding surface in the phase plane is achieved [7].

4. Application to Inverted Pendulum

To illustrate the proposed method, we study the problem of balancing an inverted pendulum on a cart. The dynamic equations of the pendulum are [4]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 + a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} \tag{15}$$

where x_1 is the angle in rad of the pendulum from vertical axis, x_2 the angular velocity in rad s^{-1}, g = 9.8 m/s^2 the acceleration due to gravity, m = 2.0 kg the mass of the pendulum, $a = (m + M)^{-1}$, M = 8.0 kg the mass of the cart, $2l$ = 1.0m the length of pendulum, and u the force applied to the cart. A TS fuzzy model used to approximate the above system is [4]:

Plant Rules:
Rule 1: IF x_1 is about 0, THEN $\dot{x} = \mathbf{A}_1 x + \mathbf{B}_1 u$
Rule 2: IF x_1 is about $\pi/2$, THEN $\dot{x} = \mathbf{A}_2 x + \mathbf{B}_2 u$
where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4l/3 - aml} & 0 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 0 \\ \dfrac{a}{4l/3 - aml} \end{bmatrix}$$
$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ \dfrac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

with $\beta = \cos(88^\circ)$. The membership functions for Rule 1 and Rule 2 are shown in Fig. 1.

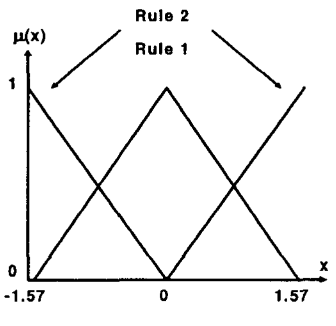

Fig. 1 Membership functions

The rules of the new fuzzy-model-based controller are:

Controller Rules:
Rule 1: IF x_1 is about 0, THEN $u = -\mathbf{K}_1\mathbf{x} + r$
Rule 2: IF x_1 is about $\pi/2$, THEN $u = -\mathbf{K}_2\mathbf{x} + r$

Design of controller parameters is categorized into two parts; one is for PDC and the other is for sliding mode control.

In order to determine the parameters of PDC, we simply choose the closed-loop eigenvalues (-2, -2) for both $\mathbf{A}_1$ - $\mathbf{B}_1\mathbf{K}_1$ and $\mathbf{A}_2$ - $\mathbf{B}_2\mathbf{K}_2$. Then, we obtain

$$\mathbf{K}_1 = [-120.6667 \quad -22.6667], \quad \mathbf{K}_2 = [-2551.6 \quad -764.0]$$

The PDC, however, cannot guarantee the stability of the closed system and deal with the tracking problem. As mentioned before we combine the sliding mode control theory to resolve these problems. To apply the proposed method to this system, we need to determine the bounds f^U, g^U, g_L, and F^U. For this system, we have

$$|f(x_1,x_2)| = \left|\frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2}{4l/3 - aml\cos^2(x_1)}\right|$$
$$\le \frac{9.8 + 0.05x_2^2}{2/3 - 0.1} = 17.2931 + 0.0882x_2^2 \equiv f^U(x_1,x_2) \tag{16}$$

$$|g(x_1,x_2)| = \left|\frac{a\cos x_1}{4l/3 - aml\cos^2 x_1}\right|$$
$$\le 0.1765 \equiv g^U(x_1,x_2) \tag{17}$$

If we require that $|x_1| \le \pi/6$, then

$$|g(x_1,x_2)| \ge \left|\frac{\frac{1}{10}\cos\frac{\pi}{6}}{\frac{4\times 0.5}{3} - \frac{1}{10}(2)(0.5)\cos^2\frac{\pi}{6}}\right| \tag{18}$$
$$= 0.1464 \equiv g_L$$

The remaining parameters for sliding mode control are chosen to be $\lambda = 5$, $K = 10$. Using these parameters we can construct the proposed stable fuzzy logic controller.

In order to show the stabilization performance of the proposed method, we apply the controller to the given system (15). Figure 2 – Figure 4 shows the simulation results for initial condition 15° (0.2618 rad). Figure 2 shows the state $x_1(t)$ (solid line) and its desired value $x_{1d} = \pi\sin(t)/30$ (dashed line) for the initial condition, Figure 3 shows the state $x_2(t)$ (solid line) and its desired value $x_{2d} = \pi\cos(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0) = [\pi/60 \quad 0]^T$ and Figure 4 shows the control $u(t)$. As seen in this figure, the response of the proposed method is satisfactory.

As mentioned above, the conventional PDC method cannot deal with the tracking problem. Figure 6 – Figure 8 show the tracking results for initial conditions $\mathbf{x}(0) = [-\pi/60 \quad 0]^T$. Figure 6 shows the state $x_1(t)$ (solid line) and its desired value $x_{1d} = \pi\sin(t)/30$ (dashed line). Figure 7 shows the state $x_2(t)$ (solid line) and its desired value $x_{2d} = \pi\cos(t)/30$ (dashed line) for the initial condition. Figure 8 shows the control $u(t)$. To eliminate chattering, we simply introduce a saturation function in the control law instead of the switching function. The width of the boundary layer is 1.

Figure 5 – Figure 7 show the tracking results for initial conditions $\mathbf{x}(0) = [-\pi/60 \quad 0]^T$. Figure 5 shows the state $x_1(t)$ (solid line) and its desired value $x_{1d} = \pi\sin(t)/30$ (dashed line). Figure 6 shows the state $x_2(t)$ (solid line) and its desired value $x_{2d} = \pi\cos(t)/30$ (dashed line) for the initial condition. Figure 7 shows the control $u(t)$. Figure 8 – Figure 10 show the

tracking results for initial conditions $\mathbf{x}(0)=[\pi/60 \quad 0]^T$. Figure 8 shows the state $x_1(t)$ (solid line) and its desired value $x_{1d}=\pi\sin(t)/30$ (dashed line). Figure 9 shows the state $x_2(t)$ (solid line) and its desired value $x_{2d}=\pi\cos(t)/30$ (dashed line) for the initial condition. Figure 10 shows the control $u(t)$.

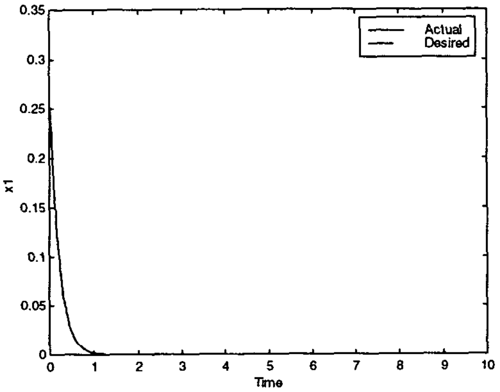

Fig. 2 The state $x_1(t)$ (solid line) and its desired value $x_{1d}=\pi\sin(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0)=[-\pi/60 \quad 0]^T$

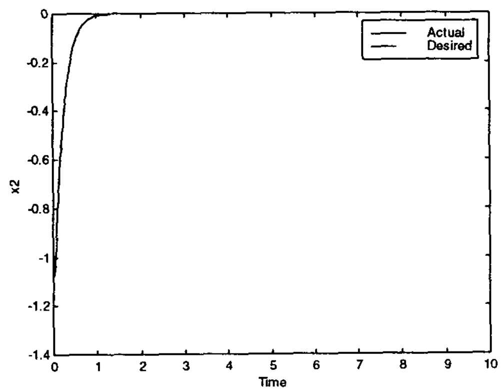

Fig. 3 The state $x_2(t)$ (solid line) and its desired value $x_{2d}=\pi\cos(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0)=[\pi/60 \quad 0]^T$

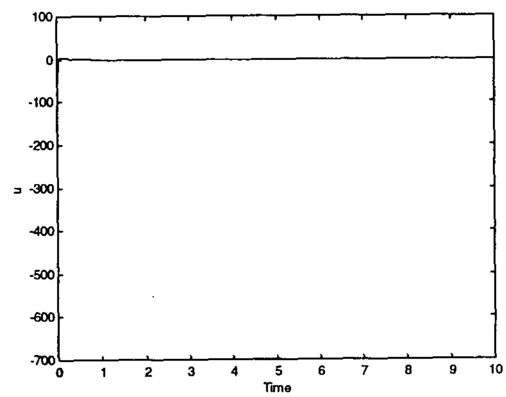

Fig. 4 The control $u(t)$ for the initial condition $\mathbf{x}(0)=[\pi/60 \quad 0]^T$

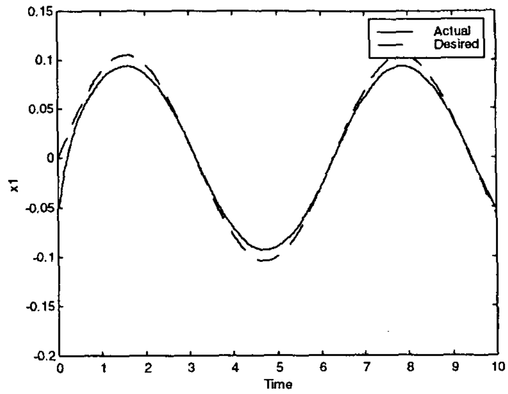

Fig. 5 The state $x_1(t)$ (solid line) and its desired value $x_{1d}=\pi\sin(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0)=[-\pi/60 \quad 0]^T$

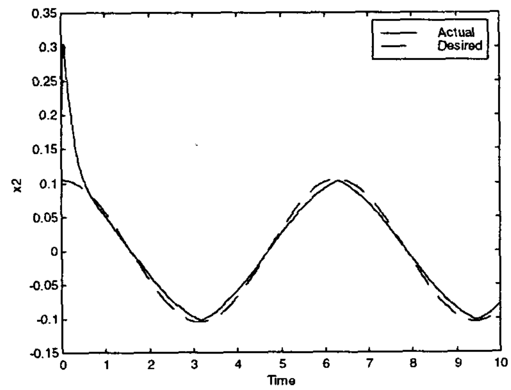

Fig. 6 The state $x_2(t)$ (solid line) and its desired value $x_{2d}=\pi\cos(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0)=[-\pi/60 \quad 0]^T$

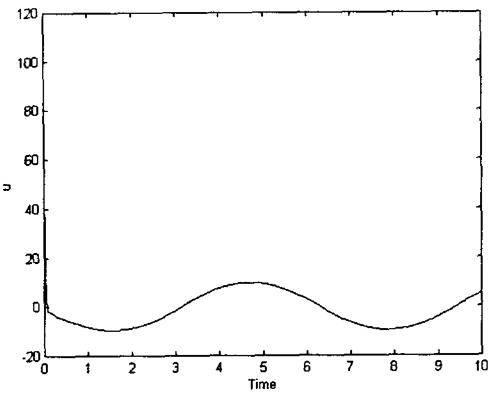

Fig. 7 The control $u(t)$ for the initial condition $\mathbf{x}(0)=[-\pi/60 \quad 0]^T$

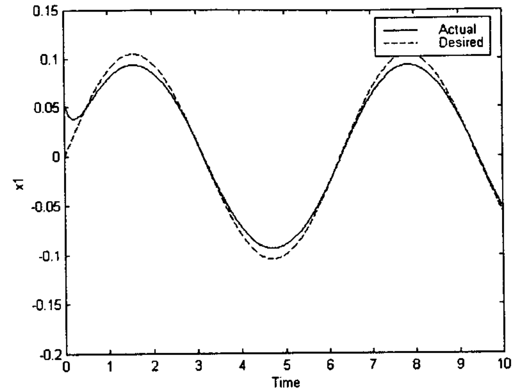

Fig. 8 The state $x_1(t)$ (solid line) and its desired value $x_{1d} = \pi \sin(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0) = [\pi/60 \ \ 0]^T$

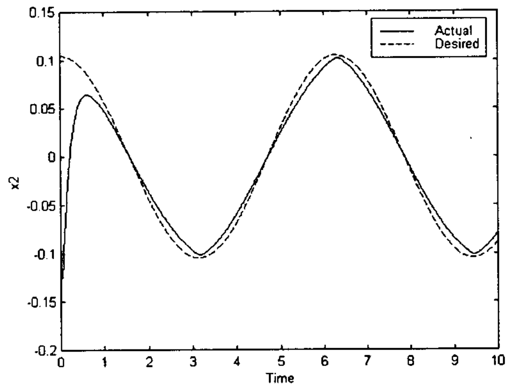

Fig. 9 The state $x_2(t)$ (solid line) and its desired value $x_{2d} = \pi \cos(t)/30$ (dashed line) for the initial condition $\mathbf{x}(0) = [\pi/60 \ \ 0]^T$

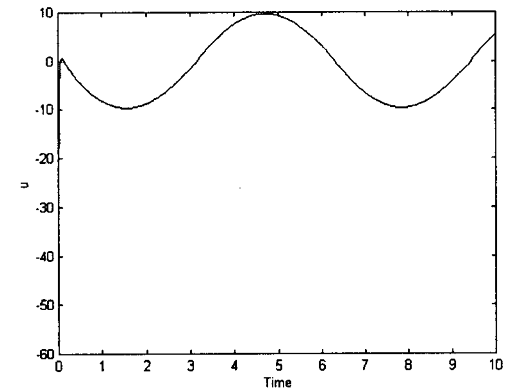

Fig. 10 The control $u(t)$ for the initial condition $\mathbf{x}(0) = [\pi/60 \ \ 0]^T$

5. Conclusion

In this paper, we propose a stable fuzzy logic controller architecture for an uncertain SISO nonlinear system. In the design procedure, we represent the fuzzy system as a family of local state space models, and construct a global fuzzy logic controller by considering each local state feedback controller. Unlike other conventional methods, we incorporate the sliding mode control theory into this approach to obtain robust tracking performance without finding a common positive definite matrix. Finally, simulation example is performed for the control of an inverted pendulum to show the effectiveness and feasibility of the proposed fuzzy control method.

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