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Hybrid State Space Self-Tuning Fuzzy Controller with Dual-Rate Sampling

Oh-Kook Kwon*, Young-Hoon Joo**, Jin-Bae Park* and L. S. Shieh***

*Dept. of Electrical Eng., Yonsei Univ., Seodaemoonku, Seoul 120-749, KOREA
Tel: +82-2-361-2773; Fax: +82-2-392-4230; E-MAIL: kooki@control.yonsei.ac.kr
** Dept. of Control & Instrumentation Eng., Kunsan Univ., Kunsan 573-701, KOREA
Tel: +82-654-469-4706; Fax: +82-654-466-2086; E-MAIL: yhjoo@ks.kunsan.ac.kr
***Dept. of Electrical & Computer Eng., University of Houston, Houston, TX77204, USA
Tel: +1-713-743-4439; Fax: +1-713-743-4444; E-MAIL: Lshieh@uh.edu

Abstract – In this paper, the hybrid state space self-tuning control technique is studied within the framework of fuzzy systems and dual-rate sampling control theory. We show that fuzzy modeling techniques can be used to formulate chaotic dynamical systems. Then, we develop the hybrid state space self-tuning fuzzy control techniques with dual-rate sampling for digital control of chaotic systems. An equivalent fast-rate discrete-time state-space model of the continuous-time system is constructed by using fuzzy inference systems. To obtain the continuous-time optimal state feedback gains, the constructed discrete-time fuzzy system is converted into a continuous-time system. The developed optimal continuous-time control law is then converted into an equivalent slow-rate digital control law using the proposed digital redesign method. The proposed technique enables us to systematically and effectively carry out framework for modeling and control of chaotic systems. The proposed method has been successfully applied for controlling the chaotic trajectories of Chua's circuit.

1. Introduction

Recently, most attention has been focused on developing techniques for the control of chaotic dynamical systems[1]. While chaos has become one of the most focusing research topics in the literature, we have witnessed rapidly growing interest in making the control system more intelligent and efficient. Among intelligent approaches, fuzzy control has enjoyed remarkable success in various applications[2]. Moreover, recent advances in fuzzy control have laid the foundation for intelligent control of various nonlinear processes, including chaotic systems. In this paper, we develop hybrid state space self-tuning fuzzy control techniques for digital control of chaotic systems.

Most real dynamic systems are described in a continuous-time framework. The rapid advances in digital control theory and the availability of high performance, low cost microprocessors have led the development of digital controllers for analog systems. Since the design of a continuous-time system using the digital counterpart is not closely coupled with the continuous aspects of the real environments, it is desirable to develop a hybrid control scheme[3].

In the past years, the adaptive control concepts[3-7] have been extended to combine a discrete-time adaptive mechanism (for updating the controller parameters) with a continuous-time control theory for self-tuning control of hybrid systems. Karwick[5] developed a state space self-tuning for pole assignments of continuous-time systems, and Helliot[6] proposed a discrete adaptation techniques for the control of continuous time processes.

For practical realizations of the developed advanced self-tuning algorithms for the adaptive control of systems, it becomes necessary to utilize dual-rate sampling schemes. A fast-rate sampling scheme is used to perform parameter identification and fast-rate controller design, while a slow-rate sampling scheme is employed to establish a slow-late controller which takes into account the performance of the analog controller and the computational delays in the identification and control processes. The conventional self-tuning schemes can hardly control nonlinear state-space systems and require more investigations. This paper proposes a hybrid state-space self-tuning fuzzy control scheme to perform digital control of non-linear state space systems.

Fuzzy inference systems employing fuzzy if-then rules can formulate the qualitative model [9]. By embedding the fuzzy inference system into the framework of a digital redesign, hybrid state-space self-tuning fuzzy control is obtained. In this work, fuzzy inference systems consist of a fuzzy modeling and fuzzy control. The particular fuzzy modeling framework employed here is the so-called Takagi-Sugeno model[11]. Once the fuzzy model representation of a chaotic system is obtained, we can apply some of the newly developed fuzzy control design techniques to the control of the chaotic system.

In this paper, new state space self-tuning fuzzy control scheme is applied to two Chua's circuit systems.

2. Fuzzy Modeling

2.1. Takagi-Sugeno Fuzzy Model

Fuzzy inference systems are known as fuzzy rule based systems, fuzzy models or fuzzy controllers when used as controllers. In this paper, Takagi-Sugeno(TS) fuzzy inference system, where local dynamics in different state space regions are represented by linear model, is used to model a chaotic system. The main characteristics of a TS fuzzy model of the system is to express the local dynamics of each fuzzy rule by a linear system model. The overall fuzzy model of the system is achieved by fuzzy blending of the linear system models[2].

The ith rule of the TS fuzzy model is of the following form:

Rule i: IF $x(kT)$ is M_1^i and ... and $x^{(n-1)}(kT)$ is M_n^i

THEN $\mathbf{x}(kT+T) = F_i\mathbf{x}(kT) + G_i u(kT)$ (1)

$(i = 1, 2, ..., r)$

where $x(kT)$ is state variables, $u(kT)$ is the control input, and r is the number of rules. M is the fuzzy sets and F_i and G_i are state matrices.

Using the defuzzification method to obtain the overall output of the dynamic fuzzy model, it can be expressed as the following :

$$\mathbf{x}(kT+T) = \frac{\sum_{i=1}^{r} w_i(kT)(F_i\mathbf{x}(kT) + G_i u(kT))}{\sum_{i=1}^{r} w_i(kT)}$$

$$= \sum_{i=1}^{r} \mu_i(kT)(F_i\mathbf{x}(kT) + G_i u(kT)) \quad (2)$$

$$= F(\mu(kT))\mathbf{x}(kT) + G(\mu(kT))u(kT)$$

where, $w_i(kT) = \prod_{j=1}^{n} M_j^i(x^{(j-1)}(kT))$

$$\mu_i(kT) = \frac{w_i(kT)}{\sum_{i=1}^{r} w_i(kT)}$$

$$\mu(kT) = (\mu_1(kT), \mu_2(kT), \cdots, \mu_n(kT))$$

and $M_j^i(x^{(j-1)}(kT))$ is the grade of membership of $x^{(j-1)}(kT)$ in M_j^i and $w_i(kT)$ is the firing strength of ith rule.

The open-loop system of (2) is

$$\mathbf{x}(kT) = \frac{\sum_{i=1}^{r} w_i(kT) F_i \mathbf{x}(kT)}{\sum_{i=1}^{r} w_i(kT)} \quad (3)$$

where it is assumed that

$$\sum_{i=1}^{r} w_i(t) > 0, \quad w_i(t) > 0, \quad i = 1,2,...,r$$

2.2. Fuzzy Modeling of Chua's Circuit

The chaotic system which is so-called *Chua's circuit* is a simple electronic system, which consists of one inductor (L), two capacitors (C_1, C_2), one linear resistor (R) and one piecewise-linear or nonlinear resistor (g). *Chua's circuit* has been shown to possess very rich nonlinear dynamics such as bifurcations and chaos [15].

The dynamic equations of Chua's circuit is described by

$$\dot{v}_{C1} = \frac{1}{C_1}\left(\frac{1}{R}(v_{C2} - v_{C1}) - g(v_{C1})\right) \quad (4)$$

$$\dot{v}_{C2} = \frac{1}{C_2}\left(\frac{1}{R}(v_{C1} - v_{C2}) + i_L\right) \quad (5)$$

$$\dot{i}_L = \frac{1}{L}(-v_{C1} - R_0 i_L) \quad (6)$$

where v_{c1}, v_{c2} and i_L are the state variables.

Consider two types of characteristic of the nonlinear resistor $g(v_{c1})$. One is the well-known piecewise-linear characteristic and the other is a cubic one.

2.2.1. Case 1: $g(v_{c1})$ is piecewise-linear.

$$g(v_{C1}) = G_b v_{C1} + \frac{1}{2}(G_a - G_b)\left(|v_{C1} + E| - |v_{C1} - E|\right) \quad (7)$$

where G_a, $G_b < 0$.

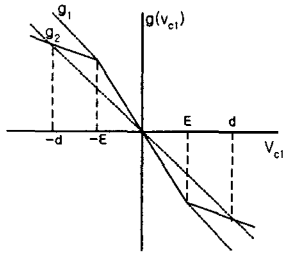

Fig. 1 Resistor characteristic in the case of piecewise-linear

We aim to obtain a fuzzy model in the open loop form (3) for Chua's circuit with characteristic (7). Assuming $v_{C1} \in [-d, d]$, $d > E > 0$, the following form to bound $g(v_{c1})$ is obtained :

$$g_1(v_{C1}) = G_a v_{C1} \quad (8)$$

$$g_2(v_{C1}) = \left(G_a + \frac{(G_a - G_b)E}{d}\right)v_{C1} = G v_{C1} \quad (9)$$

where $G \equiv G_a + \frac{(G_a - G_b)E}{d}$.

Chua's circuit become a linear system if G_a, and G_b are same. With $G_a \neq G_b$, the trapezoidal membership functions are used to model the Chua's circuit.

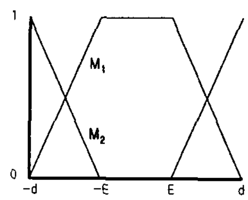

Fig. 2 Membership functions in the case of $g(v_{c1})$ is piecewise-linear

Denote $x = [v_{C1}, v_{C2}, i_L]^T$. The fuzzy inference rules can be represented by the followings :

Rule 1: IF v_{C1} is $M_1(v_{C1})$ (near 0), THEN $\dot{x}(t) = A_1 x(t)$

Rule 2: IF v_{C1} is $M_2(v_{C1})$ (near $\pm d$), THEN $\dot{x}(t) = A_2 x(t)$

where

$$A_1 = \begin{bmatrix} -\frac{1}{C_1 R} - \frac{G_a}{C_1} & \frac{1}{C_1 R} & 0 \\ \frac{1}{C_2 R} & -\frac{1}{C_2 R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} -\frac{1}{C_1 R} - \frac{G}{C_1} & \frac{1}{C_1 R} & 0 \\ \frac{1}{C_2 R} & -\frac{1}{C_2 R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}$$

2.2.2. Case 2: $g(v_{c1})$ is cubic.

$$g(v_{C1}) = av_{C1} + cv_{C1}^3 \qquad \text{(where } a<0,\ c>0\text{)} \tag{10}$$

Similarly as in 2.2.1, assuming $v_{C1} \in [-d, d]$, $d > E > 0$, the following form to bound $g(v_{c1})$ is obtained :

$$g_1(v_{C1}) = av_{C1} \tag{11}$$

$$g_2(v_{C1}) = (a + cd^2)v_{C1} = G_c v_{C1} \tag{12}$$

The membership functions are derived as :

$$M_1(v_{C1}) = 1 - \left(\frac{v_{C1}}{d}\right)^2 \tag{13}$$

$$M_2(v_{C1}) = 1 - M_1(v_{C1}) = \left(\frac{v_{C1}}{d}\right)^2 \tag{14}$$

The fuzzy inference rules can be represented by the followings :

Rule 1: IF v_{C1} is $M_1(v_{C1})$ (near 0), THEN $\dot{x}(t) = A_1 x(t)$

Rule 2: IF v_{C1} is $M_2(v_{C1})$ (near $\pm d$), THEN $\dot{x}(t) = A_2 x(t)$

where

$$A_1 = \begin{bmatrix} -\frac{1}{C_1 R} - \frac{a}{C_1} & \frac{1}{C_1 R} & 0 \\ \frac{1}{C_2 R} & -\frac{1}{C_2 R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} -\frac{1}{C_1 R} - \frac{G_c}{C_1} & \frac{1}{C_1 R} & 0 \\ \frac{1}{C_2 R} & -\frac{1}{C_2 R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}$$

3. Hybrid State Space Self-Tuning Fuzzy Control

In order to synthesize fuzzy control laws for the stabilization of nonlinear systems, the hybrid state space self-tuning fuzzy control technique is used.

3.1. Optimal control with pole placement

Consider the linear controllable continuous-time system described by

$$\begin{aligned} \dot{x}_c(t) &= \mathrm{A}x_c(t) + \mathrm{B}u_c(t) \\ y(t) &= Cx_c(t) \end{aligned} \tag{15}$$

The cost function for the system in Eqn. (3) can be expressed as

$$J = \int_0^\infty \left[x_c^T(t)Qx_c(t) + u_c^T(t)Ru_c(t)\right]dt \tag{16}$$

where Q and R are n x n nonnegative definite and m x m positive definite symmetric matrices. The feedback control law, which minimizes the performance index, is expressed as

$$u_c(t) = -K_c x_c(t) + E_c r(t) = -R^{-1}B^T P x_c(t) + E_c r(t) \tag{17}$$

where K_c is the feedback gain, E_c is the forward gain, $r(t)$ is a reference input, and P is a non-negative symmetric matrix which can be solved by Riccati equation

$$PBR^{-1}BP - PA - A^T P - Q = 0_n \tag{18}$$

with (Q, A) detectable. By this solution, the overall closed-loop system becomes

$$\dot{x}_c(t) = (A - BK_c)x_c(t) + BE_c u_c(t) \tag{19}$$

where K_c is $R^{-1}B^T P$ and the eigenvalues of A-BK_c exist in the open left-half plane of the complex s-plane. Our objective is to determine Q, R and Kc so that the closed loop system in (19) has its eigenvalues lying on or within the hatched region of Fig. 3.

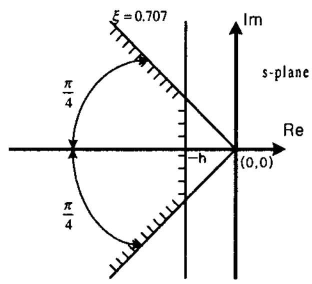

Fig. 3 Region of interest in the s-plane

Lemma 1 [4]: Let (A, B) be the pair of the given open loop system in (15). Also let $h \geq 0$ represent the prescribed degree of relative stability. Then the eigenvalues of the closed loop system $A - BR^{-1}B^T P$ lie to the left of the $-h$ vertical line, with the matrix P being the solution of the Riccati equation

$$PBR^{-1}B^T P - P(A + hI_n) - (A + hI_n)^T P = 0_n \tag{20}$$

Note that the use of the degree of relative stability h in (20) for finding the optimal control gain ($R^{-1}B^T P$) is highly recommended if the model of interest is an approximation of the original continuous-time system.

Lemma 2 [14]: Let the given stable system matrix $A \in R^{n \times n}$ in (15) have eigenvalues $\hat{\lambda}_i^-(i=1,\cdots,\hat{n}^-)$ lying in the open hatched sector of Fig. 1 in the s-plane and eigenvalues $\hat{\lambda}_i^+(i=1,\cdots,\hat{n}^+)$ outside that sector with $\hat{n}=\hat{n}^-+\hat{n}^+$. Now, consider the two Riccati equations with the assigned $R>0$

$$\hat{Q}BR^{-1}B^T\hat{Q}-\hat{Q}(-A^2)-(-A^2)^T\hat{Q}=0_{\hat{n}} \quad (21)$$

$$PB\,R^{-1}B^TP-PA-A^TP-\hat{Q}=0_{\hat{n}} \quad (22)$$

Then, the optimal closed-loop system matrix

$$A_c=A-BK_c=A-B(\gamma R^{-1}B^TP) \quad (23)$$

will enclose the invariant eigenvalues $\hat{\lambda}_i^-(i=1,\cdots,\hat{n}^-)$ and at least one additional pair of complex conjugate eigenvalues enter the open sector of Fig. 3, for the constant gain γ in (23) satisfying

$$\gamma \geq \max\left\{\frac{1}{2},\frac{b+\sqrt{b^2+ac}}{a}\right\}, \quad (24)$$

where $a=tr[(BR^{-1}B^TP)^2]$, $b=tr[BR^{-1}B^TPA]$, $c=tr[BR^{-1}B^T\hat{Q}]/2$ and $tr[\cdot]$ denotes the trace of $tr[\cdot]$. If $c=0$, all eigenvalues of A_c have been optimally placed in the desired open sector of Fig. 3.

3.2. Continuous-time design procedure

The continuous-time design procedures are described as follows:

Step 1: Let (A,B) be the given system matrices as in (15). Specify the value of h and the weighting matrix $R>0$. Set $i=0$ and denote $A_i=A$ and $\gamma_i=1$ for $i=0$. Solve Eq. (20) for P (denoted by P_i) to obtain the closed-loop system $A_{i+1}=A_i-BK_i$, where $K_i=\gamma_iR^{-1}B^TP_i$.

Step 2: Set $i:=i+1$ and solve Eq. (21) with $A:=A_i$ to find $\hat{Q}$ (denoted by $\hat{Q}_i$). If $c=tr[BR^{-1}B^T\hat{Q}_i]/2=0$, go to Step 4.

Step 3: Solve Eq. (22) with $A:=A_i$ and $\hat{Q}:=\hat{Q}_i$ to determine P (denoted by P_i) and to obtain the closed-loop system $A_{i+1}=A_i-BK_i$, where $K_i=\gamma_iR^{-1}B^TP_i$ and γ_i is a determined from (24). Go to Step 2.

Step 4: The desired optimal control law is

$$u_c(t)=-K_cx_c(t)+E_cr(t) \quad (25)$$

where $K_c=\sum_{j=0}^{i-1}K_j$ is the desired state-feedback gain and E_c is a forward gain. For tracking a constant reference input $r(t)$, the performance index in (16) can be written as

$$J=\int_0^\infty \{[Cx_c(t)-r(t)]^TQ_e[Cx_c(t)-r(t)]+u_c^T(t)Ru_c(t)]\}dt \quad (26)$$

where the weighting matrix Q_e can be chosen from the aforementioned steps as

$$Q_e=2hP_0+\sum_{j=1}^{i}[\hat{Q}_j+(r_j-1)P_jBR^{-1}B^TP_j\}r_j \quad (27)$$

The desired optimal tracking control law [15] is

$$u_c(t)=-K_ex_e(t)+E_er(t) \quad (28)$$

where $K_e=R^{-1}B^TP_e$, P_e is the solution of

$$A^TP_e+P_eA-P_eBR^{-1}B^TP_e+C^TQ_eC=0_{\hat{n}} \quad (29)$$

and

$$E_e=-R^{-1}B^T(A-BK_e)^{-T}C^TQ_e. \quad (30)$$

Note that when $C=I_{\hat{n}}$, the feedback gain K_c in (25) is identical to K_e in (28) and the forward gain E_c in (25) can be chosen as E_e in (28).

3.3. Digital redesign by state matching

For the implementation of the digital-time control law, the obtained continuous-time control law is converted into an equivalent digital law. The digital redesign method matches the closed-loop state $x_c(t)$ at $t=kT_s$ and digitally controlled state $x_d(t)$ at $t=kT_s$.

Using the bilinear transformation method with a sampling time T_s, the control law is developed in the following.

The equivalent discrete-time control law and state equation are described by

$$u_d(kT_s)=-K_dx_d(kT_s)+E_dr(kT_s) \quad \text{for} \quad kT_s\leq t<kT_s+T_s \quad (31)$$

$$x_d(kT_s+T_s)=Fx_d(kT_s)+Gu_d(kT_s) \\ y(kT_s)=Cx_d(kT_s) \quad (32)$$

where $F=e^{AT_s}$ and $G=[F-I_n]A^{-1}B$.

Therefore, the closed-loop discrete-time system with the sampling time T_s is

$$x_d(kT_s+T_s)=(F-GK_d)x_d(kT_s)+GE_dr(kT_s) \quad (33)$$

where F and G are the equivalent discrete-time state matrices, and K_d and E_d is the equivalent discrete-time feedback gain and forward gain, respectively.

Applying the block-pulse function method [4] to approximate the $u_c(t)$ in the interval kT_s and $(k+1)T_s$ results in

$$u_c(t)\approx\sum_{i=0}^{\infty}\frac{1}{2}(u_c(iT_s)+u_c(iT_s+T_s))\phi_i(t) \quad (34)$$

where $\phi(t)$ is the block-pulse function defined as follows:

$$\phi_i(t)=\begin{cases}1 & for\ iT_s\leq t<(i+1)T_s \\ 0 & otherelse\end{cases} \quad (35)$$

Using the block-pulse function (35) yield

$$x_c(kT_s+T_s)=Fx_c(kT_s)+\frac{1}{2}G[u_c(iT_s)+u_c(iT_s+T_s)] \quad (36)$$

where $F=e^{AT_s}$ and $G=(F-I_n)A^{-1}B$. Assuming $r(t)=r(kT_s)$ over each with $t=kT_s$ and $t=kT_s+T_s$, respectively. Substituting the discretized control law in (25) into the open loop system in (36) results in the following approximated closed-loop system :

$$x_c(kT_s+T_s)=\left(I_n+\frac{1}{2}GK_c\right)^{-1}\left(F-\frac{1}{2}GK_c\right)x_c(kT_s) + \left(I_n+\frac{1}{2}GK_c\right)^{-1}GE_c r(kT_s) \quad (37)$$

Equating $x_c(kT_s+T_s)$ and $x_c(kT_s)$ in (37) to respective $x_d(kT_s+T_s)$ and $x_d(kT_s)$ in (33) gives

$$F-GK_d=\left(I_m+\frac{1}{2}GK_c\right)^{-1}\left(F-\frac{1}{2}GK_c\right) \quad (38)$$

and

$$GE_d=\left(I_n+\frac{1}{2}GK_c\right)^{-1}GE_c \quad (39)$$

The desired digital gains (K_d, E_d) in (31) can be solved from (38) and (39) as follows:

$$K_d=\frac{1}{2}(I_n+\frac{1}{2}K_cG)^{-1}K_c(F+I_n) \quad (40)$$

$$E_d=(I_m+\frac{1}{2}K_cG)^{-1}E_c \quad (41)$$

3.4. Model conversions

To implement the obtained digital control lawin (31), we need to convert a fast-rate sampling model into a slow-rate sampling model is needed.

$$A=\frac{1}{T}\ln(F)\cong\frac{2}{T}H \quad (42)$$

where $H\equiv(F-I_n)(F+I_n)^{-1}$. And the matrix B can be found by

$$B=A(F-I_n)^{-1}G \quad (43)$$

The conversion of a fast-rate digital model to a slow-rate digital model can be carried out as follows.

$$F_s=e^{AT_s}=(e^{AT})^N=F^N \quad (44)$$

$$G_s=(F_s-I_n)A^{-1}B=(F_s-I_n)(F-I_n)^{-1}G \quad (45)$$

4. Simulation

Consider *Chua's circuit* with control inputs

$$\dot{v}_{C1}=\frac{1}{C_1}\left(\frac{1}{R}(v_{C2}-v_{C1})-g(v_{C1})\right)+u_1 \quad (46)$$

$$\dot{v}_{C2}=\frac{1}{C_2}\left(\frac{1}{R}(v_{C1}-v_{C2})+i_L\right)+u_2 \quad (47)$$

$$\dot{i}_L=\frac{1}{L}(-v_{C1}-R_0 i_L)+u_3 \quad (48)$$

The fuzzy rules of the model and control are as follows:

Continuous state space model

Rule 1: IF v_{C1} is $M_1(v_{C1})$, THEN $\dot{x}_c(t)=A_1x_c(t)+B_1u_c(t)$

Rule 2: IF v_{C1} is $M_2(v_{C1})$, THEN $\dot{x}_c(t)=A_2x_c(t)+B_2u_c(t)$

Continuous-time control law

Rule 1 : IF v_{C1} is $M_1(v_{C1})$, THEN $u_c(t)=-K_c^1x_c(t)$

Rule 2 : IF v_{C1} is $M_2(v_{C1})$, THEN $u_c(t)=-K_c^2x_c(t)$

In order to use the digital redesign method, the continuous state space system need to be converted into digital state space system. The fuzzy rules of the digital state space system are as follows :

Digital state space model

Rule 1: IF v_{C1} is $M_1(v_{C1})$,
THEN $x_d(kT_s+T_s)=F_1x_d(kT_s)+G_1u_d(kT_s)$

Rule 2: IF v_{C1} is $M_2(v_{C1})$,
THEN $x_d(kT_s+T_s)=F_2x_d(kT_s)+G_2u_d(kT_s)$

Digital control law

Rule 1 : IF v_{C1} is $M_1(v_{C1})$ THEN $u_d(kT_s)=-K_d^1x_d(kT_s)$

Rule 2 : IF v_{C1} is $M_2(v_{C1})$ THEN $u_d(kT_s)=-K_d^2x_d(kT_s)$

where $F=e^{AT_s}$, $G=(F-I_n)A^{-1}B$, $T_s=NT$ (T_s: slow-sampling rate, T : fast-sampling rate, N : sampling period), and K_d^1, K_d^2 are determined by (40).

Case 1 : $g(v_{c1})$ = piecewise-linear

Choose the parameter as the followings :

R = 1.439, R_0 = 0, C_1 = 0.1, C_2 = 2, L = 0.143,
G_b = -0.1, G_a = -4, E = 0.75, d = 15

Fig. 4 shows the response of Chua's circuit with a piecewise-linear resistor. The initial condition is [0 1 0] and the hybrid state space self-tuning fuzzy control method is activated at t=250. In Fig. 5, the strange attractor illustrates the limiting property of the Chua's circuit with piecewise linear characteristic. It shows the phase plane trajectory of the Chua's circuit to control.

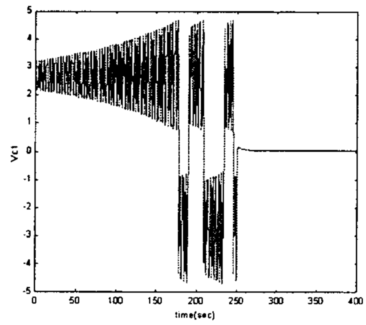

Fig. 4 Response of Chua's circuit (case 1)

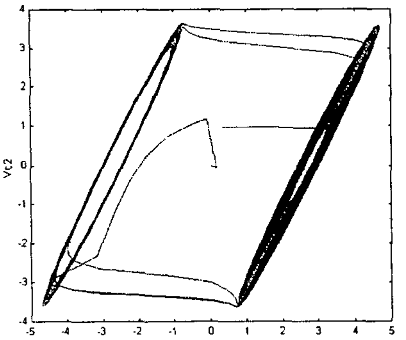

Fig. 5 State space trajectory of the Chua's circuit (case 1)

Case 2 : $g(v_{c1})$ = cubic

Choose the parameter as the followings :

$R = 1.439,\quad R_0 = 0,\quad C_1 = 1.0,\quad C_2 = 9.5,$

$L = 1.357,\quad a = -0.8,\quad c = 0.044,\quad d = 3$

Fig. 6 shows the response of Chua's circuit with cubic characteristic. The initial condition is [-1 0.8 1] and the hybrid state space self-tuning fuzzy control method is activated at t=400. Fig. 7 illustrates the phase plane trajectory of the Chua's circuit to control

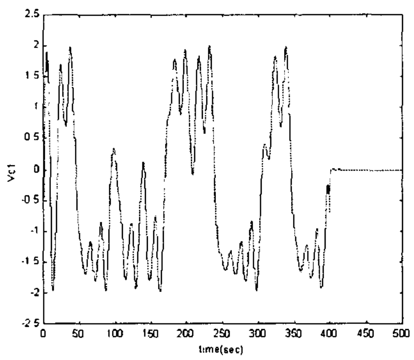

Fig. 6 Response of Chua's circuit (case 2)

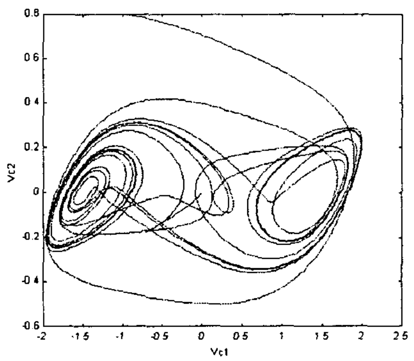

Fig. 7 State space trajectory of the Chua's circuit (case 2)

5. Discussion

The digital redesign scheme together with fuzzy inference systems is proposed in this paper. The fuzzy inference systems are used to get the discrete-time state-space model and control the applied force. This scheme has advantages of the adaptive and robust results. By the application to Chua's circuit, the performance of the proposed scheme is better than the conventional digital redesign schemes in the viewpoint of the computational efforts and performance. This scheme will be applied to the control based digital systems.

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