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A Fuzzy Sliding Mode Control of Wheeled Mobile Robot with a Differential Drive

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Abstract

In this paper we introduce a modeling of wheeled mobile robot with a differential drive derived by R.M. DeSantis and using the dynamics modeling with some disturbance term we control the wheeled mobile robot using fuzzy sliding mode control(FSMC) method. In a fuzzy control approach it is very difficult to prove the stability of the fuzzy controller. Therefore, to overcome that difficult proof of the stability in a fuzzy control method, we first propose a sliding mode controller and prove the stability of the proposed controller. Next, transforming the proposed sliding mode controller into a fuzzy sliding mode controller without changing the basic structure of the sliding mode controller, we easily obtain a fuzzy sliding mode controller(FSMC) whose stability is guaranteed without difficult stability proof prcedure of the proposed FSMC.

1 Introduction

Wheeled Mobile Robots constitute a class of mechanical systems characterized by kinematical constraints that are not integrable and cannot therefore be eliminated from the model equations. So the consequence is that the conventional robot planning and control algorithms developed for robotic manipulators without constraints are no more applicable. And there are several kinds of mobile robots whose modelings are very complex. The modelings of the mobile robots are classified and well described in [6] [5]. To know more the modeling of the mobile robots, referred to [6]. In this paper, we will use the wheeled mobile robot with a differential drive which has a front castor and a pair of rear co-axial drive wheels as shown in Fig.1, which is like type (2,0) in [6]. In this paper, we propose a fuzzy sliding mode controller for a wheeled mobile robot to track the desired path. First we construct the sliding mode controller which makes the mobile robot track the desired path in cartesian space where the large vibration of input torque exists. Therefore, to a next step, we modify the sliding mode controller to lessen above unnecessary effects using fuzzy control approach. In most of the robotic literature trajectory-tracking is meant to imply the convergence of the vehicle's state to a desired state which is itself a prescribed function of time but, though path-tracking still entails the convergence of the vehicle's state to a desired state, the desired state in the path-tracking is a function of the position and orientation of the vehicle with respect to the path to be followed.

2 A modeling of wheeled mobile robot with a differential drive

In general, the motion of a mobile wheeled robot with a differential drive is usually obtained by means of a locomotion platform equipped with a front caster and a pair of rear drive wheels. We also model the wheeled mobile robot as shown in Fig. 1. Each of drive wheels is independantly driven by a DC motor. We will assume that the motion of the mobile robot is planar, that its geometrical and physical properties are symmetrical with respect to its longitudinal axis, that the contact between tires and surface of motion is pointwise, and that no forces are exerted from the other wheels.

The dynamics model of the mobile robot with a differential drive wheels is given by the following equations, referred to [3]:

- Dynamics Modeling including the influence of the DC motors's dynamics

In the absence of lateral and longitudinal slippage

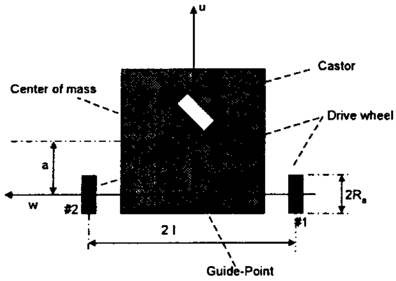

Figure 1: Vehicle's Geometry

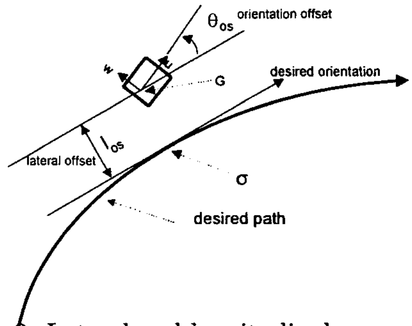

Figure 2: Lateral and longitudinal errors

the vehicle's dynamics including the motor dynamics are described as follows [3]:

$$\dot{v}_u = a\Omega^2 - \frac{K_\Phi K_b v_u}{R_e R_a^2 m} + \frac{K_\Phi}{R_e R_a m} U_1 \quad (1)$$

$$\dot{\Omega} = -\frac{ma\Omega v_u}{(j+ma^2)} - \frac{2K_\Phi K_b l^2 \Omega}{R_e R_a^2 (j+ma^2)} \quad (2)$$

$$+\frac{2K_\Phi l}{R_e R_a (j+ma^2)} U_2 \quad (3)$$

$$v_w = a\Omega \quad (4)$$

where K_ϕ, K_b, and R_e are the DC motors' characteristic parameters and denoting

$$U_1 = (V_1 + V_2)/2 \quad (5)$$

$$U_2 = (V_1 - V_2)/2 \quad (6)$$

3 The Design of a Fuzzy Sliding Mode Controller

We can measure the content to how the mobile robot follows the given path with a desired velocity in terms of velocity error (v_{os}), heading error (θ_{os}), and lateral error (l_{os}). These errors are defined as follows;

$$v_{os}(t) := v_u(t) - v_{ud}(t) \quad (7)$$

$$\theta_{os}(t) := \theta(t) - \theta_d(t) \quad (8)$$

$$l_{os}(t) := -\{x(t) - x_d(t)\} sin\theta_d(t) \quad (9)$$

$$+\{y(t) - y_d(t)\} cos\theta_d(t) \quad (10)$$

The concrete description of errors in the path-tracking is shown in Fig.2.

Note that l_{os} represents the signed distance between the "guide-point" and the desired path. The path-tracking problem is equivalent to finding the control input such that

$$\lim_{t\to\infty} [v_{os}(t), \theta_{os}(t), l_{os}(t)] = 0 \quad (11)$$

The dynamics of the path-tracking errors are described as follows [3];

$$\dot{v}_{os} = \dot{v}_u - \dot{v}_{ud} = -\dot{v}_{ud} + a\Omega^2 \quad (12)$$

$$-\frac{K_\Phi K_b v_u}{R_e R_a^2 m} + \frac{K_p hi U_1}{R_e R_a m} \quad (13)$$

$$\dot{\theta}_{os} = \Omega - \dot{\theta}_d \quad (14)$$

$$\dot{l}_{os} = (v_{ud} + v_{os}) sin\theta_{os} \quad (15)$$

3.1 The Design of the Sliding Mode Controller

A mobile robot dynamics is so complex and that the exact modeling is very difficult to obtain. There are so many dynamics elements not to be described exactly in dynamics modeling and it is very difficult or may be impossible to describe many unknown dynamics elements respectively. Therefore, we consider various unknown dynamics elements as external disturbances and represent both the external disturbances and the unknown dynamics elements as a single disturbance term for easy mathematical treatment. We assume that the unknown dynamics are bounded and the bound of that unknown dynamics is known. We denote the bound of the unknown dynamics and external disturbances as τ_d and the dynamics equation including the unknown dynamics is represented as follows;

$$\dot{v}_u = a\Omega^2 - \frac{K_\Phi K_b v_u}{R_e R_a^2 m} \quad (16)$$

$$+\frac{K_\Phi}{R_e R_a m} U_1 + \tau_{d1} \quad (17)$$

$$\dot{\Omega} = -\frac{ma\Omega v_u}{(j+ma^2)} - \frac{2K_\Phi K_b l^2 \Omega}{R_e R_a^2 (j+ma^2)} \quad (18)$$

$$+\frac{2K_\Phi l}{R_e R_a (j+ma^2)} U_2 + \tau_{d2} \quad (19)$$

Now, we will design the sliding mode controller and prove the stability of proposed controller. First, we linearize the dynamics equation to simplify the mathematical manipulation.

If the following controller

$$U_1 = \frac{R_eR_am}{K_\phi}\{-a\Omega^2 + \frac{K_\phi K_b v_u}{R_eR_a^2m} + u_{r1}\} \quad (20)$$

$$U_2 = \frac{R_eR_a(j+ma^2)}{2K_\phi l}\{\frac{mav_u\Omega}{(j+ma^2)} \quad (21)$$

$$+\frac{2K_\phi K_b l^2\Omega}{R_eR_a^2(j+ma^2)} + u_{r2}\} \quad (22)$$

is substituded into the dynamics equation 19,

Then the dynamics equations 19 are linearized as follows

$$\dot{v}_u = \tau_{d1} + u_{r1} \quad (23)$$

$$\dot{\Omega} = \tau_{d2} + u_{r2} \quad (24)$$

Now, let the sliding surface be constructed as follows;

$$s_1 = v_{os} + \lambda_1 \int v_{os}dt \quad (25)$$

$$s_2 = \dot{\theta}_{os} + \lambda_2(\theta_{os} + l_{os}) \quad (26)$$

where λ_1 and λ_2 are constants which determine the dynamics of the sliding phase.

We will show the stability of the sliding surfaces later using a special assumption. We design the sliding mode controller which satisfies the condition $s\dot{s} < 0$.

If the following sliding mode controller

$$u_{r1} = \dot{v}_{ud} - K_{s1}s_1 - K_{s2}sign(s_1) + \lambda_1^2 \int v_{os}dt \quad (27)$$

$$u_{r2} = \dot{\Omega}_d + \lambda_2^2(\theta_{os} + l_{os}) - \lambda_2\dot{l}_{os} \quad (28)$$

$$-K_{\theta 1}s_2 - K_{\theta 2}sign(s_2) \quad (29)$$

is proposed and is given to the linearized dynamics equation (24)
then
this proposed controller satisfies the sliding mode condition $s\dot{s} < 0$ Proof)
To know if the proposed controller satisfies the sliding mode condition first we differentiate $\dot{s}_1$ and s_2,

$$\dot{s}_1 = -\dot{v}_{ud} + \dot{v}_u + \lambda_1 v_{os} = -\dot{v}_{ud} + \quad (30)$$

$$a\Omega^2 - \frac{K_\phi K_b v_u}{R_eR_a^2m} + \frac{K_\phi}{R_eR_am}U_1 \quad (31)$$

$$+\lambda_1 v_{os} + \tau_{d1} \quad (32)$$

$$\dot{s}_2 = \dot{\Omega} - \dot{\Omega}_d + \lambda_2(\dot{\theta}_{os} + \dot{l}_{os}) \quad (33)$$

$$= -\frac{mav_u\Omega}{(j+ma^2)} - \frac{2K_o K_b l^2\Omega}{R_eR_a^2(j+ma^2)} \quad (34)$$

$$+\frac{2K_o l}{R_eR_a(j+ma^2)}U_2 - \dot{\Omega}_d \quad (35)$$

$$+\lambda_2(\dot{\theta}_{os} + \dot{l}_{os}) \quad (36)$$

Then the proposed controller is substituted into above derivatives of the sliding surface $\dot{s}_1$ and $\dot{s}_2$ the following result can be obtained

$$\begin{aligned}\dot{s}_1 &= -(K_{v1} - \lambda_1)s_1 - K_{v2}sign(s_1) + \tau_{d1} \\ &\le -(K_{v1} - \lambda_1)s_1 - (K_{v2} - |\tau_{d1}|)sign(s_1) < 0 \\ \dot{s}_2 &= -(K_{\theta 1} - \lambda_2)s_2 - K_{\theta 2}sign(s_2) + \tau_{d2} \\ &\le -(K_{\theta 2} - \lambda_2)s_2 - (K_{\theta 2} - |\tau_{d2}|)sign(s_2) < 0\end{aligned}$$

Therefore, we can know that the proposed controller satisfies the sliding mode condition and guarantees that any initial state of the mobile robot can get into the sliding surfaces if the following condition is satisfied;

$$K_{v1} > \lambda_1, K_{v2} > |\tau_{d1}| K_{\theta 1} > \lambda_2, K_{\theta 2} > |\tau_{d2}| \quad (37)$$

Now, we can also prove the convergence of the sliding surface using the Lyapunov theorem. We define vector s as follows

$$s = [s_1 \ s_2] \quad (38)$$

and let the Lyapunov candidate function be

$$V = \frac{1}{2}s^TQs \quad (39)$$

where we choose Q as an identity matrix. Then the derivative of the Lyapunov candidate function is

$$\dot{V} = s_1\dot{s}_1 + s_2\dot{s}_2 \le -(K_{v1} - \lambda_1){s_1}^2 \quad (40)$$

$$-(K_{v2} - |\tau_{d1}|)|s_1| - (K_{\theta 1} - \lambda_2){s_2}^2$$

$$-(K_{\theta 2} - |\tau_{d2}|)|s_2| < 0 \quad (41)$$

so, we can know the derivative of Lyapunov candidate function is negative definite. From above results we can know that any initial states converge to the proposed sliding surfaces, that is, $s_1 = 0 and s_2 = 0$. Now, we will show that the path-tracking errors converge to zero from the convergence of the sliding surfaces. We differentiate above sliding surface equation (26).

$$\dot{v}_{os} + \lambda_1 v_{os} = 0 \quad (42)$$

$$\ddot{\theta}_{os} + \lambda_2(\dot{\theta}_{os} + \dot{l}_{os}) = 0 \quad (43)$$

. Taking the Laplace transformation above first equation, we know that the velocity error converges to zero. And substituting $\dot{l}_{os}$ into the above second equation, the following equation

$$\ddot{\theta}_{os} + \lambda_2\dot{\theta}_{os} + \lambda_2(v_{ud} + v_{os})sin\theta_{os} = 0 \quad (44)$$

is obtained. As we showed the velocity error converged to zero, the equation 44 is changed as follows;

$$\ddot{\theta}_{os} + \lambda_2\dot{\theta}_{os} + \lambda_2 v_{ud} sin\theta_{os} = 0 \qquad (45)$$

and that the path-tracking heading angle error is small, $sin\theta_o s$ is approximated as θ_{os} ($sin\theta_{os} \cong \theta_{os}$). Therefore, the equation 45 is changed as follows;

$$\ddot{\theta}_{os} + \lambda_2\dot{\theta}_{os} + \lambda_2 v_{ud}\theta_{os} = 0 \qquad (46)$$

Now, because λ_2 and v_{ud} are positive numbers we can know that the θ_{os} converges to zero. Therefore, as the velocity and orientation errors are guaranteed to converge to zero, the convergence problem of the distance error will be remained as an convergence problem to be proved. However, that problem is very simple because we showed that s_1 and s_2 converged to zeros. As we has shown that θ_{os} and $\dot{\theta}_{os}$ converge to zeros, from equation $s_2 = \dot{\theta}_{os} + \lambda_2(\theta_{os} + l_{os}) = 0$, we can know that the distance error converges to zero. Now, we have proved that the velocity and orientation and distance errors are guaranteed to converge to zero, that is, the stability of the proposed controller. However, the sliding mode controller has some problems in that the chattering of the sliding states s_1, s_2 across the sliding surfaces and the vibration of the input torques even though the errors are very small. Next, to overcome above problems we modify the proposed sliding mode controller and obtain a fuzzy sliding mode controller where the gains of the proposed sliding mode controller are functions of the sliding variables $s_1$ands_2 instead of fixed constants. The simulation results will be shown later at the simulation chapter.

3.2 A Fuzzy Sliding Mode Controller

We proposed the sliding mode controller and have shown the stability of the proposed controller and now we will introduce the fuzzy control approach into the proposed controller. The motive reason of adding the fuzzy approach into the sliding mode controller is to overcome or lessen the chattering of the input torques and the chattering of the sliding states $s_1$ands_2 across the sliding surfaces. We construct a fuzzy controller for each control input and the absolute value of each control input is alwalys larger than the absolute values of each disturbances. First, we consider the controller u_{r1} and u_{r1} proposed in the sliding mode controller above section as follows;

$$\begin{aligned} u_{r1} &= -K_{v1}s_1 - K_{v2}sign(s_1) + \lambda_1^2 \int v_{os}dt. \\ u_{r2} &= \lambda_2^2(\theta_{os} + l_{os}) - \lambda_2\dot{l}_{os} \\ &\quad -K_{\theta 1}s_2 - K_{\theta 2}sign(s_2) \end{aligned}$$

To simplify the fuzzy controller, we change the proposed controller as follows;

$$\begin{aligned} U_{N1} &= \frac{R_e R_a m}{K_\phi}\{-a\Omega^2 + \frac{K_\phi K_b v_u}{R_e R_a^2 m} && (47) \\ &\quad +\lambda_1^2 \int v_{os}dt + u_{n1}\} && (48) \\ U_{N2} &= \frac{R_e R_a(j+ma^2)}{2K_\phi l}\{\frac{mav_u\Omega}{(j+ma^2)} && (49) \\ &\quad +\frac{2K_\phi K_b l^2\Omega}{R_e R_a^2(j+ma^2)}\lambda_2^2(\theta_{os}+l_{os}) && (50) \\ &\quad -\lambda_2\dot{l}_{os} + u_{n2}\} && (51) \\ &\text{and} && (52) \\ u_{n1} &= -K_{v1}s_1 - K_{v2}sign(s_1) && (53) \\ u_{n2} &= -K_{\theta 1}s_2 - K_{\theta 2}sign(s_2) && (54) \end{aligned}$$

First, we consider the fuzzy controller for the velocity error and the same procedure can be applied for the combination of lateral offset and orientation error. We will keep it in mind that the absolute value of the fuzzy controller output always have to be larger than the absolute value of the disturbance to guarantee continuously the stability of the proposed sliding mode controller. Therefore, the actual output of the fuzzy sliding mode controller is the summation of both output of the fuzzy controller and the signed bound of the disturbance, that is,

$$\begin{aligned} u_{n1} &= u_{fuzzy1} - sign(s_1)|\tau_{d1}|_{max} && (55) \\ u_{n2} &= u_{fuzzy2} - sign(s_2)|\tau_{d2}|_{max} && (56) \end{aligned}$$

we take the distances from current state to the sliding surface(D_o) and the origin of the error coordinate(D_s) as the fuzzy controller inputs as shown in Fig.3 and the outputs are the propulsion input u_{fuzzy1} for tracking the desired velocity and the steering angle input u_{fuzzy2} for tracking the desired angle($\theta_{os} = 0$) and the convergenc of the distance from current state to the desired path to zero($l_{os} = 0$). In fact, considering the steering angle input we have to make the mobile robot move near the desired path first and then try to make both the distance error and the steering angle error approach to zero. So, considering the fuzzy rules, much more weights are given to the distance error than the orientation error to achieve above objective when the distance error is large.

The fuzzy input space for D_s is divided as following term set shown in Fig.4 ; NB(negative big), NS(negative small), NZ(negative zero), PZ(positive zero), PS(positive small), PB(positive big) and the fuzzy input space for D_o is divided as the following term set shown in Fig.4; ZE(zero), SM(small),

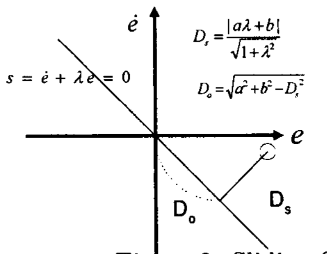

Figure 3: Sliding Surface

D_o	NB	NS	NZ	PZ	PS	PB
B	PB	PB	PB	NB	NB	NB
S	PB	PS	PS	NS	NS	NB
Z	PB	PS	PZ	NZ	NS	NB

Figure 4: Rule Base

BG(big). And the fuzzy ouput space is divided similarly as shown in Fig.4; NB(negative big), NS(negative small), NZ(negative zero), PZ(positive zero), PS(positive small), PB(positive big). The fuzzy rules are constructed by considering above points that the distance(D_s) to the sliding surface is much more important than D_o and D_o element considered only when the error and derivative of error is very small. The concrete fuzzy rules are represented as a table in Fig.4.

A defuzzification is a mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of nonfuzzy(crisp) control actions. In this paper, we choose the center of area method(COA) as a defuzzification strategy and in the case of a discrete universe, this COA method is represented as follows [2];

$$z_o = \frac{\sum_{j=1}^{n} \mu_z(w_j) \cdot w_j}{\sum_{j=1}^{n} \mu_z(w_j)} \qquad (57)$$

4 Simulation

We use a wheeled mobile robot model equipped with a differential drive.

Parameter Values of the Wheeled Mobile Robot

$a = 0.5m$: The distance between the c.o.m. and the drive wheels' axle; $l = 0.5m$: The distance between a drive wheel and the longitudinal axis; $m = 200Kg$: The mass of the wheeled mobile robot; $j = 12.5Kgm^2$: The yaw moment of inertia with respect to the c.o.m..; $R_a = 0.12m$: The radius of the drive wheels in the wheeled mobile robot; R_e = 8Homs : Resistance of the DC motor inductor; $K_\phi = K_b = 0.35$ Volt/rad/s: The characteristic parameters of the DC motor.

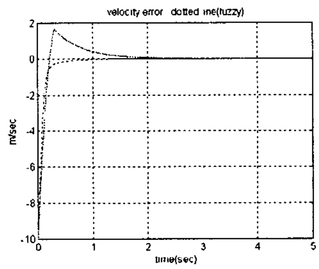

Figure 5: velocity error

We will compare the performance of the fuzzy sliding mode controller with that of the sliding mode controller in terms of the vibration of the input torques. As said above, simulations will be performed for the path-tracking of the wheelde mobile robot and we take the linear line as the desired path where the desired speed will be constant(v_{ud}= 10 m/s) and the desired orientation is 45 degrees.

Both disturbances and unmodeled dynamics are taken as a disturbance term τ_d and the disturbance terms are assumed to be a triangle functions as follow;

$$\tau_{d1} = mag\ sin(w_1 t) \qquad (58)$$

$$\tau_{d2} = mag\ sin(w_2 t) \qquad (59)$$

where mag is the magnitude of the disturbance. We design the sliding surface to be $\lambda_1 = 2$, $\lambda_2 = 2$. The gain parameters of SMC are $K_{v1} = 5$, $K_{v2} = 20$ and $K_{\theta 1} = 40.25$, $K_{\theta 2} = 30$. The path-tracking performances of the proposed controller, that is, $v_{os}, \theta_{os}, l_{os}$ is shown in Fig.5 , Fig.6 and Fig.7. Fig.5 shows the tracking performance of the desired velocity where the transient performance of proposed fuzzy SMC controller(dotted line) is better than that of proposed SMC controller. As shown in Fig.6 and Fig.7, the performances of two proposed controllers are almost the same in terms of lateral offset and orientation error. However, the performance of proposed fuzzy controller is much better than that of proposed SMC controller in terms of control input torques shown in Fig.8 and Fig.9. The vibration of input torques in SMC controller is very large and the magnitude of that is also large. But the vibrations of the input torques in fuzzy SMC are very small and the magnitudes of the input torques are also very small.

5 Conclusion

In this paper, we proposed a fuzzy sliding mode controller for a wheeled mobile robot equipped with a differential drive. We used the kinematics and dynamics equation derived by R.M. Desantis [3] [4]

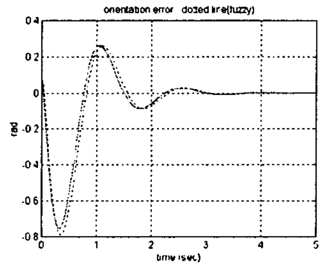

Figure 6: orientation error

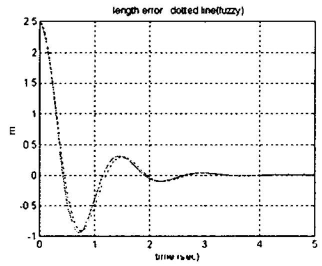

Figure 7: lateral offset

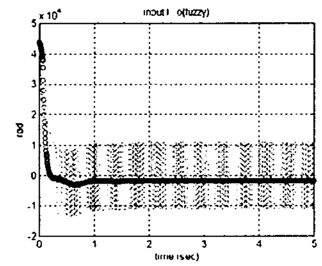

Figure 8: Input torque 1

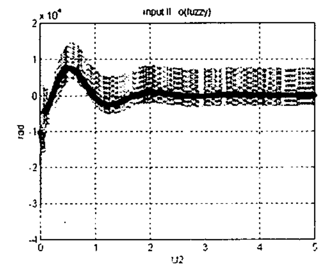

Figure 9: Input torque 2

and added the unknown dynamic term(τ_d) into the dynamics equation. We have constructed and analyzed a sliding mode controller and proved the stability of the proposed controller. And transforming the proposed controller a fuzzy controller, that is, a fuzzy sliding mode controller, we could obtain a fuzzy sliding mode controller, the stability of which was guaranteed avoiding the difficult stability issue of the proposed fuzzy controller. In this paper, we think that much contribution is given to the design of the sliding surface. In other words, using only two slidign surfaces we showed that the three control variables, position and orientation, converged to the desired values. And in the presence of external disturbance we obtain a fuzzy sliding mode(SMC) controller which guarantees a stable path-tracking.

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