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A Method for Separating Volterra Kernels of Nonlinear Systems by Use of Different Amplitude M-sequences

Hiroshi HARADA *, Eiji NISHIYAMA **,
Hiroshi KASHIWAGI * and Teruo YAMAGUCHI *
* Faculty of Engineering, Kumamoto University, Kumamoto, Japan
Tel: +81-96-342-3747, Fax: +81-96-342-3729
Email: hiroshi@mech.kumamoto-u.ac.jp
** Kumamoto National College of Technology, Kumamoto, Japan

Abstract

This paper describes a new method for separation of the Volterra kernels which are identified by use of M-sequence. One of the authors has proposed a method for identification of Volterra kernels of nonlinear systems using M-sequence and correlation technique. When M-sequences are applied to a nonlinear system, the crosscorrelation function between the input and the output of the nonlinear system includes cross-sections of high-order Volterra kernels. However, if various order Volterra kernels exixt on the obtained crosscorrelation function, it is difficult to separate the Volterra kernels.

In this paper, the authors show that the magnitude of Volterra kernels is maginified by the amplitude of M-sequence according to the order of Volterra kernels. By use of this property, each order Volterra kernel is obtained by solving linear equations.

Simulations are carried out for some nonlinear systems. The results show that Volterra kernels can be separated in each order successfully by the proposed method.

Key words : Nonlinear system, Identification, Volterra kernel, M-sequence, Crosscorrelation function

1. Introduction

One of the authors has proposed a method to identify Volterra kernels of nonlinear systems using M-sequence and correlation technique [1]~[4]. When M-sequences are applied to a nonlinear system, the crosscorrelation function between the input and the output includes cross-sections of high-order Volterra kernels. However, when the degree of M-sequence becomes low, the various order Volterra kernels overlap with each other on the crosscorrelation function. So, it is difficult to identify the Volterra kernels.

In order to solve this problem, this paper describes a new method for separation of Volterra kernels which are included in the crosscorrelation function. First, we briefly review the method for identification of Volterra kernels of nonlinear systems using M-sequence and correlation function. Then, we show that the magnitude of the Volterra kernels are maginified by the amplitude of M-sequence according to the order of Volterra kernels. Using this property, the Volterra kernels are separated in each order by solving linear equations.

Simulations are carried out for some nonlinear systems, The results show that the overlapped Volterra kernels can be separated successfully by the proposed method.

2. Identification of Volterra kernels

An example of a nonlinear system which is identified in this paper is shown in Fig. 1. This nonlinear system is called a Wiener type nonlinear system and can be represented as a cascade of a linear system and a nonlinear system. Let $u(t)$, $z(t)$ and $y(t)$ be the input and the output of the linear system and the output of the nonlinear sytem, respectively. Then the relation between $u(t)$ and $y(t)$ can be expressed by

$$y(t)=\sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty}g_i(\tau_1,\tau_2,\cdots,\tau_i)\times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2\cdots d\tau_i. \quad (1)$$

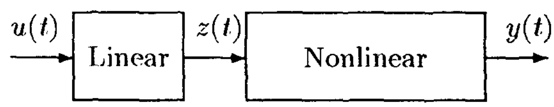

Fig. 1: An example of nonlinear system

Here, $g_i(\tau_1, \tau_2, \cdots, \tau_i)$ is i-th order Volterra kernels.

When an M-sequence is used as the input of the nonlinear system, the crosscorrelation function $\phi_{uy}(\tau)$ between the input $u(t)$ and the output $y(t)$ is given by the next equation.

$$\begin{aligned}\phi_{uy}(\tau) &= \sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} g_i(\tau_1, \tau_2, \cdots, \tau_i) \\ &\quad \times\phi_{u_1u_1\cdots u_1}(\tau_1, \tau_2, \cdots \tau_i)d\tau_1 d\tau_2\cdots d\tau_i \\ &= \sum_{i=1}^{\infty} h_i(\tau) \qquad (2)\end{aligned}$$

Here, $\phi_{u_1u_1\cdots u_1}(\tau_1, \tau_2, \cdots \tau_i)$ is i-th order autocorrelation function of M-sequence and $h_i(\tau)$ is the summation of i-th order Volterra kernel which appear on the crosscorrelation function. When the order i is 1,2,3, they can be expressed by the following equations.

$$h_1(\tau) = \Delta t g_1(\tau) \qquad (3)$$

$$h_2(\tau) = 2(\Delta t)^2\sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}\Delta t, \tau - k_{22}^{(j)}\Delta t) \qquad (4)$$

$$\begin{aligned}h_3(\tau) &= -2(\Delta t)^3 g_3(\tau, \tau, \tau) \\ &+3(\Delta t)^3\sum_{q=1}^{m_3} g_3(\tau, q, q) + 6(\Delta t)^3 \\ &\sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}\Delta t, \tau - k_{32}^{(j)}\Delta t, \tau - k_{33}^{(j)}\Delta t) \qquad (5)\end{aligned}$$

Here, Δt is a clock period of M-sequence and m_i is the number of cross-section of i-th order Volterra kernels. When the crosscorrelation function is obtained, the first and second terms of the right side of eqn. (5) overlap with the first Volterra kernel. In order to exclude these high-order Volterra kernels, it is neccesary to separate the Volterra kernels in each order.

3. Separation of Volterra kernels

In order to separate the Volterra kernels, the nonlinear system shown is approximated to the nonlinear system shown in Fig. 2. In this case,

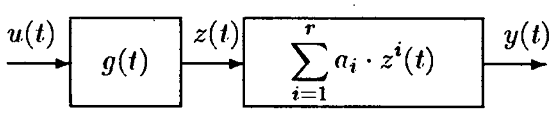

Fig. 2: Approximation of a nonlinear system

output $y(t)$ of the nonlinear system is given by the next equation.

$$\begin{aligned}y(t) &= \sum_{i=1}^{r} a_i\cdot z^i(t) \\ &= \sum_{i=1}^{r}\left\{\int_0^{\infty} g(\tau)u(t-\tau)d\tau\right\}^i \\ &= \sum_{i=1}^{r} h_i(\tau) \qquad (6)\end{aligned}$$

Then, i-th order Volterra kernel $g_i(\tau_1, \tau_2, \cdots, \tau_i)$ becomes

$$g_i(\tau_1, \tau_2, \cdots, \tau_i) = a_i\prod_{q=1}^{i} g_1(\tau_q) \qquad (7)$$

Let A be the amplitude of M-sequence which is applied to the nonlinear system. Then, the autocorrelation function $\phi_{uu\cdots u}(\tau)$ of M-sequence becomes

$$\begin{aligned}&\phi_{uu\cdots u}(\tau - \tau_1, \cdots, \tau - \tau_i) \\ &= A^i\phi_{u_1u_1\cdots u_1}(\tau - \tau_1, \cdots, \tau - \tau_i) \qquad (8)\end{aligned}$$

Let $\phi_{uy}(\tau; A)$ denote the crosscorrelation function between the input M-sequence $u(t)$, the amplitude of which is equal to A, and the output $y(t)$ of the nonlinear system. Substituting eqn. (8) into eqn. (6), $\phi_{uy}(\tau; A)$ is given as

$$\phi_{uy}(\tau; A) = \sum_{i=1}^{r} A^i\cdot h_i(\tau) \qquad (9)$$

By use of eqn. (9), the i-th order Volterra kernels $h_i(\tau)$ can be separated as follows. First, the same M-sequences which have r different amplitude A_j $(1 \le j \le r)$ are applied to the nonlinear system. The crosscorrelation functions $\phi_{uy}(\tau; A_j)$ are calculated and the following linear equations are obtained.

$$\boldsymbol{\phi}(\tau) = \boldsymbol{A}\boldsymbol{h} \qquad (10)$$

$$\boldsymbol{\phi}(\tau) = \begin{pmatrix}\phi_{uy}(\tau; A_1) \\ \phi_{uy}(\tau; A_2) \\ \vdots \\ \phi_{uy}(\tau; A_r)\end{pmatrix} \qquad (11)$$

$$\boldsymbol{A} = \begin{pmatrix}A_1 & A_1^2 & \cdots & A_1^r \\ A_2 & A_2^2 & \cdots & A_2^r \\ \vdots & \vdots & \ddots & \vdots \\ A_r & A_r^2 & \cdots & A_r^r\end{pmatrix} \qquad (12)$$

$$\boldsymbol{h}(\tau) = \begin{pmatrix}h_1(\tau) \\ h_2(\tau) \\ \vdots \\ h_r(\tau)\end{pmatrix} \qquad (13)$$

By use of eqn. (10), the i-th order Volterra kernel $h_i(\tau)$ can be obtained by solving the next equation.

$$h(\tau) = A^{-1}\phi(\tau) \tag{14}$$

4. Computer simulation

In order to ensure that i-th order Volterra kernels can be separated by use of the proposed method, some computer simulations are carried out. The results are as follows.

4..1 Polynomial system

The first example is a nonlinear system, the output of which can be represented by a polynomial of the output of a linear system. In this example, the linear system is 1st order system whose transfer function is given by the next equation.

$$G(s) = \frac{1}{1+5s} \tag{15}$$

Let $z(t)$ be the output of the linear system. Output $y(t)$ of the nonlinear system can be expressed by the next equation.

$$y(t) = z(t) + a_2 z^2(t) + a_3 z^3(t) \tag{16}$$
$$a_2 = a_3 = -10.0$$

An example of the crosscorrelation function is shown in Fig. 3. In this case, the order of M-sequence is 13 and the amplitude A of M-sequence is 2.

Taking inverse Laplace transform of eqn. (15), 1st Volterra kernel of the nonlinear system is given as

$$g_1(\tau) = \frac{1}{5}e^{-\frac{\tau}{5}}. \tag{17}$$

However, 3rd order Volterra kernels expressed by the first and the second term of eqn. (5) are

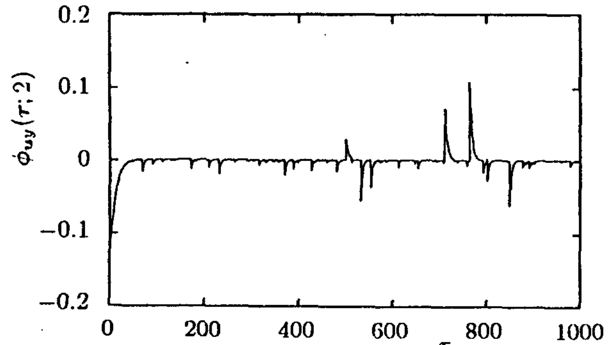

Fig. 3: Crosscorrelation function between input M-sequence and the output of the nonlinear system

overlapped with 1st order Volterra kernel. So, 1st order Volterra kernel can not be obtained from the crosscorrelation function.

In order to separate Volterra kernels by the proposed method, the amplotudes of M-sequences are chosen as $A_1 = 1, A_2 = -1, A_3 = 2$, respectively. In this case, matrix A becomes

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 2 & 4 & 8 \end{pmatrix}. \tag{18}$$

Then, i-th order Volterra kernels can be obtained by solving the next equation.

$$h(\tau) = \frac{1}{6}\begin{pmatrix} 6 & -2 & -1 \\ 3 & 3 & 0 \\ -3 & -1 & 1 \end{pmatrix}\phi(\tau) \tag{19}$$

The separated Volterra kernels $h_i(\tau)$ $(i = 1, 2, 3)$ are shown in Fig. 4.

In order to ensure that Volterra kernels can be separated by the proposed method, coefficient a_2 and a_3 are calculated by the method which we have proposed in literature [5]. The estimated coefficients $\hat{a}_2$ and $\hat{a}_3$ are

$$\hat{a}_2 = -9.99, \quad \hat{a}_3 = -9.91$$

Because the calculated ciefficients agree quite well with the true values, it is clear that Volterra kernels are separated by the proposed method.

4..2 Saturation type nonlinear system

The second example is a saturation type nonlinear system. In this case, the linear system is a 2nd order system and the transfer function $G(s)$ is given as

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{20}$$
$$\zeta = 0.5, \quad \omega_n = 0.5$$

The realtion between the output $z(t)$ of the linear system and the output $y(t)$ of the nonlinear system can be expressed by the next equation.

$$y(t) = \tan^{-1}(z(t)) \tag{21}$$

In this case, function $\tan^{-1}(-)$ is an odd function. Then, even order Volterra kernels do not exist.

$$h_{2i}(\tau) = 0 \quad (i = 1, 2, \cdots)$$

So, in order to obtain the 1st order Volterra kernel, the output $y(t)$ is approximated by the next equation.

$$y(t) = z(t) + a_3 \cdot z^3(t) + a_5 \cdot z^5(t) \tag{22}$$

Here, a_3 and a_5 are unknown constants. The amplitudes of M-sequences are chosen as $A_1 = 1, A_2 = 2, A_3 = 3$ and 1st order Volterra kernel $h_1(\tau)$ is calculated by

$$h_1(\tau) = \frac{45\phi_{uy}(\tau;1) - 9\phi_{uy}(\tau;2) + \phi_{uy}(\tau;3)}{30} \tag{23}$$

The identified 1st Volterra kernel is shown in Fig. 5. From this figure, the obtained 1st Volterra kernel agrees well with the theoretical value.

5. Conclusion

In this paper, we propose a new method for separation of Volterra kernels identified by M-sequence and crosscorrelation function. From the results of the computer simulation, i-th order Volterra kernels can be separated well by the proposed method successfully.

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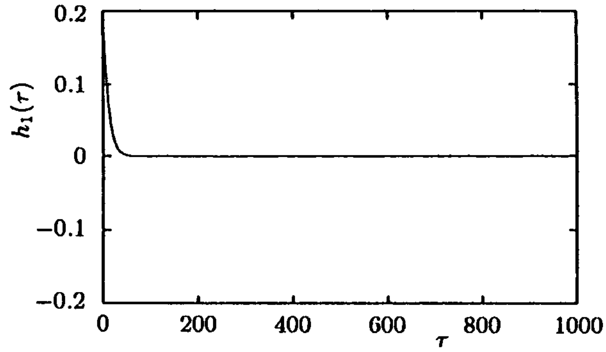
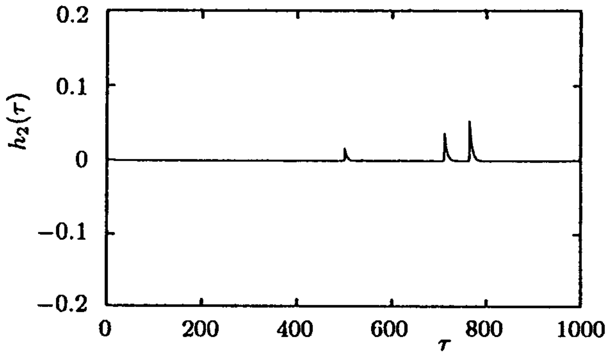
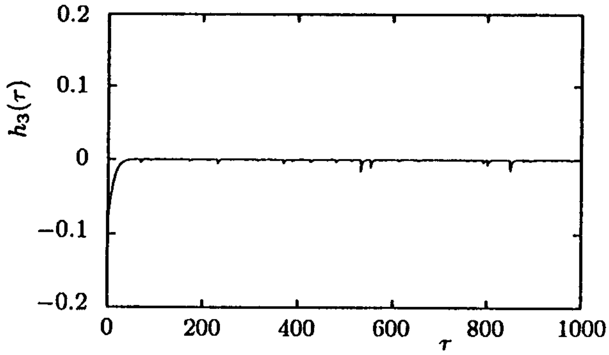


(a) 1st order Volterra kernel $h_1(\tau)$



(b) 2nd order Volterra kernels $h_2(\tau)$



(c) 3rd order Volterra kernels $h_3(\tau)$

Fig. 4: Separated Volterra kernels

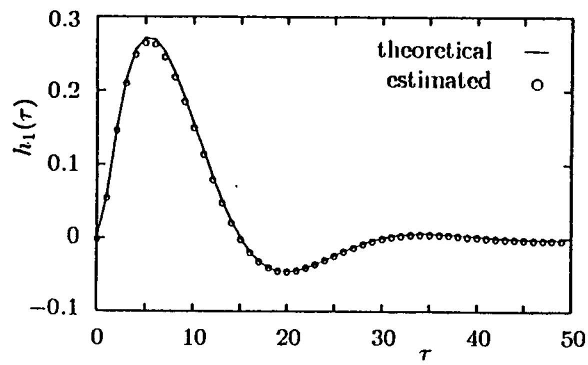


Fig. 5: Separated 1st Volterra kernel