Proceedings of the 13th KACC, October 1998

IDENTIFICATION OF HAMMERSTEIN-TYPE NONLINEAR SYSTEM

Eiji NISHIYAMA*, Hiroshi HARADA** and Hiroshi KASHIWAGI**
* Kumamoto National College of Technology, Nishigoshi-cho, Kumamoto, Japan.
** Faculty of Engineering, Kumamoto University, Kumamoto, Japan.

ABSTRACT

Many classes of nonlinear systems can be represented by Volterra kernel expansion. Therefore, identification of Volterra kernels of nonlinear system is an important task for obtaining the nonlinear characteristics of the nonlinear system. Although one of the authors has recently proposed a new method for obtaining the Volterra kernels of a nonlinear system by use of M-sequence and correlation technique, our method of nonlinear system identification is limited to Wiener-type nonlinear system and we can not apply this method to the identification of Hammerstein-type nonlinear system. This paper describes a new method for obtaining Volterra kernels of Hammerstein nonlinear system by adding a linear element in front of the Hammerstein system. First we calculate the linear element of Hammerstein system by use of conventional correlation method. Secondly, we put a linear element in front of Hammerstein system. Then the total system becomes Wiener-type nonlinear system. Therefore we can use our method on Volterra kernel identification by use of M-sequence. Thus we get the coefficients of the approximation polynomial of nonlinear element of Hammerstein system. From the results of simulation, a good agreement with theoretical considerations is obtained, showing a wide applicability of our method.

1 INTRODUCTION

The authors[1-6] have already proposed a method of identification of nonlinear system which uses polynomial approximation by use of Volterra kernels obtained from the crosscorrelation function between the input and the output.

However, this method is applicable to only Wiener-type nonlinear system.

If this method is applied to Hammerstein-type nonlinear system, we can not obtain Volterra kernels from the crosscorrelation function.

In this paper, we propose a new identification method for Hammerstein-type nonlinear system by use of pre-linear filter. By putting pre-filter in front of Hammerstein system, the pre-filter and Hammerstein-type system can be regard as Wiener-type nonlinear system plus linear element. Thus, we can apply our method of Volterra kernel identification. The simulation results show a good agreement with theoretical considerations.

2 IDENTIFICATION OF VOLTERRA KERNEL[4]

Let's consider a nonlinear system having input $u(t)$ and output $y(t)$ as shown in Fig.1. The nonlinear system can be written as in the next equation.

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \cdots, \tau_i) \times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2 \cdots d\tau_i \quad (1)$$

Here, $g_i(\tau_1, \tau_2, \cdots, \tau_i)$ is ith order Volterra kernel. We calculate the crosscorrelation function between the input $u(t)$ and $y(t)$. When the input is an M-sequence, the crosscorrelation function becomes

$$\phi_{uy}(\tau) \simeq \sum_{i=1}^{\infty} i!(\Delta t)^i \times \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)}). \quad (2)$$

Here, Δt shows the clock pulse period. And $k_{ir}^{(j)}$ ($r = 1, 2, \cdots, i$) are those integers which satisfy the next equation.

$$u(\tau)u(\tau + k_{i1}^{(j)})u(\tau + k_{i2}^{(j)}) \cdots u(\tau + k_{ii-1}^{(j)}) = u(\tau + k_{ii}^{(j)}) \quad (3)$$

m_i is the total number of $k_{ir}^{(j)}$. If $k_{ii}^{(j)}$'s are apart from each other sufficiently, we can obtain the Volterra kernels from Eq.(2).

3 DETERMINATION OF PARAMETERS OF APPROXIMATED POLYNOMIAL [3]

3.1 Approximated Polynomial

Let us consider a polynomial type nonlinear system as shown in Fig.2. This nonlinear model is shown in

Figure 1: Nonlinear system

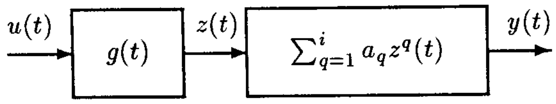


Figure 2: Polynomial type nonlinear system

Table 1: Suitable M-sequences for identifying Volterra kernels of nonlinear system[6]

deg.	poly.	1st length	2nd(df-1)	3rd(df-2,1)
6	163	11	25	-
12	10527	11	3629	595
13	24453	14	1287	7887
13	27217	24	7387	2994
14	62505	16	5955	7051
15	133257	31	10228	3516
16	260577	30	23505	48497

the next equation.

$$z(t) = g(\tau) * u(t) \tag{4}$$

$$y(t) = \sum_{q=1}^{i} a_q \cdot z^q(t) \tag{5}$$

Here $*$ denotes the convolution integral. By use of these equations, i-th Volterra kernels can be shown as follows,

$$g_i(\tau_1, \tau_2, \cdots, \tau_i) = a_i \cdot \prod_{q=1}^{i} g_1(\tau_q). \tag{6}$$

Then it is shown in reference[2],

$$g_1(\tau_1) = \phi_{uy}(\tau) \tag{7}$$

$$g_i(\tau_1, \tau_2, \cdots) = \phi_{uy}(\tau - k_{ii}^{(j)}). \tag{8}$$

The coefficients a_i's can be written as[3]

$$a_i = \frac{\phi_{uy}(\tau)}{\prod_{q=1}^{i} \phi_{uy}(\tau - k_{q1}^{(j)})}. \tag{9}$$

3.2 Selecting M-sequence

Table 1 shows suitable M-sequences for identifying the parameters of the approximated polynomial. In this table, **deg.** column shows degree of characteristic polynomial of M-sequence and **poly.** shows polynomial of M-sequence displayed in octal number. **1st length** shows the length of the interval where 1st order Volterra kernel can be accurately observed without any overlap of other kernels. **2nd(df-1)** shows the time delay at which the crosssection of 2nd Volterra kernels for one shift difference between the two arguments appears on the crosscorrelation function. **3rd(df-2,1)** shows the starting point of the crosssection of 3rd order kernel for shift 2-1.

3.3 Example of Getting Parameter

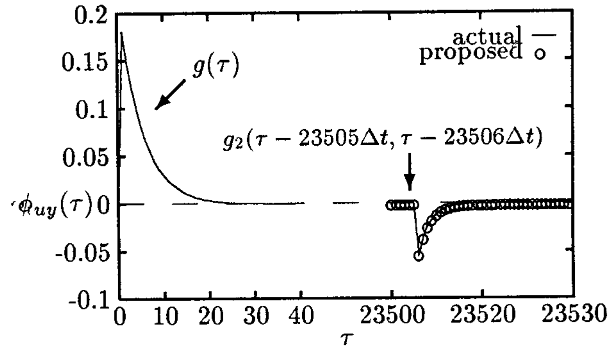


Figure 3: Crosscorrelation function of single input system

We applied this method to polynomial type nonlinear system as shown in Fig.2. In this figure, the linear element $g(t)$ is

$$G(s) = \frac{1}{1 + 5s} \tag{10}$$

and nonlinear element is assumed as $a_1 = 1.0$, $a_2 = 1.0$, $a_3 = 0.0$, $\cdots$. We use 16 degree M-sequence as an input. The crosscorrelation function between the input and output is shown in Fig.4. In this figure, solid line shows the theoretical crosscorrelation function between the input and output, and we can observe 1st order Volterra kernel from $\tau = 0$ to 30 and 2nd order from $\tau = 23506$ to 23520. By use of Eq.(9), the coefficient $\hat{a}_2$ is obtained as

$$\hat{a}_2 = 1.00018 \tag{11}$$

and by use of this coefficient, we calculate the 2nd order Volterra kernel. $\circ$ in Fig.4 is the calculated 2nd order Volterra kernel. We have a good agreement between the theoretical coefficient and the observed one. From this results, we can say that the system coefficients we used are accurately determined.

4 IDENTIFICATION OF HAMMERSTEIN TYPE NONILNEAR SYSTEM

We consider Hammerstein type nonlinear system, here we explain Wiener type and Hammerstein type nonlinear system. the nonlinear system shown in Fig.4 is call wiener-type, where input $u(t)$ is first applied to a linear element and then a nonlinear element. We can identify Weiner type nonlinear system by use of our previous research.

The nonlinear system shown in Fig.5 is called Hammer stein-type, where input $u(t)$ is first applied to a nonlinear element and then a linear element. Our method by use of M-sequcence correlation is not useful for this system. In this paper, we proposed a new method to identify a Hammerstein type nonlinear system. We consider that a nonlinear system has not memory and can be written as in the next equation

$$z(t) = \sum_{q=1}^{i} a_q \cdot \xi^q(t) \quad (a_i \in \Re). \tag{12}$$

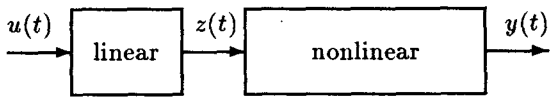


Figure 4: Wiener type nonlinear system

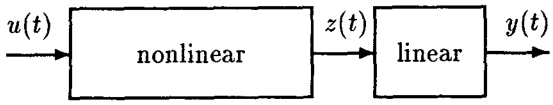


Figure 5: Hammerstein type nonlinear system

First, we calculate the linear element $g_L(\tau)$ of the nonlinear system. That is, we apply M-sequence ξ_{pm} in place of $\xi(t)$, and take the crosscorrelation function between the system output. Then we obtain $g_L(\tau)$.

Next, we get coefficient of approximated polynomial. The block diagram of this method is shown in Fig.6. In this figure, binary input $u(t)$ is changed into the multi-valued signal $\xi(t)$ via pre-linear filter. Then, we can identify the nonlinear Volterra kernel. Next, $z(t)$ becomes system output $y(t)$ via linear element $g_L(\tau_L)$. Nonlinear output $z(t)$ can be written as

$$\begin{aligned} z(t) &= \int_0^\infty g_0(\tau_0)u(t-\tau_0)d\tau_0 \\ &+ \int_0^\infty \int_0^\infty g_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \cdots . \end{aligned} \quad (13)$$

Then, system output $y(t)$ is shown as the convolution between $z(t)$ and $g_L(\tau)$. Since $u(t')$ can be defined only for $(0 < t' < t)$, we have

$$\begin{aligned} y(t) &= \int_0^t g_L(\tau_L) z(t-\tau_L)d\tau_L \\ &= \int_0^t g_L(\tau_L)\{ \\ & \int_0^t g_0(\tau_0)u(t-\tau_0-\tau_L)d\tau_0 d\tau_L \\ &+ \int_0^t \int_0^t g_2(\tau_1,\tau_2)u(t-\tau_1-\tau_L) \\ &\times u(t-\tau_2-\tau_L)d\tau_1 d\tau_2 + \cdots\}d\tau_L . \end{aligned} \quad (14)$$

Let us denote the first term of right hand side of Eq.(14) as $y_1(t)$. Then,

$$y_1(t) = \int_0^t \int_0^t g_L(\tau_L)g_0(\tau_0)u(t-\tau_0-\tau_L)d\tau_0\tau_L \quad (15)$$

and we transform τ_L to τ_1' by $\tau_0 + \tau_L = \tau_1'$,

$$y_1(t) = \int_{\tau_0}^{\tau_0+t} \int_0^t g_L(\tau_1'-\tau_0)g_0(\tau_0)u(t-\tau_1')d\tau_0 d\tau_1'. \quad (16)$$

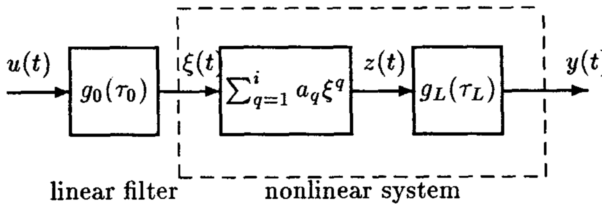


Figure 6: Hammerstein-type nonlinear system with pre-linear filter

Here $\tau_1' > \tau_0$, and $t > \tau_1'$ then,

$$y_1(t) = \int_0^t \int_0^{\tau_1'} g_L(\tau_1'-\tau_0)g_0(\tau_0)u(t-\tau_1')d\tau_0 d\tau_1'. \quad (17)$$

Therefore,

$$y_1(t) = \int_0^t g_{d1}(\tau_1')u(t-\tau_1')d\tau_1'. \quad (18)$$

Here,

$$g_{d1}(\tau_1') = \int_0^{\tau_1'} g_{01}(\tau_0)g_L(\tau_1'-\tau_0)d\tau_0. \quad (19)$$

Thus, the first order Volterra kernel of total system can be shown as the convolution between $g_0(\tau_0)$ and $g_L(\tau_L)$.

Let us denote the second term of right hand side of Eq.(14) as $y_2(t)$.

$$\begin{aligned} y_2(t) &= \int_0^t \int_0^t \int_0^t g_L(\tau_L)g_2(\tau_1,\tau_2) \\ &\times u(t-\tau_1-\tau_L)u(t-\tau_2-\tau_L)d\tau_1\tau_2\tau_L . \end{aligned} \quad (20)$$

By putting $\tau_1 + \tau_L = \tau_1'$, $\tau_2 + \tau_L = \tau_2'$, we have

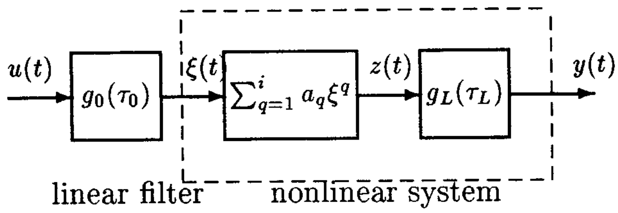
$$\begin{aligned} y_2(t) &= \int_0^t \int_{\tau_L}^{\tau_L+t} \int_{\tau_L}^{\tau_L+t} g_L(\tau_L) \\ &\times g_2(\tau_1'-\tau_1,\tau_2'-\tau_2)u(t-\tau_1')u(t-\tau_2')d\tau_1' d\tau_2' d\tau_L . \end{aligned} \quad (21)$$

Note that τ_1' and τ_2' in g_2 satisfy $\tau_0 < \tau_1'$, $\tau_0 < \tau_2'$ and τ_1 and τ_2 satisfy $\tau_1' < t$, $\tau_2' < t$. Since $u(t), g_L, g_2$ is continuous, we can change the order of integration. Thus,

$$\begin{aligned} y_2(t) &= \int_0^t \int_0^t \int_0^t g_L(\tau_L)g_2(\tau_1'-\tau_1,\tau_2'-\tau_2) \\ &\quad \times u(t-\tau_1')u(t-\tau_2')\tau_L d\tau_1' d\tau_2' \\ &= \int_0^t g_{d2}(\tau_1',\tau_2')u(t-\tau_1')u(t-\tau_2')d\tau_1' d\tau_2' \end{aligned} \quad (22)$$

where,

$$g_{d2}(\tau_1',\tau_2') = \int_0^{\tau_1',\tau_2'} g_L(\tau_L)g_2(\tau_1'-\tau_L,\tau_2'-\tau_L)d\tau_L. \quad (23)$$



Here the integration is carried out up to the smaller of τ_1' and τ_2'. The second order Volterra kernel of total system can be shown as the convolution between pre-filter and second order Volterra kernel of nonlinear element. $g_{d1}(\tau_1'), g_{d2}(\tau_1', \tau_2'), \cdots$ is displayed on the crosscorrelation function between system input and total output, so we can measure these functions. And the nonlinear system which can be represented as Eq.(12) can be written as $g_2(\tau_1, \tau_2) = a_2 g_0(\tau_1) g_0(\tau_2)$. Therefore,

$$g_{d2}(\tau_1', \tau_2') = a_2 \int_0^{\tau_1', \tau_2'} g_L(\tau_L) g_0(\tau_1' - \tau_L) g_0(\tau_2' - \tau_L) d\tau_L. \quad (24)$$

Similarly, third order Volterra kernel $g_{d3}(\tau_1', \tau_2', \tau_3')$ is

$$g_{d3}(\tau_1', \tau_2', \tau_3') = a_3 \int_0^{\tau_1', \tau_2', \tau_3'} g_L(\tau_L) \times g_0(\tau_1' - \tau_L) g_0(\tau_2' - \tau_L) g_0(\tau_3' - \tau_L) d\tau_L. \quad (25)$$

Then, coefficient a_2, a_3 can be written as

$$a_2 = g_{d2}(\tau_1', \tau_2') / \int_0^{\tau_1', \tau_2'} g_L(\tau_L) g_0(\tau_1' - \tau_L) g_0(\tau_2' - \tau_L) d\tau_L \quad (26)$$

$$a_3 = g_{d3}(\tau_1', \tau_2', \tau_3') / \int_0^{\tau_1', \tau_2', \tau_3'} g_L(\tau_L) \times g_0(\tau_1' - \tau_L) g_0(\tau_2' - \tau_L) g_0(\tau_3' - \tau_L) d\tau_L. \quad (27)$$

That is, the coefficient a_2, a_3 can be calculated by comparison between the crosscorrelation function and the convolution between pre-filter and linear element. And, in this way, we can obtain hider order coefficient $a_4, \cdots$, but we consider up to a_3 in this paper.

5 SIMULATION OF PROPOSED METHOD

We carried out numerical simulations in order to show the effectiveness of this method of identification of Hammerstein system. Fig. shows the block diagram of the simulation. We used M-sequence of 16th degree (201345(oct)).

5.1 Simulation up to second order

The nonlinear element of the simulated system is

$$z(t) = \xi(t) - 2\xi^2(t) \quad (28)$$

and the linear element is

$$G_L(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \quad (29)$$

$$(\omega_n = 1.0, \zeta = 0.7)$$

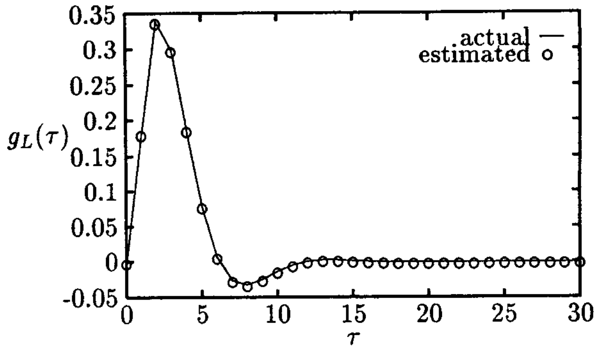


Figure 7: Linear part of this system

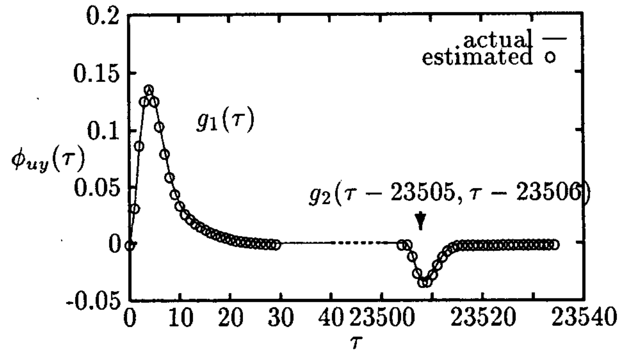


Figure 8: Crosscorrelation function of this system

The pre-linear filter in front of the nonlinear element is

$$G_0(s) = \frac{1}{s+5}. \quad (30)$$

The calculated first order Volterra kernel by use of the proposed method is shown in Fig. 7. In this figure, solid line indicates theoretical value, and o shows estimated value. Thus, calculated coefficient $\hat{a}_2$ from Eq.(26) is

$$\hat{a}_2 = -1.992 \quad (31)$$

$\hat{a}_2$ shows a good agreement with theoretical value. Fig.8 shows the crosscorrelation function comparing actual system with the convolution between measured $g_L(\tau)$ and $\hat{a}_2$.

5.2 Simulation up to third order

The nonlinear element of the simulated system is

$$z(t) = \xi(t) - 10\xi^2(t) - 10\xi^3 \quad (32)$$

and the linear element is

$$G_L(s) = \frac{1}{s+1}. \quad (33)$$

Also, pre-linear filter in front of the nonlinear element

$$G_0(s) = \frac{1}{s+5}. \quad (34)$$

Fig.9 shows the Volterra kernels on the crosscorrela-

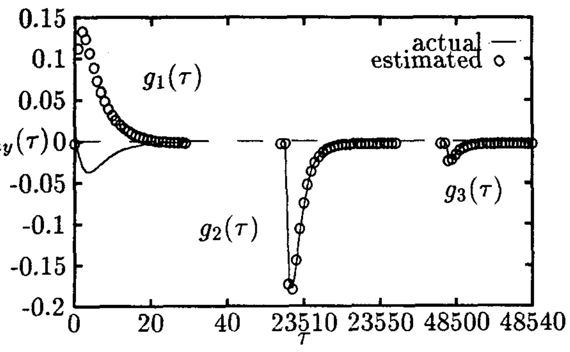


Figure 9: Crosscorrelation function of this system

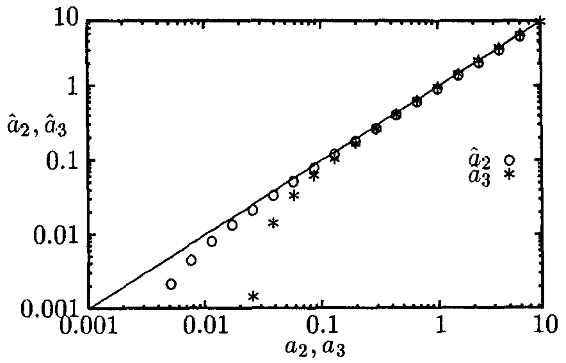


Figure 10: Accuracy of polynomial coefficient

tion function. In this figure, we see first order Volterra kernel from $\tau = 0$ to 50, second kernel from $\tau = 23505$ to 23550 and third kernel from $\tau = 48497$ to 48520. The solid line is actual value and o's are estimated values. The reason why the estimated first order Volterra kernel is different from actual values is that the actual first order kernel has the effect of odd higher kernels, Therefore the first Volterra kernel is estimated from the covolution integral of pre-filter $g_0(\tau)$ and linear element $g_L(\tau)$. Thus, calculated coefficients $\hat{a}_2$ and $\hat{a}_3$ from Eq.(26), Eq.(27) are

$$\hat{a}_2 = -9.97 \quad (35)$$

$$\hat{a}_3 = -9.64 \quad (36)$$

$\hat{a}_2$ and $\hat{a}_3$ show good agreements with theoretical values.

In this simulation, it is shown that the method of identification of Volterra kernel of total system is effective having high accuracy.

6 ACCURACY OF IDENTIFICATION

We investigated the accuracy of identified coefficients of Hammerstein type nonlinear system by use of this method, by changing a_2, a_3 independently. Fig.10 shows the relation of theoretical coefficient a_2, a_3 and estimated coefficient $\hat{a}_2$, $\hat{a}_3$. From this figure, we can say that a_2 is obtained with the accuracy of 0.01, and a_3 with 0.1.

7 CONCLUSION

We proposed a new method of identification of Volterra kernels of Hammerstein type nonlinear system. That is, we put a known pre-linear filter in front of Hammerstein system, then this system can be regard as Wiener type nonlinear system plus linear element. Therefore we can use our method on Volterra kernel identification by use of M-sequence. Then, we can identify Volterra kernel of total system from the crosscorrelation function between M-sequence input and the system output. And by use of this relation, we can identify the coefficient of the approximated polynomial of the nonlinear system.

Thus we get the coefficients of the approximation polynomial of nonlinear element of Hammerstein type nonlinear system.

From the results of simulation, a good agreement with theoretical considerations is obtained, showing a wide applicability of our method.

References

[1] Nishiyama,E., H.Kashiwagi : Identification of Volterra Kernels of Nonlinear systems by use of M-sequence Correlation *Proc. '97 SYSID* held in Kitakyushu, Japan pp.739-743, 1997.

[2] Harada,H., E.Nishiyama, and H.Kashiwagi : A Method for Identifying Voltterra Kernels of Nonlinear System Using Low-degree M-sequence *Trans. SICE*, Vol.34, No.1, pp.9-12, 1998.

[3] Kashiwagi.H. and Y.Sun : Identification of Volterra Kernels of Nonlinear System : *Trans. SICE*, Vol.31, No.8, pp.1054-1060, 1995.

[4] Nishiyama,E. : Linearization and Polynomial Modeling for Nonlinear System by use of Volterra Kernels: *Ph.D Thesis*, Graduate School of Science and Technology, Kumamoto University, 1998.

[5] Barker,H.A. and S.N.Obidegwu : Combined Cross-correlation Method for the Measurement of 2nd-order Volterra Kernel. *Proc. IEE*, Vol.120-1, pp.144-118, 1973.

[6] Schetzen,M. : *The Volterra and Wiener Theories of Nonlinear Systems*, A Wiley Intersci., Pub., 1980.