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A New Input Estimation Algorithm for Target Tracking Problem

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Abstract

In this paper, a new input estimation algorithm is proposed for target tracking problem. The unknown target maneuver is approximated by a linear combination of independent time functions and the coefficients are estimated by using a weighted least-squares estimation technique. The proposed algorithm is verified by computer simulation of a realistic two-dimensional tracking problem. The proposed algorithm provides significant improvements in estimation performance over the conventional input estimation techniques based on the constant-input assumption.

1 Introduction

A linear Kalman filter has been widely applied to target tracking problems because of its recursive structure and optimal estimation property[1]. In the presence of unknown target maneuver, the estimates of Kalman filter is biased, and thus an auxiliary estimation process should be implemented to compensate the error [2]. Two different approaches have been widely studied to handle this problem: model-based adaptive filtering and unknown input estimation.

The model-based adaptive filtering technique is pioneered by Singer [3], who suggests that the unknown input of a manned maneuverable vehicle is modeled as a correlated white noise process. This formulation suppresses the bias caused by unknown inputs to some degree but exhibits poorer performance than simple Kalman filtering when there is no target maneuver. Instead of relying on a statistical description of input maneuvers, Bar-Shalom and Birmiwal [4] propose a variable-dimension filter which introduces extra states when an input is detected. Another interesting work is done by Moose [5] who considers a multiple-model formulation in which the target acceleration is assumed to belong a time-invariant set of discrete values and the change of its value is modeled as a semi-Markov process. Model-based adaptive filtering techniques have evolved to the interacting multiple model (IMM) algorithm [6], in which the change of the plant is modeled as a Markovian parameter having a transition probability. Using a hypothesis merging technique for multiple-model filtering, the IMM algorithm calculates the Bayesian sum of the filter outputs.

Although the model-based approaches are easily implementable and computationally efficient, the filter performance may be greatly degraded in the presence of external inputs that comply with none of the models. Input estimation is a totally different approach which determines if external inputs exist, and directly estimates the magnitude of the unknown input. McAulay, et al [7] uses statistical decision theory to derive an optimum input detector. Once an input sequence is detected, then the filter is reinitialized using the most recent data. Chan, et al [8] proposes an input estimation technique using a supplementary estimation procedure to a normal Kalman filter and utilizing the least square method to calculate the input magnitude. Bogler [9] extends this method to derive a recursive algorithm based on multiple-model filtering which is inspired by [10].

The merit of input estimation is that unknown inputs are directly estimated from the available measurements regardless of the input level, without re-initializing any of filter parameters. However, the earlier works on input estimation assume that the input level is constant within the detection window. This has been the main drawback of the input estimation approaches since a realistic input may change in various fashions within the detection window. Consequently, estimation of the unknown input signal based on such a strong assumption on the input shapes gives only limited performance. Direct estimation of the input level at every sample time can be conceived as an alternative to the approaches based on the assumption of a constant-level input. However, estimation of an arbitrary input signal is computationally inefficient as the detection window size (or the number of measurement samples) increases. Note that the detection window size should be sufficiently large if the input sequences are properly estimated from noisy measurements.

In this paper, a new input estimation method is proposed to overcome the constant-level assumption on input signal. Instead of estimating the input sequence at every sample time, the proposed method approximates the input signal as a linear combination of some elementary base functions of time. This formulation greatly simplifies the estimation procedure since only the coefficient vector of the base functions needs to be estimated. Due to this feature, a large detection window can be employed without significantly increasing the computational burden. Furthermore, the proposed method is capable of decomposing the unknown input sequence by using time functions of interest. For example, if a constant and a linear functions are chosen as the base functions, then the estimated coefficient vector is likely to represent the constant and linear components of the unknown input sequence.

The estimation algorithm is derived by using the weighted least-squares estimation method. The optimal estimates of coefficient vector are determined by extremizing a cost value which is a weighted sum of filter residuals. The proposed method also includes a detection logic, which is an important part of input estimation. A generalized detection logic based on the conventional chi-square test is derived. This logic analyzes the statistical characteristics of the contribution of measurement noise to the cost value and compares the result with a threshold value calculated for a prescribed probability of false alarm(PFA). The state estimates are affected only when an input detection is declared.

As a numerical example, the new algorithm is applied to a realistic two-dimensional tracking problem. Various simulation results show that estimation performance is significantly enhanced by including linear, or quadratic components in addition to the constant-level input model. The effect of the choice of base functions on estimation accuracy is also investigated.

This paper is organized as follows: First, two different linear Kalman filters are considered; one is for the nominal system which does not consider the unknown input sequence, and the other is for the system whose unknown input is approximated by a linear combination of base time functions. Using the residuals of the two filters, an input estimation algorithm is derived using the weighted least-squares estimation technique. Next, a detection logic is presented and the update procedure is summarized. Finally, the proposed input estimation algorithm is applied to a conventional two-dimensional tracking problem for two different input scenarios, and computer simulation results are provided.

2 Problem Formulation

The mathematical model is expressed in the rectangular coordinate system. The system equations are linear time-invariant with linear measurements which are transformed from bearing-only measurements. The relative motion of target is

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + E\mathbf{w}_k, \\ \mathbf{z}_{k+1} &= H\mathbf{x}_{k+1} + \mathbf{v}_{k+1}, \\ E\left\{\mathbf{w}_k\mathbf{w}_j^T\right\} &= Q_k\delta_{kj},\ E\left\{\mathbf{v}_k\mathbf{v}_j^T\right\} = R_k\delta_{kj}, \end{aligned} \tag{1}$$

where $\mathbf{x}_k = (x_k, \dot{x}_k, y_k, \dot{y}_k)^T$ is the relative positions and velocities of the target in 2-dimensional plain, and $\mathbf{w}_k, \mathbf{v}_k$ are noise sequences with given covariance matrices. The appropriate matrices are

$$\begin{aligned} F &= \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\ G &= E = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}, \\ H &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \end{aligned}$$

A linear Kalman filter for this system is expressed as

$$\begin{aligned} \hat{\mathbf{x}}_{k+1} &= M_{k+1}\hat{\mathbf{x}}_k + N_{k+1}\mathbf{u}_k + K_{k+1}\mathbf{z}_{k+1}, \\ \bar{P}_{k+1} &= F\hat{P}_kF^T + EQ_kE^T, \\ \hat{P}_{k+1} &= [I - K_{k+1}H]\,\bar{P}_{k+1}, \end{aligned} \tag{2}$$

where $M_k \triangleq [I - K_kH]F, N_k \triangleq [I - K_kH]\,G$, and the Kalman gain matrix is computed as

$$K_{k+1} = \bar{P}_{k+1}H^T\left[H\bar{P}_{k+1}H^T + R_{k+1}\right]^{-1}. \tag{3}$$

In the case that the input vector sequence $\{\mathbf{u}_k\}$ is unknown, the Kalman filter does not work properly unless an adaptive tuning of the filter model or a supplementary estimation process of the unknown input is implemented; the latter is referred to as input estimation. In general, the filter for input estimation is composed of the basic Kalman filter and an additional input estimation algorithm to compensate the effect of unknown inputs.

Similar to the other estimation algorithms, we first construct a nominal Kalman filter which assumes that the system model has no input, i.e. $\mathbf{u}_k = 0$ in (1). The state of this filter are regarded as the nominal estimate until the presence of unknown input is detected. The nominal estimate is denoted as $\hat{\mathbf{x}}_k^s$ to discriminate it from $\hat{\mathbf{x}}_k$, which is the actual estimate we obtain when unknown inputs are present. The filter equation of the nominal Kalman filter is obtained by letting $\mathbf{u}_k \equiv 0$ in (2) as

$$\hat{\mathbf{x}}_{k+1}^s = M_{k+1}\hat{\mathbf{x}}_k^s + K_{k+1}\mathbf{z}_{k+1}. \quad (4)$$

Suppose that there are no inputs before the time $t = kT$, then $\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^s$. But if there exists an input signal $\mathbf{u}_k$ at $t = kT$, then the two state estimates will no longer be same, and the difference after n sampling times is expressed as

$$\hat{\mathbf{x}}_{k+n} = \hat{\mathbf{x}}_{k+n}^s + \sum_{i=1}^{n} M_{k+n}^{k+i} N_{k+i}\mathbf{u}_{k+i-1}, \quad (5)$$

where the transition matrix M_k^n is defined as $M_k^n \triangleq M_k M_{k-1} \cdots M_{n+1}$. The residuals of the two filters at time $t = (k+n)T$ are defined as [1]

$$\begin{aligned} \Im_{k+n} &\triangleq \mathbf{z}_{k+n} - H_{k+n}\hat{\mathbf{x}}_{k+n}, \\ \Im_{k+n}^s &\triangleq \mathbf{z}_{k+n} - H_{k+n}\hat{\mathbf{x}}_{k+n}^s. \end{aligned} \quad (6)$$

From (5), the two residuals are related as

$$\Im_{k+n} = \Im_{k+n}^s - \sum_{i=1}^{n} H_{k+n} M_{k+n}^{k+i} N_{k+i}\mathbf{u}_{k+i-1}. \quad (7)$$

3 Estimation of Unknown Target Maneuver

Consider a detection window of N samples. To simplify the expression, the following notations are defined.

$$A_{k+n}^{k+i} \triangleq H_{k+n} M_{k+n}^{k+i} N_{k+i}, \quad (8)$$

$$B_{k+n} \triangleq H_{k+n}\hat{P}_{k+n}H_{k+n}^T + R_{k+n}, \quad (9)$$

where A_{k+n}^{k+i} represents the contribution of the input $\mathbf{u}_{k+i-1}$ to the measurement $\mathbf{z}_{k+n}$, and B_{k+n} is the covariance matrix of $\Im_{k+n}$. The cost value is defined as

$$J(k,N) \triangleq -\frac{1}{2}\sum_{n=1}^{N} \Im_{k+n}^T B_{k+n}^{-1} \Im_{k+n}. \quad (10)$$

Now, assume that the input sequence is expressed as a linear combination of p time functions as

$$\mathbf{u}_k = \sum_{j=1}^{p} \mathbf{a}_j b_j(t_k), \quad (11)$$

where $\mathbf{a}_j \in \Re^l$ is a constant-coefficient vector of a scalar function $b_j(t_k)$. Using (7) and (11), the cost becomes

$$J(k,N) = -\frac{1}{2}\sum_{n=1}^{N}\left(\Im_{k+n}^s - \sum_{i=1}^{n} A_{k+n}^{k+i} \sum_{j=1}^{p}\mathbf{a}_j b_j(t_{k+i-1})\right)^T B_{k+n}^{-1} \left(\Im_{k+n}^s - \sum_{i=1}^{n} A_{k+n}^{k+i}\sum_{j=1}^{p}\mathbf{a}_j b_j(t_{k+i-1})\right) \quad (12)$$

The estimation of the unknown input is performed by calculating the optimal coefficient vectors $\hat{\mathbf{a}}_j, j = 1,\ldots,p$, which maximize (due to sign convention) the cost value defined by (12).

The estimation process is straightforward. First, differentiate the cost value with respect to $\mathbf{a}_\alpha, \alpha = 1,\ldots,p$, to obtain

$$\begin{aligned} &\sum_{n=1}^{N}\left(C_{k+n}^{\alpha}\right)^T B_{k+n}^{-1}\Im_{k+n}^s - \\ &\quad \sum_{n=1}^{N}\left(C_{k+n}^{\alpha}\right)^T B_{k+n}^{-1}\sum_{\beta=1}^{p} C_{k+n}^{\beta}\hat{\mathbf{a}}_\beta = 0, \\ &\quad \alpha,\beta = 1,\ldots,p, \end{aligned} \quad (13)$$

where

$$C_{k+n}^{\alpha} \triangleq \sum_{i=1}^{n} A_{k+n}^{k+i} b_\alpha(t_{k+i-1}). \quad (14)$$

Observe that $C_{k+n}^{\alpha} \in \Re^{m\times l}$ is the sum of the contribution of $b_\alpha(t_i), t_i \in [t_k, t_{k+n-1}]$, to the measurement $\mathbf{z}_{k+n}$. Now define

$$\omega_\alpha \triangleq \sum_{n=1}^{N}\left(C_{k+n}^{\alpha}\right)^T B_{k+n}^{-1}\Im_{k+n}^s \in \Re^l, \quad (15)$$

$$\gamma_{\alpha\beta} \triangleq \sum_{n=1}^{N}\left(C_{k+n}^{\alpha}\right)^T B_{k+n}^{-1} C_{k+n}^{\beta} \in \Re^{l\times l}, \quad (16)$$

then (13) is simplified as

$$\sum_{\beta=1}^{p}\gamma_{\alpha\beta}\hat{\mathbf{a}}_\beta = \omega_\beta, \quad \alpha = 1,\ldots,p. \quad (17)$$

For further simplification, we define

$$\begin{aligned} \mathbf{\Gamma} &\triangleq \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1p} \\ \vdots & \ddots & \vdots \\ \gamma_{p1} & \cdots & \gamma_{pp} \end{bmatrix} \in \Re^{lp\times lp}, \\ \mathbf{\Omega} &\triangleq \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_p \end{bmatrix} \in \Re^{lp}, \ \hat{\mathbf{a}} \triangleq \begin{bmatrix} \hat{\mathbf{a}}_1 \\ \vdots \\ \hat{\mathbf{a}}_p \end{bmatrix} \in \Re^{lp}. \end{aligned} \quad (18)$$

Then, (17) is rewritten as

$$\mathbf{\Gamma}\hat{\mathbf{a}} = \mathbf{\Omega}. \tag{19}$$

The solution of equation (19) can be obtained by simply inverting the matrix $\mathbf{\Gamma}$ as

$$\hat{\mathbf{a}} = \mathbf{\Gamma}^{-1}\mathbf{\Omega}. \tag{20}$$

4 Detection and Update

The most useful tool for the detection logic is the chi-square test. First, assume that no input has been applied to the system. Then the cost value of (10) is a chi-square random variable. A further investigation of this value can be made by substituting the estimated coefficient vectors into (12). A straightforward calculation by substituting (20) into (12) results in

$$J^o = -\frac{1}{2}\sum_{n=1}^{N}(\Im^s_{k+n})^T B^{-1}_{k+n}\Im^s_{k+n} + \frac{1}{2}\mathbf{\Omega}^{\mathbf{T}}\mathbf{\Gamma}^{-1}\mathbf{\Omega}. \tag{21}$$

The first term of (21) is independent to the estimation algorithm. The second term is, therefore, solely interesting for the detection purpose. We define this term as a new cost value

$$J^o_1 \triangleq \mathbf{\Omega}^{\mathbf{T}}\mathbf{\Gamma}^{-1}\mathbf{\Omega}. \tag{22}$$

The statistics of the new cost value is characterized by the following lemma.

Lemma 1 The cost value J^o_1 is a chi-square random variable of lp degree of freedom.

(The proof is omitted.) ∎

Suppose λ is the threshold value used for the detection policy expressed as an inequality condition

$$J^o_1 = \mathbf{\Omega}^{\mathbf{T}}\mathbf{\Gamma}^{-1}\mathbf{\Omega} > \lambda. \tag{23}$$

Then the detection threshold λ can be determined by the prescribed probability of false alarm (PFA), which represents how probably happen false alarms due to the measurement noise, as

$$\lambda = \chi^2_{\mathrm{PFA},lp}. \tag{24}$$

The value can be obtained from the statistical table as in [11].

Once the input is detected, the filter state vector and covariance matrix are updated using the following equations.

$$\hat{\mathbf{x}}^{\mathrm{up}}_{k+n} = \hat{\mathbf{x}}^s_{k+n} + \sum_{j=1}^{p} D^j_{k+n}\hat{\mathbf{a}}_j \tag{25}$$

$$\Im^{\mathrm{up}}_{k+n} = \Im^s_{k+n} - \sum_{j=1}^{p} C^j_{k+n}\hat{\mathbf{a}}_j \tag{26}$$

$$\hat{P}^{\mathrm{up}}_{k+n} = \hat{P}_{k+n} + \sum_{i=1}^{p}\sum_{j=1}^{p} D^i_{k+n}\zeta_{ij}\left(D^j_{k+n}\right)^T \tag{27}$$

$$B^{\mathrm{up}}_{k+n} = B_{k+n} + \sum_{i=1}^{p}\sum_{j=1}^{p} C^i_{k+n}\zeta_{ij}\left(C^j_{k+n}\right)^T \tag{28}$$

for all $n = 1, \ldots, N$, where

$$D^j_{k+n} \triangleq \sum_{i=1}^{n} M^{k+i}_{k+n}N_{k+i}b_j(t_{k+i-1}) \in \Re^{n\times l}, \tag{29}$$

is the sum of the contribution of input base function $b_j(t_i)$, $t_i \in [t_k, t_{k+n-1}]$, to the filter state vector, and is related to C^j_{k+n} as

$$C^j_{k+n} = HD^j_{k+n}. \tag{30}$$

5 Simulation

The proposed algorithm is verified by computer simulation of a realistic two-dimensional tracking problem. The base functions used for simulations are displayed in Fig.1, where b_1 is constant, b_2 is linear, and b_3 is quadratic.

The proposed algorithm is tested with two input scenarios illustrated in Fig.2. Scenario I is the case that the target executes a constant $3g$ maneuver in both axes from $t = 20$ sec to $t = 25$ sec. Scenario II considers a more realistic case in which the target turns 180^o to evade the closing intercepter. The initial values for both scenarios are chosen as

$$x_0 = \hat{x}_0 = (0,\ 200,\ 0,\ 200)^T,$$
$$\hat{P}_0 = 100I,$$

that is, the target starts from the origin with a velocity of 200 m/sec in both x and y axes. The noise covariance matrices are assumed to be constant with respect to time as $R = 100I$ and $Q = I$. PFA is set as 0.005.

We considered three different input estimation; Scheme I corresponds to the well-known input estimation method of Chan, et al [8], which considers b_1 only. Scheme II has two base functions b_1 and b_2 and Scheme III includes all three base functions. The filter performance of each scheme for each of the two input scenarios is computed by a Monte Carlo simulation with 20 runs. The simulation results are given in Figs. 3 and 4.

The input is constant in Scenario I so that Scheme I is expected to be appropriate, but its performance falls short of that of Schemes II and III. Scheme I produces an input estimate smaller than the actual input level, which can happen if the input starts in the middle of the detection window, as shown in Fig.5. With Schemes II and III, therefore, the input profile can be approximated better than with Scheme I even for the case of constant input. Estimation accuracy may be improved by using more of higher-order polynomial time functions. This is why the average RMS errors are much smaller with Schemes II and III.

It is observed that Scheme III does not perform better than Scheme II during the maneuver. But the RMS error of Scheme III becomes much smaller for the post-maneuver period. This implies that the ability of compensating the unknown input sequence can be enhanced by using more base functions, in general. However, the use of more base functions may result in computational inefficiency without a significant improvement in performance. Extensive numerical simulation must precede the selection of optimal base functions for a trade-off between estimation performance and computational efficiency.

6 Conclusion

A new input estimation algorithm of linear stochastic systems is proposed in this paper. The proposed method approximates the unknown input as a sum of elementary time functions and estimates the optimal coefficient vector by using the weighted least-squares method. This approach enables us to work with a more general formulation than the conventional methods considering constant-input cases only. Simulation results show that the proposed algorithm using time-varying base functions provides better estimation performance during and after the unknown target maneuver than the previous input estimation techniques considering constant inputs only.

The main advantages of the proposed method may be summarized as follows: fast-varying unknown input sequences can be easily handled to obtain better estimation accuracy; estimation of the coefficient vector is computationally more efficient than estimation of the input sequence itself. Moreover, the proposed algorithm can be applied to a general class of time-varying linear stochastic systems. Not only to the target tracking problem considered in this paper, the proposed algorithm may also be applied to fault detection and identification of a linear stochastic system.

References

[1] Maybeck, P. S., *Stochastic Models, Estimation, and Control: Volume 1, Ch. 5*, Academic Press, Inc., London, 1982.

[2] Bar-Shalom, Y., and Fortmann, T. E., *Tracking and Data Association*, Academic Press, Inc., Orlando, Florida, 1988.

[3] Singer, R. A., "Estimating optimal tracking filter performance for manned maneuvering targets," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-6, No. 4, pp. 473-483, July 1970.

[4] Bar-Shalom, Y., and Birmiwal, K., "Variable dimension filter for maneuvering target tracking," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. ASE-18, No. 5, pp. 621-629, 1982.

[5] Moose, R. L., "An adaptive state estimation solution to the maneuvering target problem," *IEEE Trans. on Automatic Control*, Vol. AC-20, No. , pp. 359-362.

[6] Blom, H. A. P., and Bar-Shalom, T., "The interacting multiple model algorithm for systems with markovian switching coefficients," *IEEE Trans. on Automatic Control*, Vol. 33, No. 8, pp. 780-783, 1988.

[7] McAulay, R. J., and Denlinger, E., "A decision-directed adaptive tracker," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-9, No. 2, pp. 229-236, 1973.

[8] Chan, Y. T., Hu, A. G. C., and Plant, J. B., "A kalman filter based tracking scheme with input estimation," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. ASE-15, No. 2, pp. 237-244, 1979.

[9] Bogler, P. L., "Tracking a maneuvering target using input estimation," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. ASE-23, No. 3, pp. 298-310, 1987.

[10] Magill, D. T., "Optimal adaptive estimation of sampled stochastic processes," *IEEE Trans. on Automatic Control*, Vol. AC-10, No. 4, pp. 434-439, 1965.

[11] Devore, J. L., *Probability and Statistics for Engineering and the Science*, 3rd ed., Brooks/Cole Publishing Company, Pacific Grove, CA, p. 676, 1991.

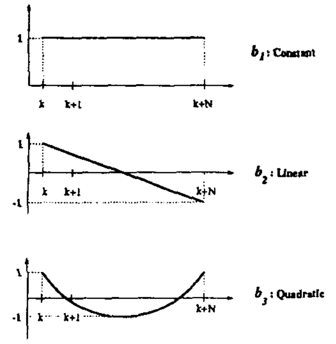
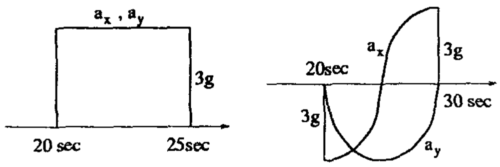


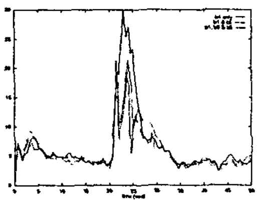
Figure 1: Time functions b_α



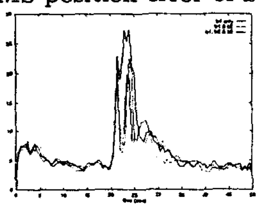
(a) Scenario I

(b) Scenario II

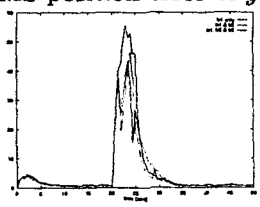
Figure 2: Input scenarios



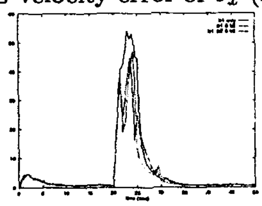
(a) RMS position error of x (m)



(b) RMS position error of y (m)

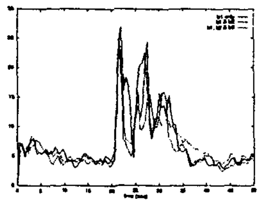


(c) RMS velocity error of v_x (m/sec)

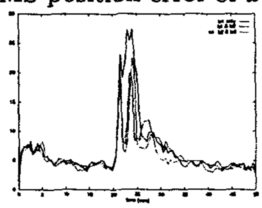


(d) RMS velocity error of v_y (m/sec)

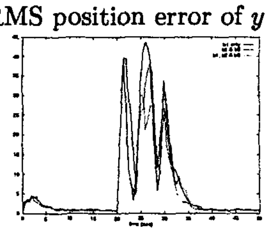
Figure 3: RMS errors of Scenario I



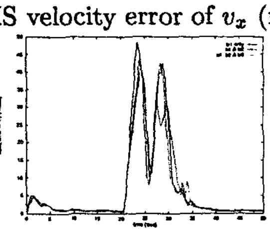
(a) RMS position error of x (m)



(b) RMS position error of y (m)



(c) RMS velocity error of v_x (m/sec)



(d) RMS velocity error of v_y (m/sec)

Figure 4: RMS errors of Scenario II

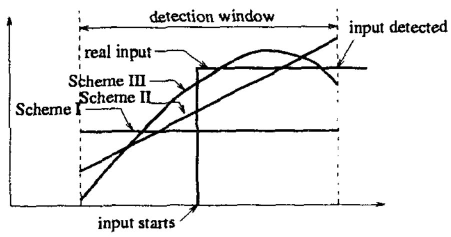


Figure 5: Approximation of unknown input in Scenario I