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PIDA Controller Design by CDM

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Abstract

A design of PIDA (Proportional-Integral-Derivative-Acceleration) controller for the third-order plant using the CDM (Coefficient Diagram Method) is presented. Using CDM, the closed-loop system with the designed PIDA controller can be made stable and satisfied both transient and steady state response specifications without any adjustment. The effect of output step disturbance can also be fastly rejected. The fast step response of the controlled system can be achieved by reducing the equivalent time constant. The MATLAB's simulation results show that the performances of the designed controlled system using CDM is better than the performances of the controlled system using PIDA controller designed by its own technique.

1. Introduction

Most of the plants in the industry are type 0 with three to five first-order lags or one first-order lag plus dead time. Type 1 plants with one or two first-order lags are also found in the industry [1]. Almost of the plants are designed and to be controlled for obtaining the desired performances by using the PID (Proportional-Integral-Derivative) controller with well tuned parameter [2]. The PID controller has been traditionally applied to the typical second-order plant. But for the third or higher order plant, it is quite difficult to get the desired performances because the order of the plant is greater than the number of zeros provided by the PID controller [3]. S. Jung and R.C. Dorf have proposed a structure of PIDA (Proportional-Integral-Derivative-Acceleration) controller for the third-order plant by locating two dominant roots and one root at the real axis just below the dominant roots with one negligible root located far from the origin of the real axis to meet the desired transient response. The desired characteristic equation is obtained and equated to the characteristic equation of the closed-loop transfer function of the controlled system. Then the values of the parameters of the PIDA controller can be obtained. But the step response does not satisfy the desired transient specification due to the effect of the zeros of the designed PIDA controller. However, the controller gain can be adjusted to obtain the desired specifications.

This paper presents a design of PIDA controller for the third-order plant using the CDM (Coefficient Diagram Method), which is proposed by Shunji Manabe [4], [5]. The configuration of the PIDA controller is designed base on the stability and the speed of the controlled system. Stability and speed are designed from the standard stabilty index and the equivalent time constant respectively. When the settling time of the controlled system has been selected, the equivalent time constant is obtained. The stability index and the equivalent time constant specify the coefficients of the characteristic polynomial. These coefficients are related to the controller parameters algebraically in explicit form. Hence, the transient and the steady state performances of the controlled system can be obtained.

Step responses of the controlled system with PIDA controller using CDM are compared to the step responses of the controlled system with the PIDA controller using Jung and Dorf technique. The results show that the step response of the controlled system with the PIDA controller using CDM has no overshoot and reaches the desired settling time without any adjustment.

2. CDM

The CDM is used to design the controller so that the step response satisfies stability, fast response and robustness requirements [4], [5]. Generally, the order of the controller designed by CDM is less than the order of the plant.

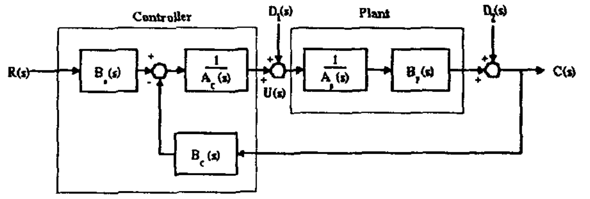

Fig. 1 CDM standard block diagram for the SISO system.

The standard block diagram of the CDM design for the SISO (Single-Input Single-Output) is shown in Fig. 1. The transfer function of the plant in the polynomial form in each block is

$$A_p(s) = p_k s^k + p_{k-1} s^{k-1} + \dots + p_0 \tag{1a}$$

$$B_p(s) = q_m s^m + q_{m-1} s^{m-1} + \dots + q_0 \tag{1b}$$

and the controller polynomials are

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1} s^{\lambda-1} + \dots + l_0 \tag{2a}$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1} s^{\lambda-1} + \dots + k_0 \tag{2b}$$

$$B_a(s) = k_0 \tag{2c}$$

where $\lambda < k$ and $m <$ k. $B_a(s)$ is a prefilter and has to be set to k_o so that the step response has zero steady-state error. Since the transfer function of the controller has two numerators, then it is called as a two-degree-of-freedom (2DOF) system.

The characteristic polynomial of the control system shown in Fig. 1 is given in the following forms

$$\begin{aligned} P(s) &= A_c(s)A_p(s) + B_c(s)B_p(s) \\ &= a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \\ &= \sum_{i=0}^{n} a_i s^i \end{aligned} \tag{3}$$

where $a_0, a_1 \dots, a_n$ are the coefficients of the characteristic polynomial. The stability index γ_i, the equivalent time constant τ and stability limit γ_i^* are defined as follows

$$\gamma_i = \frac{a_i^2}{a_{i+1} a_{i-1}} \tag{4}$$

$$\tau = \frac{a_1}{a_0} \tag{5}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0, \gamma_n = \infty \tag{6}$$

where $i = 1,\dots,n-1$. To meet the specifications, the equivalent time constant τ and the standard values of the stability index γ_i are chosen as follows

$$t_s = 2.5\text{\textasciitilde}3\,\tau \tag{7}$$

$$\gamma_{n-1} = \dots = \gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5 \tag{8}$$

In general, settling time $t_s = 2.5\tau$, and stability index $\gamma_1 = 2.5$, $\gamma_2 = \gamma_3 = 2$ are strongly recommended due to stability and the step response requirement. However, it is not necessory to make $t_s = 2.5\tau$ and $\gamma_4 \sim \gamma_{n-1} = 2$. The condition for the stability index can be relaxed to

$$\gamma_i > 1.5\gamma_i^* \tag{9}$$

The standard values stated in (8) can be used to design the controller if the following condtion is satisfied

$$p_k / p_{k-1} > \tau / (\gamma_{n-1}\gamma_{n-2} \cdots \gamma_1) \tag{10}$$

where p_k and p_{k-1} are the coefficients of the the plant at kth and (k-1)th. If the above condition is not satisfied, we can first increase γ_{n-1} then γ_{n-2} and so on, until (10) is satisfied. From (4)-(6), the coefficient a_i can be written as

$$\begin{aligned} a_i &= a_0 \tau^i \frac{1}{\gamma_{i-1} \cdots \gamma_2^{i-2} \gamma_1^{i-1}} \\ &= a_0 \tau^i \prod_{j=1}^{i-1} \frac{1}{(\gamma_{i-j})^j} \end{aligned} \tag{11}$$

Then the characteristic polynomial is expressed as

$$P(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{12}$$

3. PIDA Controller Design by CDM

The transfer function of the PIDA controller for the third-order plant proposed by Jung and Dorf [3] is given by

$$\begin{aligned} G_C(s) &= k_p + \frac{k_i}{s} + \frac{k_d}{(s+d)} s + \frac{k_a}{(s+d)(s+e)} s^2 \\ &= K \frac{(s+a)(s+b)(s+z)}{s(s+d)(s+e)} \end{aligned} \tag{13}$$

Since $d, e >> a, b, z$ the poles d, e can be neglected, hence

$$G_c(s) = \frac{k_3 s^3 + k_2 s^2 + k_1 s + k_0}{s} \tag{14}$$

where $k_3 = K$, $k_2 = K(a+b+z)$, $k_1 = K[(a+b)z+ab]$, $k_0 = K(abz)$. By comparing with (2a)-(2b), the coefficients of $A_c(s)$ of the controller in (14) are $l_3 = l_2 = l_0 = 0$ and $l_1 = 1$ or $A_c(s) = s$, and the coefficients of $B_c(s)$ remains the same. Since $A_c(s)$ is specified as s, this restriction virtually makes the plant to be fourth-order. Thus a third-order controller, one order less than the plant is sufficient.

The design procedures to obtain the PIDA controller parameters by using CDM are summarized as follows.

1) Determine the equivalent time constant τ from the desired settling time t_s.
2) Determine the proper values of the stability index γ_i from the standard stability index stated in (8).
3) From (3), derive the characteristic polynomial with the PIDA controller stated in (14) and equate to the characteristic polynomial obtained from (12) with τ and γ_i found from 1) and 2). Then all parameters k_3, k_2, k_1 and k_0 of the controller are obtained.
4) Set $B_a(s) = k_0$

4. Simulation Results

In this section, the MATLAB's simulation results of the controlled system with the PIDA controller designed by CDM are shown. The simulation results show:

1) The step responses due to the variation of the equivalent time constant τ.
2) The step responses with the constant output disturbance.
3) The step responses due to the parameters change in the plant.
4) Comparison of the step responses of AC motor model using the PIDA controller designed by CDM and by Jung and Dorf technique.

<u>The step responses due to the variation of the equivalent time constant τ</u>

The desired specification for designing the controller is $t_s \leq 2$ seconds (2% criterion).

<u>Example of type 0, 3rd order plant</u>

$$G_p(s) = \frac{1}{(s+1)(s+3)(s+6)}$$

From (7), the equivalent time constant τ is 0.8 second. Determine the stability index from (8) with the condition in (9). The values of the stability index are

$$\gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5$$

The parameters k_3, k_2, k_1 and k_0 of the PIDA controller shown in (14) are obtained by equating the following characteristic polynomial

$$\begin{aligned} P(s) &= A_c(s)A_p(s) + B_c(s)B_p(s) \\ &= s^4 + (10+k_3)s^3 + (27+k_2)s^2 + \\ &\quad (18+k_1)s + k_0 \end{aligned}$$

to the standard form of the characteristic polynomial

$$P(s) = s^4 + 12.5s^3 + 78.13s^2 + 244.14s + 305.18$$

derived from (12). Therefore, $k_0 = 305.18$, $k_1 = 226.14$, $k_2 = 51.13$, $k_3 = 2$. The transfer function of the controller is

$$\begin{aligned} G_{c10}(s) &= \frac{k_3 s^3 + k_2 s^2 + k_1 s + k_0}{s} \\ &= \frac{2.5s^3 + 51.13s^2 + 226.14s + 305.18}{s} \end{aligned}$$

and $B_a(s) = 305.18$; for $t_s = 2$ seconds.

The unit step responses with no overshoot are shown in Fig. 2. It is shown that the settling time is less than 2 seconds as desired. Faster step response could also be obtained by decreasing the equivalent time constant τ while the standard stability index γ_i remains the same. If the desired settling times t_s are 1.5 and 1.0 seconds, the equivalent time constant τ are 0.6 and 0.4 second, respectivly. That is

$$G_{c20}(s) = \frac{6.67s^3 + 111.89s^2 + 560.70s + 964.51}{s}$$

and $B_a(s) = 964.51$; for $t_s = 1.5$ seconds.

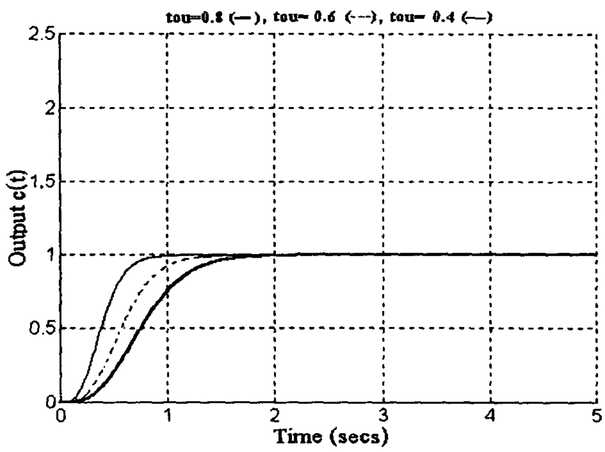

Fig. 2 Step responses due to the variation of τ for type 0.

$$G_{c30}(s) = \frac{15s^3 + 285.5s^2 + 1935.13s + 4882.81}{s}$$

and $B_a(s) = 4882.81$; for $t_s = 1$ second.

The coefficient diagram of type 0 with the designed controllers $G_{c10}(s)$, $G_{c20}(s)$ and $G_{c30}(s)$ is shown in Fig. 3. Note that when the curve a_i is left-end-down, the equivalent time constant τ is small and the step response is fast as shown in Fig. 2.

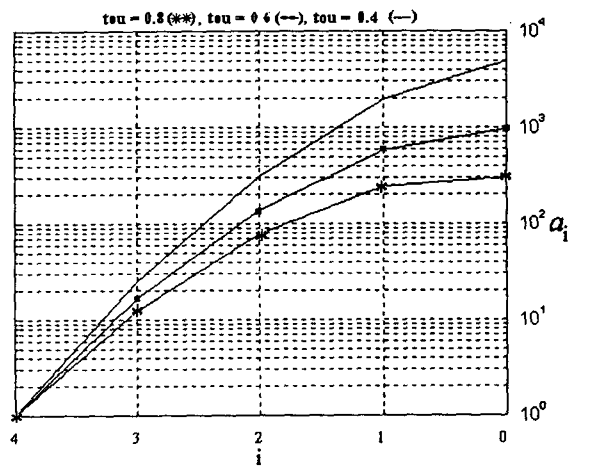

Fig. 3 Coefficient diagram with the variation of τ.

Example of type 1, 3[rd] order plant

$$G_p(s) = \frac{1}{s(s+1)(s+7)}$$

With the same specification desired,

$$G_{c11}(s) = \frac{4.5s^3 + 71.13s^2 + 244.14s + 305.18}{s}$$

and $B_a(s) = 305.18$; for $t_s = 2$ seconds.

When desired settling time t_s is changed to 1.5 seconds and 0.95 second, the equivalent time constant τ will be changed to 0.6 second and 0.38 second respectively. Then

$$G_{c21}(s) = \frac{8.67s^3 + 131.89s^2 + 578.70s + 964.51}{s}$$

and $B_a(s) = 964.51$; for $t_s = 1.5$ seconds.

$$G_{c31}(s) = \frac{18.32s^3 + 339.26s^2 + 2278.03s + 5994.81}{s}$$

and $B_a(s) = 5994.81$; for $t_s = 0.95$ second.

Step responses of the controlled systems are shown in Fig. 4 which are quite similar to the step responses shown in Fig. 2.

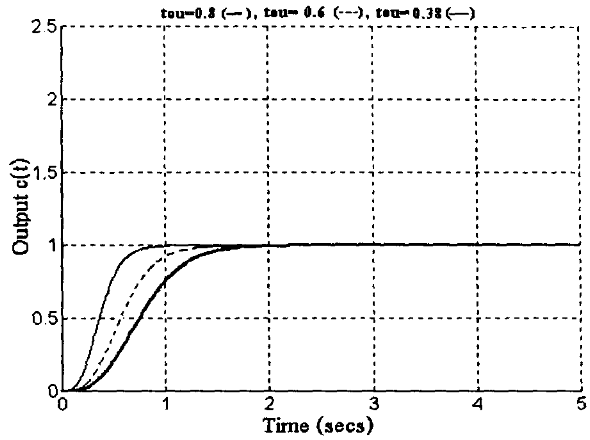

Fig. 4 Step responses due to the variation of τ for type 1.

Example of type 2, 3[rd] order plant

$$G_p(s) = \frac{1}{s^2(s+1)}$$

When the desired t_s are 2.0, 1.5 and 0.9 seconds, the step responses are almost the same as the type 0 and type 1 plant as shown in Fig. 5. The transfer functions of the controller are

$$G_{c12}(s) = \frac{11.5s^3 + 78.13s^2 + 244.14s + 305.18}{s}$$

$$G_{c22}(s) = \frac{15.67s^3 + 138.89s^2 + 578.70s + 964.51}{s}$$

$$G_{c32}(s) = \frac{26.78s^3 + 385.80s^2 + 2679.18s + 7442.18}{s}$$

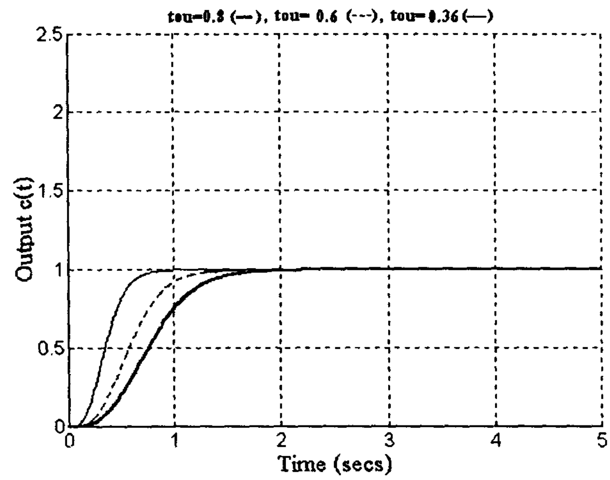

Fig. 5 Step responses due to the variation of τ for type 2.

The step responses with the constant output disturbance

In this section, the responses of the controlled system with constant output disturbance are considered. In order to compare the controlled system with the PIDA conroller designed by CDM and the system with the one is designed by Jung and Dorf technique, the settling time t_s = 1.06 seconds for type 0 is considered. Fig. 6 shows the step responses where the reference input is unit step and the constant output disturbance entering at 2 seconds is also unit step. It is clearly shown that the transient response of the designed system using CDM has no overshoot and reaches the settling time as desired without any adjustment. The effect of the constant output disturbance is also fast rejected.

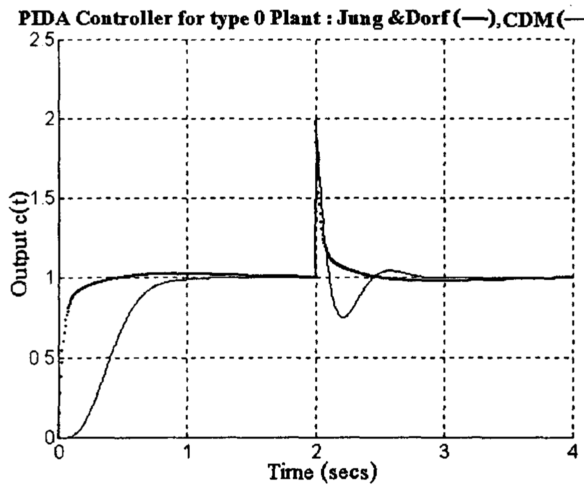

Fig. 6 Step responses with the constant output disturbances.

The step responses due to the parameters change in the plant

The same PIDA controller designed by CDM and by Jung and Dorf technique with the desired specification of settling time t_s = 1.06 seconds is used to control the ±75% purturbed type 0 plant. The ±75% parameter change of the type 0 yield

$$G_p(s) = \frac{1}{(s+1.75)(s+5.25)(s+10.5)} \qquad (+75\%)$$

and

$$G_p(s) = \frac{1}{(s+0.25)(s+0.75)(s+1.5)} \qquad (-75\%)$$

In Fig. 8, the variation of the step responses due to the change of parameters of the plant are quite siginficant large, when compare to the step responses of its nominal plant. But there is not so significant change occured in the step responses of the system with the controller by CDM as shown in Fig. 7.

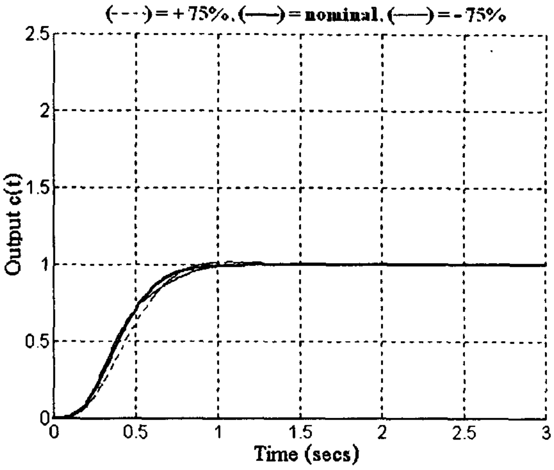

Fig. 7 System responses with a PIDA controller by CDM.

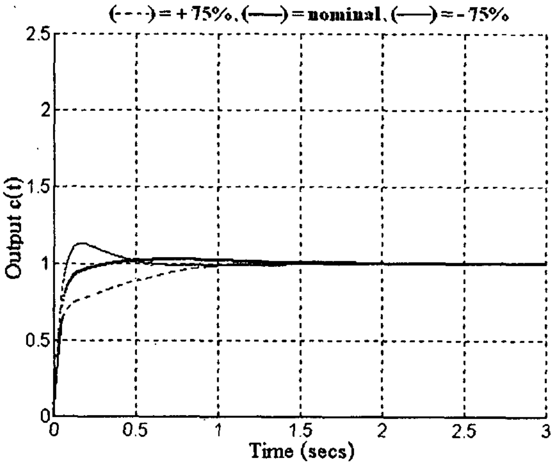

Fig. 8 System responses with a designed PIDA controller by Jung and Dorf technique.

Comparison of the step responses of AC motor model using the PIDA controller designed by CDM and by Jung and Dorf technique

The stability index γ_1 = 2.5, γ_2 = 2, γ_3 = 2.5 and the settling time t_s = 1.18 seconds (for the condition $t_s = 3\tau$, τ = 0.39 second) are used to design the PIDA controller by CDM for AC motor model [3] which has the structure as

$$G_p(s) = \frac{168.0436}{s(s^2 + 25.921s + 168.0436)}$$

Hence, the controller designed by CDM is obtained as

$$G_c(s) = \frac{0.035s^3 + 1.40s^2 + 15.28s + 38.85}{s}$$

Fig. 9 shows the step responses of the AC motor model with the PIDA controller designed from both techniques.

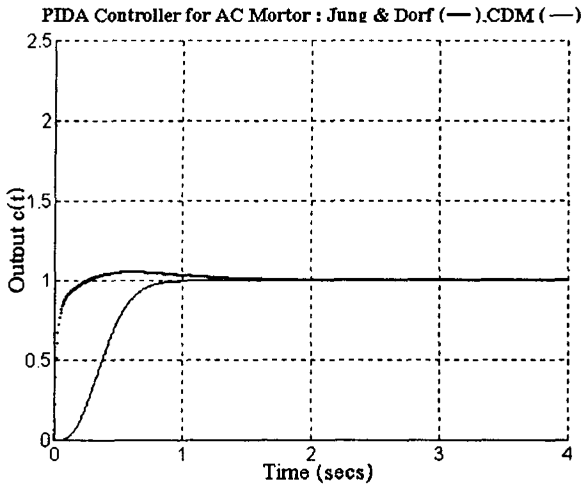

Fig. 9 Step responses of AC motor with PIDA controller.

The step response of the AC motor model with the designed controller using CDM has no overshoot and reaches the settling time without any adjustment. From the structure of the designed controller, the gain (K = 0.035) is smaller than the controller gain (K = 0.238) required by Jung and Dorf.

5. Conclusion

The designed PIDA controller using CDM has been proposed in this paper. Using CDM, the controller can be designed to satisfy the desired specifications. Fast rejection of the effect dued to constant output disturbance is also obtained. Furthermore, fast step response of the controlled system can also be designed efficiently by decreasing the equivalent time constant τ. However, this cause high gain in the system which should be carefully considered. Moreover, the simulation results in Fig. 7 also shows that when the parameters of plant have changed, the step responses are almost the same as the nominal plant.

6. References

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