Proceedings of the 13th
KACC, October 1998

PID × (n-1) Stage PD Controller for SISO Systems

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Abstract

A design technique based on the root locus approach for the SISO (Single-Input Single-Output) systems using PID (Proportional-Integral-Derivative) × (n-1) stage PD as a controller for the n^{th} order plant is presented. The controller is designed based on transient and steady state response specifications. This controller can be used instead of a conventional PID controller. The overall system is approximated as a stable and robust second order system. The desired performances are achieved by increase the gain of the controller. In addition, the controller gain can be adjusted to obtain faster response with a little overshoot. The simulation results show the merits of this approach.

1. Introduction

Most industrial plants are type 0 and consist of three to five first order lags or dead time plus one first order lag [1]. The type 1 plant that consists of one to two first order lags is also often met in industry. The PID controller is widely used by applying the well-known Ziegler-Nichols tuning method [2]. Clearly, the PID controller is properly applied in the typical second order plant, but it is quite difficult to use only the PID controller for the third or higher order plant because the order of the plant is greater than the number of zeros provided by the PID controller [3,4,5]. Moreover, the tuning methods sometimes require trial and error procedure, and the original Ziegler-Nichols settings do not always produce the best results to meet the transient response requirements because of the ¼ decay ratio criterion.

This paper presents a design technique based on the root locus approach for the n^{th} order plant $G_p(s)$ to satisfy transient and steady state response specifications. The PID × (n-1) stage PD is used as a controller $G_c(s)$. With this controller, the overall system becomes proper system, and is designed to be an approximated second order system. Of the two poles of $G_c(s)G_p(s)$, one is located at the origin and the other is located nearest the origin; these are defined as significant poles. If the poles of $G_c(s)G_p(s)$ are located at the origin, they are also defined as significant. The remaining (n-1) poles are considered as insignificant poles in both instances. Due to the transfer function of $G_p(s)$ usually determined through testing and physical modeling, linearization of a non-linear plant, or the uncertain parameters concerned, which cause the location of the poles may not be exact. Then (n-1) zeros of the controller are arbitrarily placed near the left-hand side of those insignificant poles of $G_c(s)G_p(s)$ in order to reduce the effect of these poles. The desired locations of two dominant closed-loop poles s_d are determined from the transient response specifications. The double zeros of $(s+z_c)^2$ of the controller must contribute the necessary angle to force the root locus to go through s_d. The location of the double zeros of $(s+z_c)^2$ and the gain K_c at s_d can be determined graphical or numerical computations when this procedure is not provided by pseudo quantitative feedback theory [6]. The other (n-1) closed-loop poles are located between the (n-1) pair of the open-loop pole-zeros. Hence, the amplitudes of the transient responses of these (n-1) closed-loop poles are very small and negligible although the exact pole-zeros cancellations do not occur [7]. However, the transient response does not completely satisfy the specifications because of the effect of the double zeros of $(s+z_c)^2$. By this technique, all of the root loci located on the left half of the s-plane, and the significant root locus is a circle shape. Then the gain K_c can be increased to reduce maximum overshoot and obtain the desired specification. Faster response with a little overshoot can also be achieved by further increase the gain K_c. Consequently, the system can be made stable and robust. Note that the effect of output disturbance is rapidly eliminated.

MATLAB's simulation results have indicated that when the plant has ±25% parameter deviation, the system performances remain essentially unchanged. Moreover, when the gain K_c is increased, the effect of the uncertain parameters is also decreased.

2. Structure of the SISO System

The structure of the SISO system is shown in Fig.1, where a unity feedback is assumed.

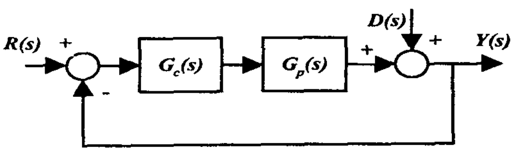

Fig. 1 Structure of the SISO system

When the PID controller is applied to a high order plant with step input, steady state error $e_{ss}(t)$ is zero, but the transient response does not meet the specifications. To meet the specifications and robustness, the PID × (n-1) stage PD controller for the n^{th} order plant is defined as

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)\prod_{j=1}^{n-1} K_{pj}\left(1 + T_{dj}s\right) = \frac{K_{c1}}{s}(s+z_c)^2\prod_{j=1}^{n-1}(s+z_j), \quad (1)$$

where K_p, T_i and T_d are proportional gain, integral time and derivative time of the PID controller; K_{pj} and T_{dj} are proportional gain and derivative time of the (n-1) stage PD controller; $-z_c$ is the real double zeros ($T_i = 4T_d$) of the PID controller; $-z_j$ is the real zeros of the (n-1) stage PD controller; K_{c1} is the gain of the controller.

The structure of the n^{th} order plant are often found and classified into type 0 and type 1. The transfer function is given by

$$G_p(s) = \frac{K}{(s+p_d)\prod_{i=1}^{n-1}(s+p_i)}, \quad (2)$$

where $-p_d$ is the real pole located nearest the origin (type 0), and $p_d = 0$ (type 1); $-p_i$ (i=1,2,....,n-1) is the real poles.

The open-loop transfer function of the closed-loop system is

$$G_c(s)G_p(s) = \frac{K_c(s+z_c)^2\prod_{j=1}^{n-1}(s+z_j)}{s(s+p_d)\prod_{i=1}^{n-1}(s+p_i)}, \quad (3)$$

where $K_c = KK_{c1}$.

As mentioned previously, one pole located at the origin and the other located nearest the origin, or both located at the origin, these are considered significant poles, and the remaining (n-1) poles are insignificant poles. The double zeros of $(s+z_c)^2$ are used to force the root locus to go through s_d. The (n-1) zeros are placed near the left–hand side of the (n-1) poles in order to reduce their effect. Hence from (3),

$$G_c(s)G_p(s) = \frac{K_c(s+z_c)^2\prod_{i=1}^{n-1}(s+p_i+\varepsilon_i)}{s(s+p_d)\prod_{i=1}^{n-1}(s+p_i)}, \quad (4)$$

where $-z_j = -(p_i + \varepsilon_i)$ (j=1,2,....,n-1; i=1,2,....,n-1), ε_i is a small real number.

From (4), in case of the type 0 plant, if some of $-p_i = -p_d$, only one pole of those multiple poles is considered as a significant pole.

3. Design Procedures

The design procedures to meet the specifications are as follows:

1) The damping ratio (ζ), undamped natural frequency (ω_n) and s_d are determined from the transient response specifications in (5).

$$P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}\,\%,\quad t_s = 4/\zeta\omega_n\ (\pm 2\%),\quad s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}. \quad (5)$$

where $P.O.$ stand for percent overshoot and t_s is the settling time.

2) Place the location of s_d, all poles and the (n-1) zeros of the $G_c(s)G_p(s)$ from (4) and (5) in the s-plane.

3) Find the sum of the angles at s_d with all of the open-loop poles and the (n-1) zeros of $G_c(s)G_p(s)$ by graphical or by numerical computations. Then determine the necessary angle of $\angle(s_d + z_c)^2$ to be added so that the total sum of the angles satisfies (6).

$$\begin{aligned}&\left[\angle(s_d + z_c)^2 + \sum_{i=1}^{n-1}\angle(s_d + p_i + \varepsilon_i)\right]\\&-\left[\angle s_d + \angle(s_d + p_d) + \sum_{i=1}^{n-1}\angle(s_d + p_i)\right]\\&= \pm(2k+1)\pi, \qquad k = 0,1,2,...\end{aligned} \tag{6}$$

4) Determine the location of the double zeros of $(s+z_c)^2$ using the angle of $\angle(s_d + z_c)^2$ found in (6).

5) Determine the gain K_c at s_d from

$$K_c = \frac{|s_d||(s_d + p_d)|\prod_{i=1}^{n-1}|(s_d + p_i)|}{|(s_d + z_c)|^2\prod_{i=1}^{n-1}|(s_d + p_i + \varepsilon_i)|}. \tag{7}$$

6) The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{K_c(s + z_c)^2\prod_{i=1}^{n-1}(s + p_i + \varepsilon_i)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)\prod_{i=1}^{n-1}(s + p_i + \delta_i)}, \tag{8}$$

where $\prod_{i=1}^{n-1}(s + p_i + \delta_i)$ are real or complex closed-loop poles located on (or close to) the negative real axis between the insignificant poles $\prod_{i=1}^{n-1}(s + p_i)$ and zeros $\prod_{i=1}^{n-1}(s + p_i + \varepsilon_i)$, δ_i is a small real or complex number.

Since all of the (n-1) closed-loop poles located near the open–loop zeros, it can be shown that the coefficients of these closed-loop poles are proportional to (ε_i -δ_i), which is a very small number. This implies that, although the poles at $-p_i$ can not be cancelled, the resulting transient responses due to these closed-loop poles have insignificant amplitudes, and their effect can be neglected [7]. If pole-zeros cancellations in (8) are considered, the closed-loop system can be approximated as a second order system as

$$\frac{Y(s)}{R(s)} \cong \frac{K_c(s + z_c)^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \tag{9}$$

It is evident that the transient response does not completely satisfy the desired specifications because greater overshoot occurs due to the effect from the double zeros of $(s+z_c)^2$. By this technique, all of the root loci located on the left half of the s-plane, and the significant root locus is a circle shape while the others are on (or close to) the negative real axis. This means that all of the roots of the characteristic equation located on the left half of the s-plane for all positive values of the gain K_c. Then the gain K_c can be adjusted to reduce the maximum overshoot and obtain the desired specifications. Moreover, if it is required, faster response with a little overshoot is achieved by further adjust the gain K_c higher than the designed value. Consequently, the system is made stable and robust.

4. Effect of the Output Disturbance

If the output disturbance $D(s)$ occurs in the system, with the absence of the reference input $R(s)$, the output $Y(s)$ due to the disturbance is

$$\frac{Y(s)}{D(s)} = \frac{1}{1 + G_c(s)G_p(s)}. \tag{10}$$

The output disturbance has an important effect on the step response at the initial state. However, the effect of output disturbance is rapidly eliminated because of this controller. Moreover, the effect of output disturbance can also be reduced when the gain K_c is increased higher than the designed value.

5. Simulation Results

Example of type 0, 4th order plant

$$G_p(s) = \frac{K}{(s+1)(s+2)^2(s+3)}.$$

The desired specifications for step input are

$P.O. \le 5$ %, $t_s(\pm 2\%) \le 1$ sec, $e_{ss}(t) = 0$.

From the desired specifications,

$\zeta = 0.690$, $\omega_n = 5.796$ rad/sec, s_d = - 4 $\pm j$ 4.195.

The controller is first set as

$$G_c(s) = \frac{K_{c1}}{s}(s+z_c)^2(s+2.2)(s+2.3)(s+3.4).$$

Determine the angle and location of the double zeros of $(s+z_c)^2$ in according to the s_d,

$$\angle(s_d+z_c)^2 = 90.175°, \quad -z_c = -8.182.$$

The gain K_c at $s_d = 0.906$ is determined. Hence,

$$G_c(s)G_p(s) = \frac{0.906(s+8.182)^2(s+2.2)(s+2.3)(s+3.4)}{s(s+1)(s+2)^2(s+3)}.$$

Figure 2 shows the root locus plot of the closed-loop system. It is clear that all of the roots of the characteristic equation are located on the left half of the s-plane for all positive values of the gain K_c, and when the gain K_c is increased, the *P.O.* is decreased.

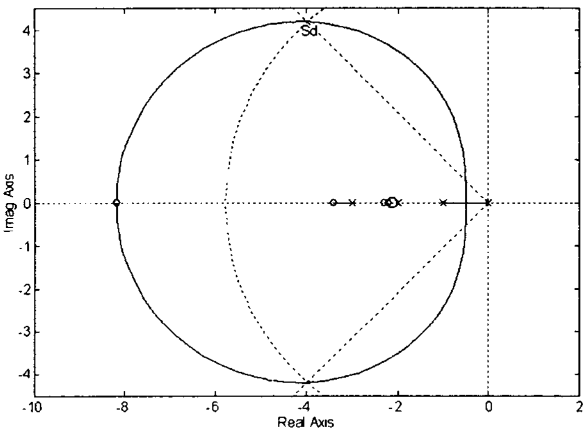

Fig. 2 Root locus plot of the closed-loop system

The step response of the controlled system (type 0) for the designed K_c = 0.906 (*P.O.* = 9.5 %, t_s= 0.75 sec, $e_{ss}(t)$ = 0) with the effect of 50% output disturbance occurred at t = 3 sec., is shown in Fig. 3. With the designed K_c, the maximum overshoot is greater than the desired specification, but the other are obtained.

Figure 4 shows the step responses of the controlled system (type 0) that satisfy all the specifications when the gain K_c is adjusted to 2 (*P.O.* = 5 %, t_s= 0.6 sec, $e_{ss}(t)$ = 0), and K_c = 20 (*P.O.* = 0.5 %, t_s= 0.15 sec, $e_{ss}(t)$ = 0), with the effect of 50% output disturbance at t = 3 sec. When K_c is increased, faster response is obtained.

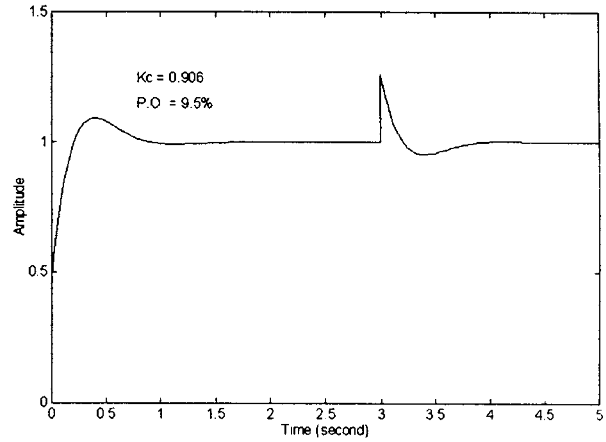

Fig. 3 Step response of the controlled system (type 0)

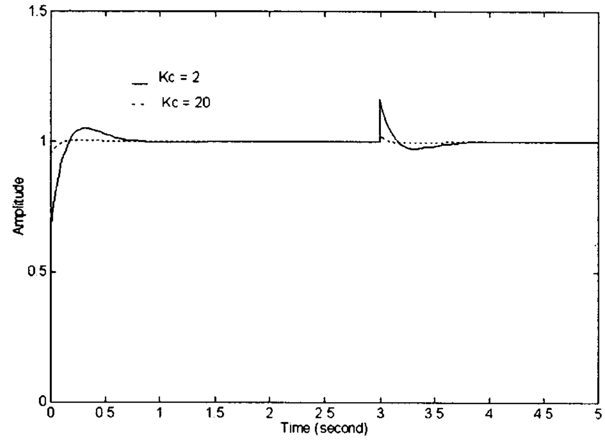

Fig. 4 Step responses of the controlled system (type 0)

<u>Example of type 1, 4th order plant</u>

$$G_p(s) = \frac{K}{s(s+1)(s+3)(s+4)}.$$

When the same specifications are desired,

$$G_c(s)G_p(s) = \frac{1.13(s+7.614)^2(s+1.2)(s+3.3)(s+4.4)}{s^2(s+1)(s+3)(s+4)}.$$

Figure 5 shows the step responses of the controlled system (type 1) for K_c= 1.13 (*P.O.* = 11 %, t_s= 0.75 sec, $e_{ss}(t)$ = 0), with the effect of 50% output disturbance at t = 3 sec. When K_c is adjusted to 3, the step response that satisfy all the specifications (*P.O.* = 4 %, t_s = 0.55

sec, $e_{ss}(t) = 0$), is also shown for comparison. It is noted that when K_c is increased, the effect of output disturbance is decreased.

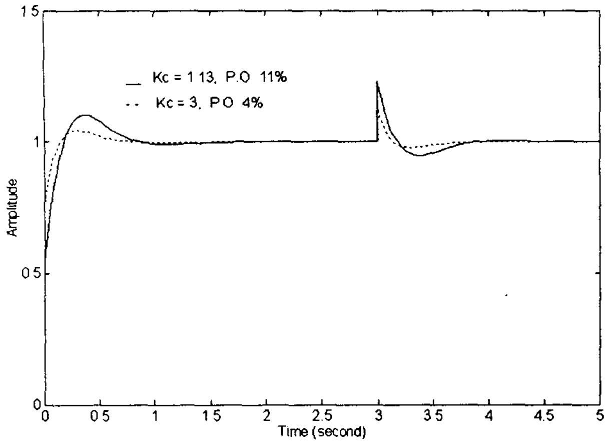

Fig. 5 Step responses of the controlled system (type 1)

Example of a plant with uncertain parameters

If ±25% parameter deviation occurs, the transfer functions of $G_c(s)G_p(s)$ in the previous example (type 0) with controller parameters remain unchanged are

$$G_c(s)G_p(s) = \frac{K_c(s+8.182)^2(s+2.2)(s+2.3)(s+3.4)}{s(s+1.25)(s+2.5)^2(s+3.75)}; \quad (+25\%),$$

and

$$G_c(s)G_p(s) = \frac{K_c(s+8.182)^2(s+2.2)(s+2.3)(s+3.4)}{s(s+0.75)(s+1.5)^2(s+2.25)}; \quad (-25\%).$$

The step response of the controlled system (type 0) without parameter deviation compared to the step responses with ±25% parameter deviation for the gain $K_c = 0.906$ and $K_c = 2$, are shown in Fig. 6 and Fig. 7 respectively. It can be concluded that the system performances remain essentially unchanged when the gain $K_c \geq 2$.

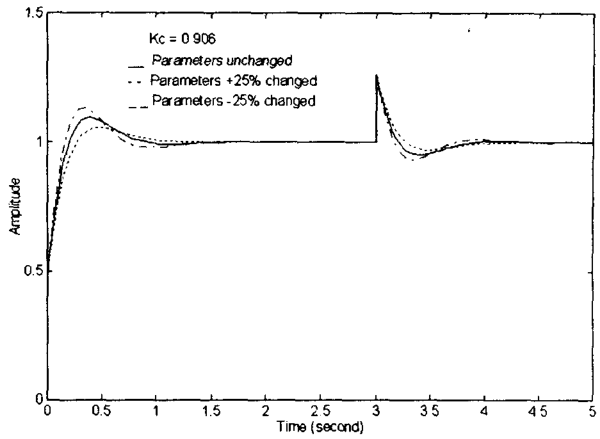

Fig. 6 Step responses of the controlled system (type 0) with parameters unchanged and ±25% changed

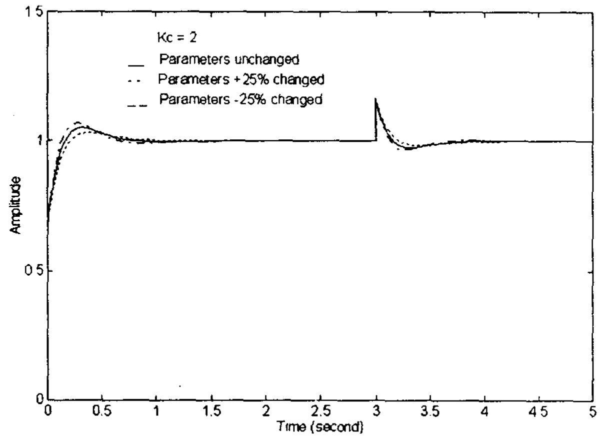

Fig. 7 Step responses of the controlled system (type 0) with parameter unchanged and ±25% changed.

6. Conclusion

The PID × (n-1) stage PD controller designed by the root locus technique has been proposed in this paper. The merits of this controller are that it can be applied to the high order plant instead of the conventional PID controller. With this proposed controller, the controlled system can be approximated as a second order system. For all positive values of the gain K_c, all of the roots of the characteristic equation of the controlled system are located on the left half of the s-plane. By increasing the gain K_c higher than the designed value, the step response is satisfied both the transient and steady state response specifications.

Faster response with a little overshoot can also be obtained. Furthermore, the designed value of the gain K_c of this proposed controller is small, then only low gain amplifier is needed. That is, the controlled can be made stable and robust. Moreover, the controller rapidly eliminates the effect of the output disturbance. All these merits are confirmed by the simulation results which clearly show that the controller can be applied to the plant with ±25% parameter deviation.

7. References

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