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Frequency Domain Properties of EALQR with Indefinite Q

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Abstract

The previously developed control design methodology, EALQR(Eigenstructure Assignment/LQR), has better performance than that of conventional LQR or eigenstructure assignment. But it has a constraint for the weigting matrix in LQR, that is the weighting matrix could be indefinite for high-order systems. In this paper, the effects of the indefinite weighting matrix in EALQR on the frequency domain properties are analyzed. The robustness criterion and quantitative frequency domain properties are also presented. Finally, the frequency domain properties of EALQR has been analyzed by applying to a flight control system design example.

1. Introduction

In general, it is well known that the optimal LQR solution has some very strong robustness properties; in particular, at each channel of the plant input there is an infinite increasing gain margin and a phase margin of plus or minus 60 degrees[1]. Several methods for designing steady-state continuous regulators have been discussed. The plant and weighting matrices are assumed to be time-invariant, with (A, B) reachable and $(A, \sqrt{Q})$ obserable[2]. LQR design problem is defined as a problem of determination of the design parameters, *i.e.*, state weighting matrix Q and control input weighting matrix R in the algebraic Riccati equation. The closed-loop eigenstructure(eigenvalues and eigenvectors) is determined by the designed matrices Q and R.

Eigenstructure assignment[3-6] is well-suited for incorporating the classical specifications on damping, settling time, and mode or disturbance decoupling into a modern multivariable control framework. Eigenstructure assignment provides the advantage of allowing great flexibility in shaping closed-loop system responses by allowing specification of closed-loop eigenvalues and eigenvectors, but has disadvantage that stability-robustness is not guaranteed. Since there the same relation between EA with LQR characteristics and LQR design with EA, these become known as the EALQR problem.

Some investigators deal with LQR control methodology with pole assignment[7-11]. Recently LQR control methodology with eigenstructure assignment has developed. In 1998, Choi and Seo[12] introduced EALQR(LQR Design with Eigenstructure Assignment Capability) as a kind of LQR design methodology. The EALQR has the advantages of the existing LQR and eigenstructure assignment methods. The method of the transformation matrix via block controller is utilized to develop the scheme. The EALQR guarantees that the desired eigenvalues are assigned exactly and the corresponding desired eigenvectors are assigned in the least square sense according to the conditions of the given system. In addition, EALQR also has more freedom with nondiagonal element for the weighting matrix. But, EALQR could have a state weighting matrix Q with some negative eigenvalues for high-order systems.

The standard LQR has been assumed that the state weighting matrix Q is postive semidefinite symmetric and input weighting matrix R is postive definite symmetric. The definiteness of the weighting matrices are closely related to the robustness property of the given system. But, Molinari[7], Ohta and Kakunuma[8], and Al-sumi and Stevens[9] indicated that indefinite Q is needed if we want to design an LQR that has a capability of assigning eigenvalues. Since indefinite Q was used for obtaining an LQR with eigenstructure assignment capability, it is needed to analyze the frequency properties for the proposed controller, EALQR. If the EALQR does not have the guaranteed LQR frequency domain properties, the value of EALQR control methodology will be degraded although any assignment of eigenstructure could be possible. As all the eigenvalues of Q have been positive, R becomes I. If negative eigenvalue of indefinie Q will be small, then R approach to I. Thus, if we condisdr design method with $R \approx I$, then it is possible that all the eigenvalues of Q can be positive in the 3rd order system. But, there is a tradeoff between eigenvectors and weighting matrices, so, it is impossible that all the eigenvalues of Q can be postive in the high-order(higher than order 4) system.

In this paper, the effects of the indefinite Q in EALQR on the frequency domain properties are analyzed. The robustness criterion and quantitative frequency domain properties are also presented. Finally, the frequency domain properties of EALQR has been analyzed by applying to a flight control system design example.

2. Problem Formulation

Consider a linear time invariant multi-variable controllable system

$$\begin{aligned} \dot{x} &= Ax + Bu, &(1)\\ u &= -Kx, &(2)\end{aligned}$$

where x, u denote the N, m dimensional state variable, and control input vector, respectively. The matrices A, B and K are system, input, and gain matrices, respectively. The conventional LQR problem is to find the optimal control input u^* that minimizes the following cost function with the positive semidefinite symmetric Q and positive definite R matrices.

$$\frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt. \tag{3}$$

The gain matrix, $K = R^{-1}B^T P$, of LQR can be obtained by solving the following matrix Riccati equation.

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \tag{4}$$

Fig. 1 shows the LQR state-feedback configuration corresponding to the control law of $u = -Kx$.

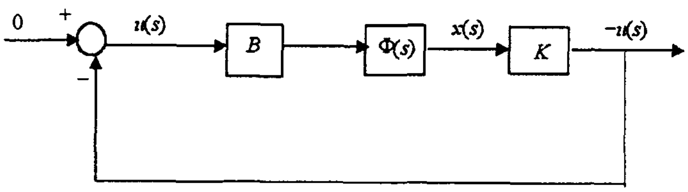

Fig. 1. LQR control system structure

LQR's loop TFM(transformation function matrix) $G_{LQ}(s)$, senstivity TFM $S_{LQ}(s)$, and closed TFM $C_{LQ}(s)$ are given by

$$\begin{aligned} G_{LQ}(s) &= K\Phi(s)B &(5)\\ S_{LQ}(s) &= [I + G_{LQ}(s)]^{-1} &(6)\\ C_{LQ}(s) &= [I + G_{LQ}(s)]^{-1}G_{LQ}(s) &(7)\end{aligned}$$

where, $\Phi(s) = (sI - A)^{-1}$.

Based on the multivariable Nyquist stability theorem, we drive a frequency domain equality for LQR[2,13], as follows:

$$[I + G_{LQ}(-s)]^T R[I + G_{LQ}(s)] = R + H(s) \tag{8}$$

where, $G_{LQ}(s) = R^{-1}B^T P(sI - A)^{-1}B$, $H(s) = [(-sI - A)^{-1}B]^T Q[(sI - A)^{-1}B]$.

IF $Q \geq 0$, then all the eigenvalues of $H(s)$ are greater than 0, *i.e.*, all the singular values of $\sqrt{H(s)}$ are always greater than 0. Thus, the frequency domain inequality is given by

$$[I + G_{LQ}(-s)]^T R[I + G_{LQ}(s)] \geq R. \tag{9}$$

If we assume $R = \rho I, \rho > 0$ in *Eq.*(9), then

$$[I + G_{LQ}(-s)]^T [I + G_{LQ}(s)] \geq 1. \tag{10}$$

We have the (Kalman) inequality

$$\sigma_{min}[I + G_{LQ}(s)] \geq 1 \quad (0\text{db}). \tag{11}$$

Eq.(11) will guarantee the satisfaction of stability-robustness condition. But, If Q has any negative eigenvalue, *i.e.*, Q is indefinite, then $H(s) \geq 0$ is not guaranteed. Thus, if Q is indefinite, then all eigenvalues of $H(s)$ can not be guaranteed positive or zero. In this case, since the maximum singular value of $\sqrt{H(s)}$ cannot be achieved, $H(s)$ cannot be separated $\sqrt{H(s)}^T \sqrt{H(s)}$. The frequency domain properties of LQR could not be guaranteed, because the frequency domain inequality in *Eq.*(11) cannot be guaranteed. If the EALQR does not have the guaranteed LQR frequency domain properties, the value of EALQR control methodology will be degraded although any assignment of eigenstructure could be possible.

The positive and negagive singular values of $H(s)$ are computed by using the eigenvalues of Q which is obtained by spectral decomposition. The difference of each singular values determines the sign of $H(s)$.

3. Frequency Domain Properties of EALQR with Indefinite Q

The matrix singular value is defined as

$$\sigma(A) = \sqrt{\lambda(A^*A)}, \quad \sigma^2(A) = \lambda(A^*A) \tag{12}$$

where, σ, superscript $*$ and $\lambda(A^*A)$ are defined as a singular value, complex conjugate matrix, and eigenvalues of A^*A, respectively. If $\sigma(A) = \sqrt{\lambda(A^*A)}$, then $A \geq 0$.

For separating the indefinite Q to two parts, *i.e.*, positive eigenvalues and negative eigenvalues, we decompose the Q using spectral decomposition as follows:

$$Q = \Phi\Lambda\Psi^T \tag{13}$$

where, Φ, Ψ and Λ are right modal matrix, left modal matrix, and diagonal matrix with eigenvalues of Q. Because Q is *Hermitian* in *Eq.*(13), we formally state the following important result.

Theorem 1 (*Spectral theorem for Hermitian matrices*)[13] Let $Q \in M_n$ be given. Then $Q = \Phi\Lambda\Psi^T = \Phi\Lambda\Phi^T$ if and ony if Q is real and Hermitain. Where, $\Psi \in M_n$ and $\Lambda \in M_n$ are a unitary matrix and a real diagonal matirix, respectively. M_n is an $n \times n$ dimensional matrix.

Theorem 2

If $Q \equiv (X^TX + Y^TY)$, then

$$\lambda(X^TX + Y^TY) = \lambda(X^TX) + \lambda(Y^TY)$$

where, $Q \in M_n$ and real diagonal.

Proof :

Let us decompose the Q using spectral decomposition.

$$Q \equiv \Phi\Lambda\Phi^T = \sum_{i=1}^{N} \phi_i\lambda_i\phi_i^T.$$

Then,

$$\begin{aligned}\lambda(\Phi\Lambda\Phi^T) &= \lambda[\Phi\sqrt{\Lambda}^T\sqrt{\Lambda}\Phi^T] \\ &= \lambda[(\sqrt{\Lambda}\Phi^T)^T(\sqrt{\Lambda}\Phi^T)] \\ &= \lambda[\sum_{i=1}^{N}(\sqrt{\lambda_i}\phi_i^T)^T(\sqrt{\lambda_i}\phi_i^T)] \\ &= \sum_{i=1}^{N}\lambda[(\sqrt{\lambda_i}\phi_i^T)^T(\sqrt{\lambda_i}\phi_i^T)].\end{aligned}$$

Thus,

$$\lambda[\sum_{i=1}^{N}(\phi_i\sqrt{\lambda_i}\phi_i^T)] = \sum_{i=1}^{N}[\lambda(\phi_i\sqrt{\lambda_i}\phi_i^T)].$$

If we assume $N = 2$, then we have

$$\lambda[\sum_{i=1}^{2}(\sqrt{\lambda_i}\phi_i^T)^T(\sqrt{\lambda_i}\phi_i^T)] = \lambda[(\sqrt{\lambda_1}\phi_1^T)^T(\sqrt{\lambda_1}\phi_1^T) + (\sqrt{\lambda_2}\phi_2^T)^T(\sqrt{\lambda_2}\phi_2^T)]$$

$$\sum_{i=1}^{2}\lambda[(\sqrt{\lambda_i}\phi_i^T)^T(\sqrt{\lambda_i}\phi_i^T)] = \lambda[(\sqrt{\lambda_1}\phi_1^T)^T(\sqrt{\lambda_1}\phi_1^T)]+\lambda[(\sqrt{\lambda_2}\phi_2^T)^T(\sqrt{\lambda_2}\phi_2^T)]$$

$H(s)$ is given by the following form by using the decomposed Q.

$$\begin{aligned}H(s) &= B^T(-sI - A)^{-1}\Phi\Lambda\Phi^T(sI - A)^{-1}B \\ &= \sum_{i=1}^{N} B^T(-sI - A)^{-1}\phi_i\lambda_i\phi_i^T(sI - A)^{-1}B\end{aligned} \quad (14)$$

In $Eq.$(14), eigenvalues of $H(s)$ are equal to the sum of eigenvalues with each decomposed term, *i.e.*,

$$\begin{aligned}&\lambda[B^T(-sI - A)^{-1}\Phi\Lambda\Phi^T(sI - A)^{-1}B] \\ &= \sum_{i=1}^{N}\lambda[B^T(-sI - A)^{-1}\phi_i\lambda_i\phi_i^T(sI - A)^{-1}B].\end{aligned} \quad (15)$$

If all the eigenvalues of Λ are positive, *i.e.*, $\Lambda = \sqrt{\Lambda}^T\sqrt{\Lambda}$, $Eq.$(15) is rewritten by

$$\sigma_i^2[\sqrt{\Lambda}\Phi^T(sI - A)^{-1}B] = \sum_{i=1}^{N}\sigma_i^2[\sqrt{\lambda_i}\phi_i^T(sI - A)^{-1}B]. \quad (16)$$

Since conventional LQR always yields $Eq.$(16), both $\sigma_i^2[\sqrt{\Lambda}\Phi^T(sI - A)^{-1}B] \geq 0$ and $H(s) \geq 0$ are also guaranteed.

But for the diagonal matrix of eigenvalues with indefinite Q, $\Lambda \neq \sqrt{\Lambda}^T\sqrt{\Lambda}$, we have the following relation.

$$\begin{aligned}&\lambda[B^T(-sI - A)^{-1}\Phi\Lambda\Phi^T(sI - A)^{-1}B] \\ &= \sum_{i=1}^{r}\lambda_p[B^T(-sI - A)^{-1}\phi_i\lambda_i\phi_i^T(sI - A)^{-1}B] \\ &- \sum_{i=1}^{N-r}\lambda_n[B^T(-sI - A)^{-1}\phi_i\lambda_i\phi_i^T(sI - A)^{-1}B]\end{aligned} \quad (17)$$

where, $\lambda_p(\cdot)$, $\lambda_n(\cdot)$ and r are the postive eigenvalues, the negative eigenvalues, and the number of positive eigenvalues. According to the singluar values theorem, the right hand side of $Eq.$(17) can be rewritten by

$$\begin{aligned}&\sum_{i=1}^{r} \sigma_p^2[\sqrt{\lambda_i}\phi_i^T(sI - A)^{-1}B] \\ &- \sum_{i=1}^{N-r} \sigma_n^2[\sqrt{\lambda_i}\phi_i^T(sI - A)^{-1}B] \geq 0\end{aligned} \quad (18)$$

where, $\sigma_p(\cdot)$ and $\sigma_n(\cdot)$ are the singluar values of $\lambda_p(\cdot)$ and $\lambda_n(\cdot)$, respectively.

$Eq.$(18) yields positive $H(s)$. Therefore, if all the eigenvalues of Q are negative or the sum of negative eigenvalues are greater than positive eigenvalues, then the inequality in $Eq.$(18) does not hold. Thus, if *Eq.(18)* holds, then LQR frequency properties are guaranteed, athough the weighting matrix Q is indefinite. Also, although $Eq.$(18) does not hold, if the difference of each value is very small, then LQR frequency properties are changed slightly.

Assume $R \neq \rho I$ in general, $Eq.$(10) can be rewritten by

$$\sigma_{min}(\sqrt{R}[I + G_{LQ}(s)]) \geq \sigma_{max}(\sqrt{R + H(s)}). \quad (19)$$

If we design a controller by EALQR, then $Eq.$(19) becomes

$$\sigma_{min}(\sqrt{R}[I + G_{LQ}(s)]) \approx \sigma_{min}(\sqrt{R})\sigma_{min}[I + G_{LQ}(s)]. \quad (20)$$

Also,

$$\sigma_{max}[\sqrt{R + H(s)}] = \sqrt{\sigma_{max}^2(\sqrt{R}) + \lambda[H(s)]}. \quad (21)$$

Substituting $Eqs.$(20), (21) to $Eq.$(19), then

$$\sigma_{min}(\sqrt{R})\sigma_{min}[I + G_{LQ}(s)] \geq \sqrt{\sigma_{max}^2(\sqrt{R}) + \lambda[H(s)]} \quad (22)$$

where, $\lambda[H(s)] = \sum_{i=1}^{r} \sigma_p^2[\sqrt{\lambda_i}\phi_i^T(sI - A)^{-1}B] - \sum_{i=1}^{N-r} \sigma_n^2[\sqrt{\lambda_i}\phi_i^T(sI - A)^{-1}B]$.

From *Eq.*(22), we have

$$\sigma_{min}[I + G_{LQ}(s)] \geq \sqrt{\frac{\sigma_{max}^2(\sqrt{R})}{\sigma_{min}^2(\sqrt{R})} + \frac{\lambda[H(s)]}{\sigma_{min}^2(\sqrt{R})}} \quad (23)$$

$$\sigma_{min}[I + G_{LQ}(s)] \geq \sqrt{a+b} \quad (20\log_{10}\sqrt{a+b} \ \text{ db}) \quad (24)$$

where, $a = \frac{\sigma_{max}^2(\sqrt{R})}{\sigma_{min}^2(\sqrt{R})}$, $b = \frac{\lambda[H(s)]}{\sigma_{min}^2(\sqrt{R})}$, and $a \geq 1$. If Q is positive semidefinite or if *Eq.*(18) is positive although Q is indefinite, then $b \geq 0$.

Since the senstivity TFM, $S_{LQ}(s)$, is $[I + G_{LQ}(s)]^{-1}$, we can obtain

$$\sigma_{max}[S_{LQ}(j\omega)] \leq \frac{1}{\sqrt{a+b}} \quad (20\log_{10}\frac{1}{\sqrt{a+b}} \ \text{ db}). \quad (25)$$

Eq.(25) means that the maximum singular value of LQR senstivity TFM is always smaller than $20\log_{10}\frac{1}{\sqrt{a+b}}$db.

Using the relations for $G_{LQ}(s)$ in *Ref.*[14], we investigated guaranteed stability-robustness bound.

$$(I + G_{LQ})^{-1} = ((G_{LQ}^{-1} + I)G_{LQ})^{-1} = G_{LQ}^{-1}(I + G_{LQ}^{-1})^{-1} \quad (26)$$

and

$$(I + G_{LQ}^{-1})(I + G_{LQ}^{-1})^{-1} = I, \quad (27)$$

$$(I + G_{LQ}^{-1})^{-1} + G_{LQ}^{-1}(I + G_{LQ}^{-1})^{-1} = I. \quad (28)$$

From *Eq.*(27) and *Eq.*(28),

$$(I + G_{LQ}^{-1})^{-1} = I - (I + G_{LQ})^{-1}. \quad (29)$$

From *Eq.*(29), singular value inequality is given by

$$\begin{aligned} \sigma_{max}[(I + G_{LQ}^{-1}(j\omega))^{-1}] &\leq 1 + \sigma_{max}[(I + G_{LQ}(j\omega))^{-1}] \\ &= 1 + \sigma_{min}^{-1}[I + G_{LQ}^{-1}(j\omega)] \\ &\leq 1 + \frac{1}{\sqrt{a+b}} \end{aligned} \quad (30)$$

or

$$\sigma_{min}[I + G_{LQ}^{-1}(j\omega)] \geq \frac{\sqrt{a+b}}{1+\sqrt{a+b}}. \quad (31)$$

To obtain the singular value inequality for the closed-loop TFM, we first get the following equation by using *Eq.*(26).

$$C_{LQ}(s) = G_{LQ}(s)[I + G_{LQ}(s)]^{-1} = [I + G_{LQ}^{-1}(s)]^{-1}. \quad (32)$$

Thus,

$$\begin{aligned} \sigma_{max}[C_{LQ}(j\omega)] &= \sigma_{max}[I + G_{LQ}^{-1}(j\omega)]^{-1} \\ &= \sigma_{min}^{-1}[I + G_{LQ}^{-1}(j\omega)]. \end{aligned} \quad (33)$$

Using *Eq.*(31) and *Eq.*(33), the singular value inequality for stability-robustness can be obtained by

$$\sigma_{max}[C_{LQ}(j\omega)] \leq 1 + \frac{1}{\sqrt{a+b}}. \quad (34)$$

Eq.(34) means that the stability-robustness of LQR is guaranteed for the following modeling error bound, $\sigma_{max}[E(j\omega)] < \frac{\sqrt{a+b}}{1+\sqrt{a+b}}$. Where, $E(j\omega) = G(j\omega)^{-1}[G_A(j\omega) - G(j\omega)]$. $G_A(j\omega)$ and $G(j\omega)$ are actual and nominal systems, respectively. Fig. 2 shows the frequency domain properties for the standard LQR which are shown in *Eqs.*(25), (34).

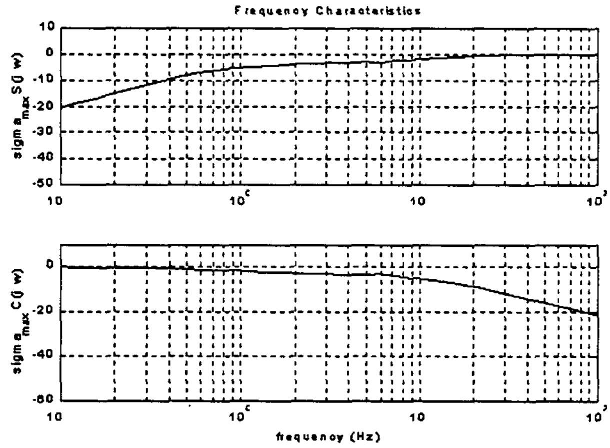

Fig. 2. Frequency domain characteristics of conventional LQR

4. Application to a Flight Control System Design

The algorithm for EALQR is presented in *Ref.*[12]. It is worthwhile to discuss the frequency domain properties of EALQR.

A linear model[11] of a fighter aircraft under consideration is the linearized two input 4th continuous controllable system as follows. The aircraft is trimmed at $Mach = 1.5$ and $h = 10,000ft$. The assumed angle of attack is $\alpha = 0.86deg$, and the locally linearized lateral directional equatons of motion is given by

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.493 & 0.015 & -1.000 & 0.020 \\ -61.176 & -7.835 & 4.991 & 0.000 \\ 31.804 & -0.235 & -0.994 & 0.000 \\ 0.000 & 1.000 & -0.015 & 0.000 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} -0.002 & 0.002 \\ 8.246 & 1.849 \\ 0.249 & -0.436 \\ 0.000 & 0.000 \end{bmatrix} \begin{bmatrix} \delta_{df} \\ \delta_r \end{bmatrix},$$

where β, p, r and ϕ represent the sideslip angle, pitch rate, yaw rate, and roll angle, whereas δ_{df} and δ_r are the deflection angles of differential flap and rudder, respectively. The eigenvalues of the open-loop system are as follows:

$$\Lambda^{open} = [\,-0.7555 \pm 5.8067i, \quad -7.8181, \quad 0.007\,].$$

The specified closed-loop eigenvalues are -8.00(roll), -0.05(sprial), and $-4.88 \pm 3.66i$ (dutch roll). That is,

$$\Lambda^d = [\,-8, \quad -4.88 \pm 3.66i, \quad -0.05\,].$$

The desired left modal matrix Ψ^d and its normalized form Ψ^d_{nor} are selected arbitrarily through the relation of $\psi_i^T \psi_i = \delta_{ij}$. The guide line for determining the derived left modal matrix is given in *Ref.*[5].

$$\Psi^d = \begin{bmatrix} 0.6 & 0.6+0.6i & 0.6-0.6i & 0.8737 \\ 0.4 & 0.4+0.4i & 0.4-0.4i & 0.1263 \\ 0.0000 & 0 & 0 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0 \end{bmatrix},$$

$$\Psi^d_{nor} = \begin{bmatrix} 0.8321 & 0.5883+0.5883i & 0.5883-0.5883i & 0.9897 \\ 0.5547 & 0.3922+0.3922i & 0.3922-0.3922i & 0.1431 \\ 0.0000 & 0 & 0 & 0.0000 \\ 0 & 0.0000 & 0.0000 & 0 \end{bmatrix}.$$

According to the design procedure of the EALQR algorithm, the achievable normalized left modal matrix Ψ^a_{nor} can be obtained in the least square sense by

$$\Psi^a_{nor} = \begin{bmatrix} -0.5473 & 0.1086+0.0454i & 0.1086-0.0454i & 0.9897 \\ -0.0820 & 0.1029-0.0032i & 0.1029+0.0032i & 0.1431 \\ 0.6540 & 0.6936-0.7005i & 0.6936+0.7005i & 0.0000 \\ -0.5158 & 0.0454-0.0400i & 0.0454+0.0400i & 0.0000 \end{bmatrix}.$$

The directions of each vector of the achievable left modal matrix is placed near the best possible directions of each desired left eigenvector in the least square sense, and the desired closed-loop eigenvalues are assigned exactly. The feedback gain matrix K and the weighting matrices Q and R can be obtained by

$$K = \begin{bmatrix} -1.5459 & -0.0592 & 4.1769 & -0.6326 \\ -2.9178 & 0.2789 & -17.0252 & 4.3767 \end{bmatrix},$$

$$R = \begin{bmatrix} 1 & 0.0000 \\ 0.0000 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} -363.2 & 17.2 & -1145. & 292.7 \\ 17.2 & -0.4 & 7. & -0.3 \\ -1145. & 7. & 396.1 & -87.4 \\ 292.7 & -0.3 & -87.4 & 19.6 \end{bmatrix}.$$

Eigenvalues of the weighting matrix Q are given by

$$\lambda(Q) = [\,-1218.7, \quad 1270.2, \quad 1.9, \quad -1.4\,].$$

Since there are negative eigenvalues, Q is indefinite. The maximum singular values of sensitivity TFM and closed-loop TFM in this case have the bound as follows:

$$\begin{aligned} \sigma_{max}[S_{LQ}(j\omega)] \le \frac{1}{\sqrt{a+b}} &= \frac{1}{\sqrt{1.0000 - 0.0434}} \\ &= 1.0224 \\ &= 0.1927 \quad \text{(db)} \end{aligned}$$

$$\begin{aligned} \sigma_{max}[C_{LQ}(j\omega)] \le 1 + \frac{1}{\sqrt{a+b}} &= 2.0224 \\ &= 6.1173 \quad \text{(db)} \end{aligned}$$

and, the resulting stability-robustness bound for the modelling error is

$$\sigma_{max}[E(j\omega)] < 0.4945.$$

Although the maximum magnitude of $\lambda[H(s)]$ is negative, senstivity characteristics of frequency domain can not drop because the maximum maganitude is small.

Fig. 3 shows frequency characteristics of the flight control system. We can see that each maximum singular value change under each bound in Fig. 3. The performance of disturbance rejection and command following in the low frequencies, and low-sensitivity performance for the high frequencies are good.

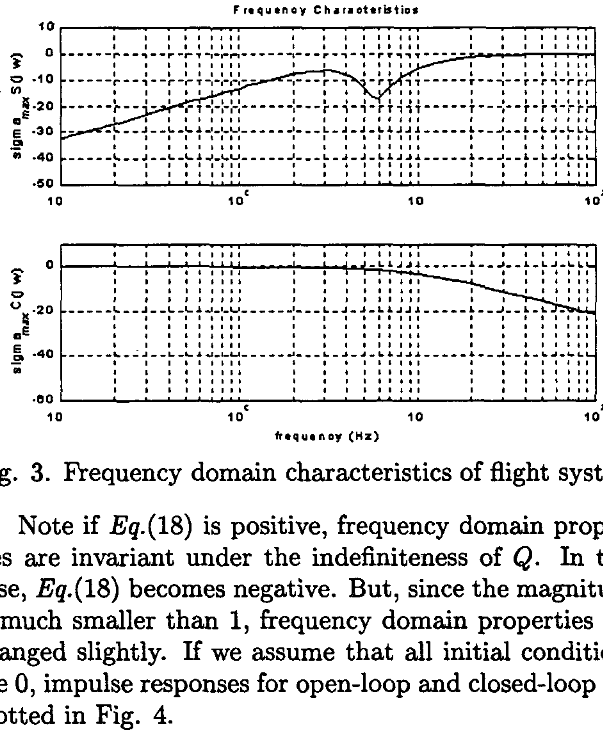

Fig. 3. Frequency domain characteristics of flight system

Note if *Eq.*(18) is positive, frequency domain properties are invariant under the indefiniteness of Q. In this case, *Eq.*(18) becomes negative. But, since the magnitude is much smaller than 1, frequency domain properties are changed slightly. If we assume that all initial conditions are 0, impulse responses for open-loop and closed-loop are plotted in Fig. 4.

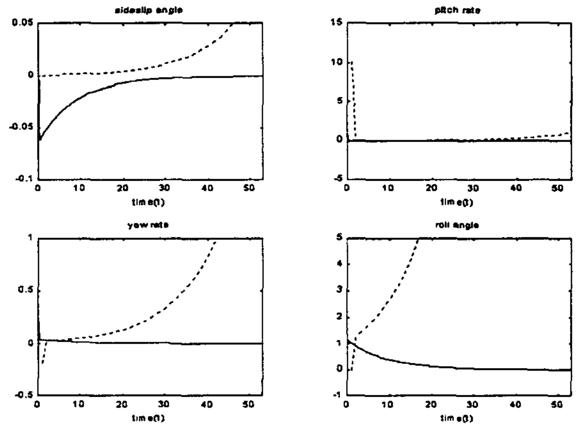

Fig. 4. Impulse responses
(dased line : open-loop, solid line : closed-loop)

5. Conclusions

There is the same relation between EA with LQR characteristics and LQR design with EA. This is known as the EALQR problem. EALQR has the advantages of the existing LQR and eigenstructure assignment methods. But, EALQR could have a state weighting matrix Q with some negative eigenvalues for high-order systems. Because of the indefiniteness of Q, there is a tradeoff between LQR and eigenstructure assignment.

In this paper, the effects of the indefinite Q in EALQR on the frequency domain properties are analyzed. The robustness criterion and quantitative frequency domain properties are also presented. Finally, the frequency domain properties of EALQR has been analyzed by applying to a flight control system design example.

Since there exists the case that an achievable subspace of eigenstructure does not belong to an achievable subspace of LQR, further discussion of a subspace of both EA and LQR shoud be exploited.

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