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Control of the flexible system under irregular disturbance by using of 「random gain」

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Abstract

A control strategy for flexible structure under irregular disturbance by using of 「random gain」 is developed and implemented. System equation is transformed to stochastic domain by F-P-K approach from physical domain. A controller is designed in the stochastic domain, accordingly system is controlled by 「random gain」 in time domain. In the paper, a new control technique is successfully employed for flexible system under white noise, and the result is verified by Monte-Carlo simulation and compared with the performance via LQR controller.

1. Introduction

Dynamic systems are often exposed to various external disturbance in the nature. Especially, random disturbance is the most general case. These include: aerospace systems excited by atmospheric and boundary layer turbulence and jet noise; aircraft and vehicles subjected to track induced vibrations; ground based structures excited by earthquakes and wind; and offshore structures excited by wind and hydrodynamic wave-induced loads. In each cases the physical variables exhibit random fluctuations in both space and time. In recent, dynamic systems become more complex and larger. [1]

As the result, random disturbance/noise effect on system is considered more in the system design. Accordingly its control in more proper way draw lot of attention. Many controller design technique were tried to reject or suppress random disturbance and achieved reliable result. Lot of stochastic control technique such as LQR, LQG, H_∞ and etc. are performed good disturbance /noise rejection, which are designed in time domain or frequency domain. Nevertheless these don't use enough noise/ disturbance information. LQR design does not use any input random signal information. LQG uses only correlation value on signal in estimation; such as Kalman filter and H_∞ filter. H_∞ controller use noise maximum norm. As a alternative controller design method for extended use of random signal information, new concept for stochastic controller design is proposed recently. Han, Kim, and Heo(1995) showed the ability of stochastic controller in a stochastic observer design by using of Fokker - Planck - Kolmogrov(F-P-K) approach[2], and Heo and Han(1996) developed the stochastic controller by using of GA based fuzzy controller in probabilistic domain[3]. An experimental study for the control of base excited flexible beam model was carried on with piezo actuator and sensor in 1996. Wind tunnel model for realistic random disturbance has been carrying on since 1997.[4][5]

System dynamics in time domain is transformed to stochastic domain by F-P-K procedure. Transformed system is represented in the form of differential equation in terms of moments by Ito's rule. New stochastic controller is designed in stochastic domain. In the paper, stochastic control strategy for flexible system under irregular disturbance is developed by using of PI controller in stochastic domain. Stochastic version of this classic control strategy is applied and verified, and it's result is compared to that of LQR controller.

2. System Equation

In the paper, flexible beam structural system under Gaussian white noise is considered. It may be written 2nd order ODE form as Eqn. (1) after modal analysis.

$$\ddot{y}+2\zeta\omega\dot{y}+\omega^2 y=p\dot{z}(t)+bV(t) \qquad (1)$$

where $z(t)$: uncorrelated white noise type base disturbance with 0 mean

$V(t)$: uncorrelated white noise type control input with 0 mean

Gaussian white noise type disturbance and control input have the following relationship.

$$\begin{aligned} \dot{z}(t) &= \frac{dB_z(t)}{dt} \\ V(t) &= \frac{dB_V(t)}{dt} \\ E[dB_z^2(t)] &= 2\pi D_z \triangle t \\ E[dB_V^2(t)] &= 2\pi D_V \triangle t \end{aligned} \qquad (2)$$

where $B_z(t)$ and $B_V(t)$ are Brownian precesses, D_z and D_V are PSD(power spectral density) of the disturbance and control signal, respectively.

Introducing the coordinates transformation Eqns.(3), Eqn.(1) can be written in the Ito's stochastic differential equation Eqns.(4).

$$\begin{aligned} y &= X_1 \\ \dot{y} &= X_2 \end{aligned} \qquad (3)$$

$$\begin{aligned} dx_1 &= x_3 dt \\ dx_2 &= \{-\omega^2 x_1 - 2\zeta\omega x_2 + p\dot{z}(t) + bV(t)\}dt \\ &= \{-\omega^2 x_1 - 2\zeta\omega x_2\}dt + pdB_z + bdB_V \end{aligned} \qquad (4)$$

The evolution of transitional joint probability density of the response coordinates $P(x,t)$ can be described by the FPK equation

$$\begin{aligned} \frac{\partial}{\partial\tau}P(X,\tau) &= -\sum_{i=1}^{2}\frac{\partial}{\partial X_i}[a_i(X,\tau)P(X,\tau)] \\ &+\frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\partial^2}{\partial X_i\partial X_j}[b_{ij}(X,\tau)P(X,\tau)] \end{aligned} \qquad (5)$$

The general differential equation of moments is

$$\begin{aligned} &\frac{\partial}{\partial\tau}E[X_1^i X_2^j] \\ &= \int\int_{-\infty}^{\infty} X_1^i X_2^j \frac{\partial}{\partial\tau}P(X,\tau)dX_1 dX_2 \end{aligned} \qquad (6)$$

Thus the first moment equations and the second moment equations from Eqn.(6) are obtained as follows

$$\begin{aligned} \dot{m}_{10} &= m_{01} \\ \dot{m}_{01} &= -\omega^2 m_{10} - 2\zeta\omega m_{01} \\ \dot{m}_{20} &= 2m_{11} \\ \dot{m}_{11} &= -\omega^2 m_{20} - 2\zeta\omega m_{11} + m_{02} \\ \dot{m}_{02} &= -2\zeta\omega m_{02} + 2\pi(p^2 D_z + b^2 D_V) \end{aligned} \qquad (7)$$

where M_{ij} : $E[X_1^i\ X_2^j]$

Since the disturbance or noise under consideration is assumed to have zero mean, the first dynamic moment equations in Eqns.(7) are conversed to zero in steady state.

3. Controller Design

Stochastic Controller design

For controller design purpose only the second dynamic moment equation will be adopted.

Then Eqn.(7) can be simplified to matrix form like

$$\dot{m} = A_m m + \tilde{P}_m D_z + \tilde{B}_m D_V \quad (8)$$

where

$$A_m = \begin{bmatrix} 0 & 2 & 0 \\ -\omega^2 & -2\zeta\omega & 1 \\ 0 & 0 & -2\zeta\omega \end{bmatrix} \text{ system matrix}$$

$$\tilde{P}_m = \begin{bmatrix} 0 \\ 0 \\ 2\pi p^2 \end{bmatrix} \text{ disturbance gain matrix,}$$

$$\tilde{B}_m = \begin{bmatrix} 0 \\ 0 \\ 2\pi b^2 \end{bmatrix} \text{ control gain matrix}$$

D_z : PSD of disturbance

D_V : PSD of control signal

In the paper, we use PI(proportional and Integral) controller with moment state feedback, and it is designed to minimize system moment response in stochastic domain. For effective gain tuning m02 moment is focused

LQR controller design

The system can be rewritten in state-space form,

$$\dot{x} = Ax + Bv + \Gamma\ddot{z} \quad (9)$$
$$v = -K_c x$$

where B is control gain matrix and Γ is disturbance gain matrix. Disturbance $\ddot{z}$ and control signal v are Gaussian white noise with zero-mean and uncorrelated in time.

Control signal u is linear function of the state, K_c is optimal state-feedback matrix which is determined in LQ sense. The optimal state-feedback gain matrix K_c is given by minimizing the cost

$$J = \lim_{T\to\infty} E\left\{ \int_0^T (x^T Q x + v^T R v)\, dt \right\} \quad (10)$$

where $Q = Q^T \geq 0$ and $R = R^T = >0$ so it is

$$K_c = R^{-1} B^T P_c \quad (11)$$

where P_c satisfies the algebraic Riccati equation

$$A^T P_c + P_c A - P_c B R^{-1} B^T P_c + Q = 0 \quad (12)$$

4. Result

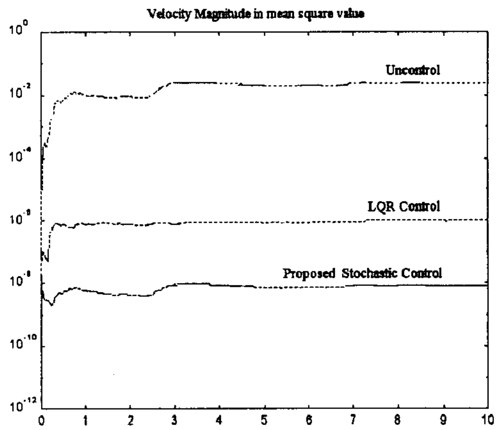

Fig. 1] Comparison of velocity mean square response

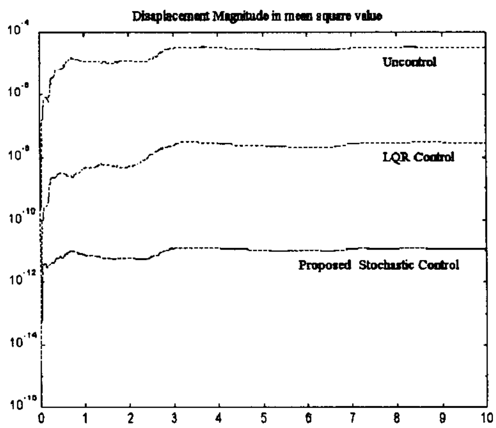

Fig 2] Comparison of displacement mean square response

Responses of system are shown in Fig.1 and Fig.2. As is seen in the figures, response under the control of m02 moment reveals good result.

Comparing with LQR controller, proposed controller reduce the displacement response about 100 times and the velocity response about 10 times.

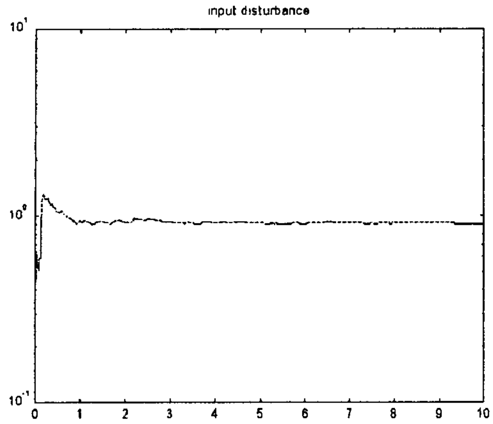

Fig. 3] Mean square Disturbance

Fig.3 show the disturbance magnitude. As shown in Fig. 4 interestingly enough, control force of two controller are tuned almost same mean square value.

Fig. 5, and Fig.6 are power spectral response in frequency domain. Fig. 5 show that typical band limited white noise response, system output makes peak at natural frequency. Fig. 6 have no peak in response.

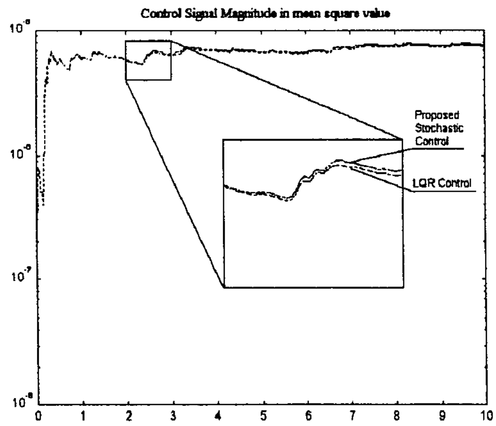

Fig. 4] Control Force in mean square value

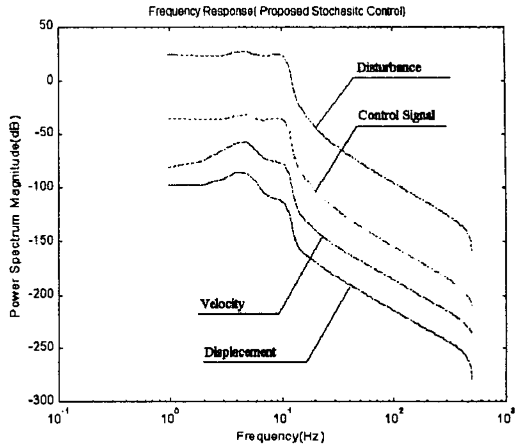

Fig 5] Frequency response of proposed Stochastic Controller

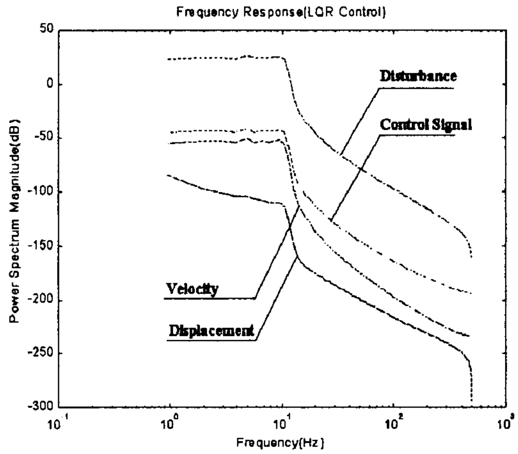

Fig. 6] Frequency response of LQR Controller

Fig. 7 and Fig. 8 are time domain response via Monte-Carlo simulation.

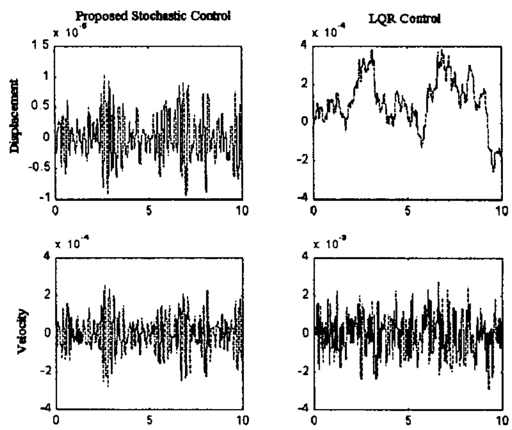

Fig 7] Time Response of System(1)

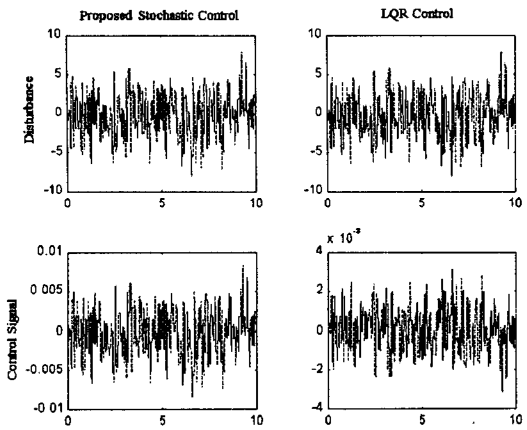

Fig 8] Time response of System (2)

	Control Force	Displacement	Velocity
uncontrol	0	3.274e-5	2.255e-2
Proposed sto-chastic control	7.588e-1	1.199e-11	8.252e-9
LQG control	7.433e-1	2.866e-8	9.401e-7

[table 1] steady state mean square response

5. Conclusion

Although the stochastic controller that mentioned in the paper is designed in simple classical way in stochastic domain, the performance of the proposed controller is better than that of LQR. Major merit of new method is easy to deal the random signal which is described with constant or simple function in terms of PSD value. In stochastic domain, controller can be designed by using of already developed controller such as PID.

In the system, control PSD value calculated in stochastic domain is realized by Monte-Carlo method in time domain. Accordingly physical system is controlled by 『Random Gain』 and random type control force in time domain. Controller gains are tuned along with random disturbance and system parameter as well. It improve system ability to suppress random disturbance. Much of the work should be digged out is laid ahead.

Acknowledgement

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Reference

[1] R. A. Ibrahim, "*Parametric Random Vibration*", Research Studies Press, Letchworth, England, 1985.

[2] Minsung Kim, Jungyoup Han and Hoon Heo, "A New Approach to Stochastic Control of Randomly Disturbed System(I)", *1995 Int'l Mechatronics Conference Chejudo 1~3 Dec.*, 1995.

[3] Jungyoup Han and Hoon Heo," Optimal Design of Smart Actuator by using of GA for the Control of a Flexible Structure Experiencing White Noise Disturbance", *1996 K.S.N.V.E spring conference*, 1996.

[4] Hoon Heo, Dae Jung Kim, JaeHyuk Yang and Yun Hyun Cho "Experimental Study on New stochastic Control Technique", Journal of Dynamic System and Control (A.S.M.E. : in preparation)

[5] Hoon Heo, Dae Jung Kim, JaeHyuk Yang, and Yun Hyun Cho, "Stochastic Control of a Thin Beam Structure under Turbulent Flow.", IEEE Transactions on Automatic Control (IEEE. : in preparation)

[6] Hoon Heo, Yun Hyun Cho, Dae Jung Kim and JaeHyuk Yang, "A Study on the Stochastic Control of Dynamic System.", Journal of Guidance, Control, and Dynamics (AIAA : in preparation)