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Development of a Robust Nonlinear Prediction-Type Controller

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Abstract

In this paper, a robust nonlinear prediction-type controller (RNPC) is developed for the continuous time nonlinear system whose control objective is composed of system output and its desired value. The basic control law of RNPC is derived such that the future response of the system is first predicted by appropriate functional expansions and the control law minimizing the difference between the predicted and desired responses is then calculated. RNPC which involves two controls, i.e., the auxiliary and robust controls into the basic control, shows the stable closed loop dynamics of nonlinear system of any relative degree and provides the robustness to the nonlinear system with parameter/modeling uncertainty. Simulation tests for the position control of a two-link rigid body manipulator confirm the performance improvement and the robustness of RNPC.

1. Introduction

Model predictive controllers have received great attention and the theories and applications have been developed extensively during past two decades[1][2][3]. Almost all of model predictive control researches have, however, been focused on the discrete time linear system models which are represented by explicit form of input and output relationship. For nonlinear systems, the prediction concepts used in the adaptive control rather than the predictive control have also been configured on the mathematical models of explicit form of input and output relationships[4][5], in which the functional expansions of Volterra and Wiener series provide the basic framework for nonlinear explicit model.

As a different approach, Lu[6] suggested one method in which the prediction concept can be configured directly on the dynamic nonlinear system models, which have no necessity to be transformed into the explicit mathematical models. The basic control law of nonlinear prediction-type controller (NPC) based on the implicit form of nonlinear system models is derived in such a way that the future response of the system is predicted by appropriate functional expansions suggested by Lu[6] and then, the control law is produced for minimizing the control objective which is a energy function composed of the predicted and desired responses. It has a very simple form and can be obtained optimally even under the control input constraint. In NPC, the prediction time interval has not only a role of determining the prediction interval but also of adjusting closed loop dynamics. It does not disturb the system stability whether its value is given by a large or small value but is concerned with the convergence of tracking error. In this paper, we call this controller as nonlinear prediction-type controller rather than nonlinear predictive controller because its internal mechanism is slightly different from existing model predictive controllers.

The control objective in NPC may be composed of system outputs or system states. In many applications, especially control of robot manipulator and underwater vehicles, the control objective with system outputs is sufficient for asymptotic tracking of all outputs and states. When the control objective is constructed with system output and its desired value, the closed loop stability of nonlinear system with the basic control law is guaranteed for systems whose relative degree is less than 5. For systems with relative degree higher than 4, other penalties such as time derivatives of output error may be incorporated into the control objective in order to obtain the closed loop stability[6]. This results in more complex form of control law and increases weighting factors (or matrices for MIMO system) which, in turn, adds the difficulty in finding the optimal values of weighting factors.

In order to remedy the problems described above, a robust nonlinear prediction-type controller is developed. In RNPC, an auxiliary control and a robust control are involved into the nonlinear prediction-type controller. By means of the auxiliary control, the closed loop dynamics is always stable for the relative degree higher than 4. Incorporation of the robust control gives the asymptotic stability of the system with parameter/modeling uncertainty without requiring any critical restrictions.

2. Development of Basic Control Law

In this section, the basic control law of NPC is derived. The closed loop dynamics of the nonlinear system with the basic control law is investigated and the problem occurred from using this control law is also investigated.

Consider the nonlinear system with system order n such as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathrm{G}(\mathbf{x})\mathbf{u}, \tag{1a}$$

$$\mathbf{y} = \mathbf{c}(\mathbf{x}), \tag{1b}$$

where $\mathbf{x} \in \mathrm{X} \subset \mathrm{R}^n$ is the state, $\mathbf{u} \in \mathrm{U} \subset \mathrm{R}^p$ the control input,

$y \in R^p$ the output. The functions $\mathbf{f}: R^n \to R^n$, $G: R^n \to R^{n\times p}$, and $\mathbf{c}: R^n \to R^p$ are supposed to be sufficiently smooth and have finite magnitudes, respectively. The domain X contains the origin and U is a compact set. It is assumed that $p \le n$.

Let r_i be the relative degree of *i*th component of output. Then, the dynamic equation of *i*th component of system output, y_i, to input is represented as

$$y_i^{(r_i)} = L_f^{r_i}(c_i) + \sum_{j=1}^{p} L_{g_j}(L_f^{r_i-1}(c_i))u_j, \quad (2)$$

where c_i and u_i are *i*th components of output function and input vector, respectively. $L_f^{r_i}(c_i)$ is the r_ith order Lie derivative of c_i with respective to **f**, and the term $L_{g_j}(L_f^{r_i-1}(c_i))$ in Eq.(2) is represented as

$$L_{g_j}(L_f^{r_i-1}(c_i)) = \frac{\partial(L_f^{r_i-1}(c_i))}{\partial \mathbf{x}} \mathbf{g}_j,$$

where $\mathbf{g}_j$ is the $n\times 1$ column vector of G.

When the above procedure is carried out repeatedly from i=1 to i=p and the resulting forms are collected to represent the compact form, the dynamic equation of system output to input is

$$\mathbf{y}^r = \mathbf{p}(\mathbf{x}) + W(\mathbf{x})\mathbf{u}, \quad (3)$$

where $\mathbf{y}^r = \left[y_1^{(r_1)} \cdots y_p^{(r_p)}\right]^T$, $\mathbf{p}(\mathbf{x}) = \left[L_f^{r_1}(c_1) \cdots L_f^{r_p}(c_p)\right]^T$ and $W(\mathbf{x})$ is a pxp nonsingular matrix, in which *i*th raw vector $\mathbf{w}_i(\mathbf{x})$ is defined as

$$\mathbf{w}_i(\mathbf{x}) = \left[L_{g_1}(L_f^{r_i-1}(c_i)) \cdots L_{g_p}(L_f^{r_i-1}(c_i))\right], \text{ for } i=1,\cdots,p.$$

As suggested by Lu[6], the future output at t+h ($h \ge 0$) from current time t is expanded such as

$$\mathbf{y}(t+h) \cong \mathbf{z}(t) + \Lambda(h)\left[\mathbf{p}(\mathbf{x}) + W(\mathbf{x})\mathbf{u}(t)\right], \quad (4a)$$

where h is called the prediction time interval, $\mathbf{y}(t+h) = [y_1(t+h) \cdots y_p(t+h)]^T$, $\mathbf{z}(t) = [z_1(t) \cdots z_p(t)]^T$, and $\Lambda(h)$ is the pxp diagonal constant matrix whose diagonal elements are represented as $\frac{h^{r_i}}{r_i!}$ (i=1,···,p).

In Eq.(4a), *i*th component of $\mathbf{z}(t)$ is represented as

$$z_i(t) = \lambda_i(h)\phi_i(t),$$

where $\lambda_i(h) = \left[1 \quad h \quad \frac{h^2}{2!} \quad \cdots \quad \frac{h^{r_i-1}}{(r_i-1)!}\right]$: $1\times r_i$ vector,

$\phi_i(t) = \left[y_i(t) \; \dot{y}_i(t) \; \cdots y_i^{(r_i-1)}(t)\right]^T$: $r_i\times 1$ vector.

The desired future output of the system is also approximated as

$$\mathbf{y}_d(t+h) \cong \mathbf{z}_d(t) + \Lambda(h)\mathbf{y}_d^r(t), \quad (4b)$$

where $\mathbf{y}_d(t+h) = [y_{d,1}(t+h) \cdots y_{d,p}(t+h)]^T$,

$\mathbf{z}_d(t) = [z_{d,1}(t) \cdots z_{d,p}(t)]^T$, and $\mathbf{y}_d^r(t) = \left[y_{d,1}^{(r_1)}(t) \cdots y_{d,p}^{(r_p)}(t)\right]^T$.

In Eq.(4b), *i*th component of $\mathbf{z}_d(t)$ is represented as

$$z_{d,i}(t) = \lambda_i(h)\phi_{d,i}(t)$$

where $\phi_{d,i}(t) = \left[y_{d,i}(t) \; \dot{y}_{d,i}(t) \cdots y_{d,i}^{(r_i-1)}(t)\right]^T$.

The future output error, $\mathbf{e}(t+h)$, is defined as

$$\mathbf{e}(t+h) = \mathbf{y}(t+h) - \mathbf{y}_d(t+h)$$

$$= \mathbf{z}_e(t) + \Lambda(h)\mathbf{p}(\mathbf{x}) + \Lambda(h)W(\mathbf{x}) - \Lambda(h)\mathbf{y}_d^r(t), \quad (5)$$

where *i*th component of $\mathbf{z}_e$ is represented as $z_{e,i}(t) = \lambda_i(h)\phi_{e,i}(t)$ and $\phi_{e,i}(t) = \left[e_i(t) \; \dot{e}_i(t) \cdots e_i^{(r_i-1)}(t)\right]^T$.

The control objective for output error to be minimized has the form of

$$J(t+h) = \frac{1}{2}\mathbf{e}(t+h)^T Q_w \mathbf{e}(t+h), \quad (6)$$

where Q_w is the pxp positive definite weight matrix which is usually diagonal matrix.

The control input is produced from the fact that the control input obtained at time t should minimize the control objective at time t+h, namely, $\partial J(t+h)/\partial \mathbf{u}(t)=0$. The control input derived in this way, by use of Eqs.(4) and Eq.(5), is as follows

$$\mathbf{u}(t) = -[(\Lambda(h)W(\mathbf{x}))^T Q_w (\Lambda(h)W(\mathbf{x}))]^{-1} \cdot (\Lambda(h)W(\mathbf{x}))^T Q_w [\mathbf{z}_e(t) + \Lambda(h)\mathbf{p}(\mathbf{x}) - \Lambda(h)\mathbf{y}_d^r(t)]. \quad (7)$$

As can be seen in Eq.(7) with a few more manipulation by use of the fact that $W(\mathbf{x})$ is nonsingular, the prediction time interval which resides in $\Lambda(h)^{-1}$ has a role of controller gain. If h is tuned to a small value, the control input is produced by a large value for a small output error and if h is set to a large value, the case is reversed.

The closed loop dynamics by inserting the control input of Eq.(7) into Eq.(3) is expressed as follows

$$\mathbf{y}^r(t) = -\Lambda(h)^{-1}\mathbf{z}_e(t) + \mathbf{y}_d^r(t). \quad (8)$$

If a $p\times 1$ vector $\mathbf{e}^r(t)$ is defined as

$$\mathbf{e}^r(t) = \begin{bmatrix} e_1^{(r_1)}(t) \\ \vdots \\ e_p^{(r_p)}(t) \end{bmatrix} = \mathbf{y}^r(t) - \mathbf{y}_d^r(t),$$

then, Eq.(8) can be expressed as

$$\mathbf{e}^r(t) + \Lambda(h)^{-1}\mathbf{z}_e(t) = 0. \quad (9)$$

The *i*th component of error dynamics in Eq.(9) is represented as

$$e_i^{(r_i)}(t) + \frac{r_i}{h}e_i^{(r_i-1)}(t) + \cdots + \frac{r_i!}{2!\cdot h^{r_i-2}}\ddot{e}_i(t) + \frac{r_i!}{h^{r_i-1}}\dot{e}_i(t) + \frac{r_i!}{h^{r_i}}e_i(t) = 0. \quad (10)$$

As can be seen in Eq.(10), the prediction time interval has, in this case, a role of determining the convergence rate of error dynamics. From Eq. (10), the error dynamics has pole(s) in open negative half complex plane for $r_i \le 4$ and it is unstable for the relative degree r_i greater than 5. Hence, the control input in Eq.(7) obtained from Eq.(6) is confined to the nonlinear system which has the relative degree of each component of output less than five.

3. Development of Control Law of RNPC

In order to remedy the problem described in the previous section, Lu[6] suggested that if the system has the *i*th relative degree r_i greater than 4, the control objective is modified in such a way that it includes other penalties from the derivative of *i*th future output error to (r_i-1)th order derivative until closed loop dynamics is stable. This procedure requires more and more weight matrices, and this makes the control input be complex and also increases the difficulty in tuning the weight matrices.

In this section, a simple and very efficient method is suggested in order for the closed loop dynamics with the control input of the form of Eq.(7) to be stable regardless of the order of the relative degree of any component of output. Moreover, in order for the closed loop dynamics to have the asymptotic convergence under the parameter/modeling uncertainty, the robust control is developed.

3-1. Auxiliary Control

As we investigate carefully the behavior of error dynamics in Eq.(9), the error dynamics is time invariant and not affected by the nonlinear functions of the system. Therefore, the error dynamics can be changed via slight modification of the structure of control input of Eq.(7) in such a way that canceling out the existing linear term and inserting additional linear term into the control input makes error dynamics always stable for any order of relative degree of each component of system output.

Let the auxiliary control $v(t)$ be defined as

$$\mathbf{v}(t) = -\mathbf{z}_e(t) + \Lambda(h)\bar{\mathbf{v}}(t), \tag{11}$$

where ith component of $\bar{\mathbf{v}}(t)$ is represented as $\bar{v}_i(t) = \lambda_{v,i}(h)\phi_{e,i}(t)$ and $\lambda_{v,i}(h) = \begin{bmatrix} \frac{1}{h^{r_i}} & \frac{r_i}{h^{r_i-1}} & \cdots & \frac{r_i}{h} \end{bmatrix}$.

Then, we make the following theorem.

Theorem 1: For the systems of Eqs.(1) which have total relative degree, r, the following control input, $\mathbf{u}_c(t)$,

$$\mathbf{u}_c(t) = -[(\Lambda(h)W(\mathbf{x}))^T Q_w(\Lambda(h)W(\mathbf{x}))]^{-1}(\Lambda(h)W(\mathbf{x}))^T Q_w \cdot[\mathbf{z}_e(t) + \Lambda(h)\mathbf{p}(\mathbf{x}) - \Lambda(h)\mathbf{y}_d^r(t) + \mathbf{v}(t)]. \tag{12}$$

achieves input/output linearization and asymptotic tracking of any given future output $\mathbf{y}_d(t+h)$ for any $h>0$, $Q_w>0$, and $W(\mathbf{x})$ being nonsingular is guaranteed, for any r_i ($i=1,\cdots p$).

Proof: When Eq.(12) is inserted into the control input $\mathbf{u}(t)$ in Eq.(3), the closed loop system is represented as

$$\mathbf{y}^r(t) = \mathbf{p}(\mathbf{x}) - \Lambda(h)^{-1}\left[\mathbf{z}_e + \Lambda(h)\mathbf{p}(\mathbf{x}) - \Lambda(h)\mathbf{y}_d^r + \mathbf{v}(t)\right]$$
$$= \mathbf{y}_d^r(t) - \bar{\mathbf{v}}(t).$$

Then, the above equation results in

$$\mathbf{e}^r(t) + \bar{\mathbf{v}}(t) = 0.$$

The ith component of the above error dynamics is

$$e_i^{(r_i)}(t) + \frac{r_i}{h}e_i^{(r_i-1)}(t) + \cdots + \frac{r_i(r_i-1)}{h^{r_i-2}}\ddot{e}_i(t) + \frac{r_i}{h^{r_i-1}}\dot{e}_i(t) + \frac{1}{h^{r_i}}e_i(t) = 0, \text{ (for } i=1,\cdots,p).$$

From the above result, the closed loop dynamics is linear and time invariant and hence, input/output linearization can be achieved.

Moreover, the above equation can be represented equivalently as

$$\left(s + \frac{1}{h}\right)^{r_i} e_i(t) = 0,$$

where s is the complex variable or differentiation operator. Since the above equation has r_ith order of poles at $-\frac{1}{h}$, this dynamics is always stable regardless of by what value the relative degree r_i may be given ■

3-2. Robust Control for Parameter/Modeling Uncertainty

For the case that the nonlinear system has the parameter/modeling uncertainty, the dynamic equation of Eq.(3) is rearranged by dividing the known functions and the unknown functions such as

$$\mathbf{y}^r(t) = \hat{\mathbf{p}}(\mathbf{x}) + \Delta\mathbf{p}(\mathbf{x}) + \left[\hat{W}(\mathbf{x}) + \Delta W(\mathbf{x})\right]\mathbf{u}(t)$$
$$= \hat{\mathbf{p}}(\mathbf{x}) + \hat{W}(\mathbf{x})\mathbf{u}(t) + \mathbf{d}(t). \tag{13}$$

where $\hat{\mathbf{p}}$ and $\hat{W}$ are the exactly known functions, $\Delta\mathbf{p}$ and ΔW are the unknown functions due to the parameter/modeling uncertainty, and $\mathbf{d}$ is the collection of the unknown functions.

In this paper, the robust control is derived based on the assumption that the bound of $\mathbf{d}(t)$ can be acquired by the following form

$$|d_i(t)| \le d_{max,i} < \infty, \forall t \ge 0, \text{ for } i=1,\cdots,p. \tag{14}$$

Let $\hat{\mathbf{u}}_c(t)$ be the control input based on the known functions $\hat{\mathbf{p}}$ and $\hat{W}$, and this control is represented as

$$\hat{\mathbf{u}}_c(t) = -[(\Lambda(h)\hat{W}(\mathbf{x}))^T Q_w(\Lambda(h)\hat{W}(\mathbf{x}))]^{-1}(\Lambda(h)\hat{W}(\mathbf{x}))^T Q_w \cdot[\mathbf{z}_e(t) + \Lambda(h)\hat{\mathbf{p}}(\mathbf{x}) - \Lambda(h)\mathbf{y}_d^r(t) + \mathbf{v}(t)] \tag{15}$$

The closed loop dynamics obtained by inserting Eq.(15) into Eq.(13) is

$$\mathbf{e}^r(t) + \bar{\mathbf{v}}(t) = \mathbf{d}(t). \tag{16}$$

As can be seen in Eq.(16), the asymptotic convergence of tracking error to zero can not be achieved when the nonlinear system has the parameter/modeling uncertainty. In other for asymptotic tracking to be achieved, the control input of Eq.(15) is modified as

$$\hat{\mathbf{u}}_c(t) = -[(\Lambda(h)\hat{W}(\mathbf{x}))^T Q_w(\Lambda(h)\hat{W}(\mathbf{x}))]^{-1}(\Lambda(h)\hat{W}(\mathbf{x}))^T Q_w \cdot[\mathbf{z}_e(t) + \Lambda(h)\hat{\mathbf{p}}(\mathbf{x}) - \Lambda(h)\mathbf{y}_d^r(t) + \mathbf{v}(t) + \Lambda(h)\mathbf{r}(t)] \tag{17}$$

where $\mathbf{r}(t)$ in the most right hand side of Eq.(17) is the robust control.

Then, the closed loop dynamics is represented as

$$\mathbf{e}^r(t) = -\bar{\mathbf{v}}(t) + \mathbf{d}(t) - \mathbf{r}(t). \tag{18}$$

In order to derive the robust control based on the Lyapunov redesign method, the error dynamics of Eq.(18) is to be rearranged into a type of canonical vector form. To see the transformation procedure of Eq.(18) into a canonical vector form, let us consider, for example, the nonlinear system of n=2 and r_i=2 (i=1,2). The error dynamics of Eq.(18), in this case, becomes

$$\begin{bmatrix} \ddot{e}_1(t) \\ \ddot{e}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{2}{h}\dot{e}_1(t) - \frac{1}{h^2}e_1(t) \\ -\frac{2}{h}\dot{e}_2(t) - \frac{1}{h^2}e_2(t) \end{bmatrix} + \mathbf{d}(t) - \mathbf{r}(t). \tag{19}$$

Define the augmented error vector, $\mathbf{e}_v$, as

$$\mathbf{e}_v = \begin{bmatrix} e_1 & e_2 & \dot{e}_1 & \dot{e}_2 \end{bmatrix}^T.$$

Then, the error dynamics of Eq.(19) can be transformed into the canonical form such as

$$\dot{\mathbf{e}}_v(t) = \Gamma\mathbf{e}_v(t) + B(\mathbf{d}(t) - \mathbf{r}(t)), \tag{20}$$

where Γ and B are 4×4 and 4×2 matrices defined as, respectively

$$\Gamma = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{h^2} & 0 & -\frac{2}{h} & 0 \\ 0 & -\frac{1}{h^2} & 0 & -\frac{2}{h} \end{bmatrix}, \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Expanding the procedure described above to higher order system , the error dynamics of general system as represented in Eq.(18) is represented as a canonical form of Eq.(20) with the r×1 augmented error vector $\mathbf{e}_v$, the r×r matrix Γ, and the r×p matrix B, in which r means total relative degree.

At this point, define the Lyapunov function candidate V as

$$V = \frac{1}{2}\mathbf{e}_v^T P \mathbf{e}_v \tag{21}$$

where the p×p symmetric positive definite matrix P satisfies the Lyapunov equation

$$\Gamma^T P + P\Gamma = -Q, \text{ for } Q>0.$$

The time derivative of V is represented, by use of Eq.(20) and Lyapunov equation, such as

$$\dot{V} = -\frac{1}{2}\mathbf{e}_v^T Q\mathbf{e}_v + \mathbf{e}_v^T PB(\mathbf{d}(t) - \mathbf{r}(t)). \tag{22}$$

In Eq.(22), the term $\mathbf{e}_v^T PB$ is a 1×p vector and defined as a p-raw vector **m** such as

$$\mathbf{m} = [m_1 \cdots m_p] = \mathbf{e}_v^T PB. \tag{23}$$

Then, Eq.(22) is again represented as

$$\dot{V} = -\frac{1}{2}\mathbf{e}_v^T Q\mathbf{e}_v + \sum_{i=1}^{p}\left[m_i d_i(t) - m_i r_i(t)\right]$$

$$\leq -\frac{1}{2}\mathbf{e}_v^T Q\mathbf{e}_v + \sum_{i=1}^{p}\left[|m_i||d_i(t)| - m_i r_i(t)\right]$$

If we set

$$r_i(t) = \text{sgn}(m_i) d_{max,i}, \quad \text{for } i=1,\cdots,p \tag{24}$$

where the function *sgn* is a sign function defined as

$$\text{sgn}(s) = \begin{cases} 1 & \text{if}(s>0) \\ -1 & \text{if}(s<0) \end{cases},$$

then the time derivative of V becomes

$$\dot{V} \leq -\frac{1}{2}\mathbf{e}_v^T Q\mathbf{e}_v. \tag{25}$$

From the above result, we make the following theorem.

Theorem 2: Suppose the nonlinear system with parameter/modeling uncertainty is represented by Eq.(13) and the bound of uncertainty can be acquired by Eq.(14). Then, the control law of Eq.(17) in which the robust control **r**(t) is given by Eq.(24) guarantees the asymptotic output tracking, i.e., $\mathbf{y}(t) \rightarrow \mathbf{y}_d(t)$ as $t \rightarrow \infty$.

Proof: The control law of Eq.(17) and the robust control of Eq.(24) make the time derivative of the Lyapunov function defined as Eq.(21) become Eq.(25). Since Q is positive definite, $\dot{V}$ has a negative or zero value. Therefore, the Lyapunov function V in Eq.(21) is the non-increasing function. If the initial value of Lyapunov function, V(0), is finite, then V(t) is finite for ∀t>0. From Eq.(21), $\mathbf{e}_v$ is also finite for ∀t>0 ($\mathbf{e}_v \in L_\infty$). By examining Eq.(20), $\dot{\mathbf{e}}_v$ is finite ($\dot{\mathbf{e}}_v \in L_\infty$) for ∀t≥0 because $\mathbf{e}_v$ and other terms such as Γ, B, and (**d**(t)-**r**(t)) are finite for ∀t≥0. By integrating both sides of Eq.(25) from t=0 to t=∞, the following inequality is obtained such as

$$\int_0^\infty \mathbf{e}_v(\tau)^T Q\mathbf{e}_v(\tau)d\tau \leq 2\left[V(0) - V(\infty)\right] < \infty.$$

From the above result, the augmented error $\mathbf{e}_v$ is square integrable and hence, $\mathbf{e}_v \in L_2$. The fact that $\mathbf{e}_v \in L_2 \cap L_\infty$ and $\dot{\mathbf{e}}_v \in L_\infty$ concludes, by Barbalat's lemma[7], that $\mathbf{e}_v(t) \rightarrow \mathbf{0}$ as $t \rightarrow \infty$. $\lim_{t\to\infty} \mathbf{e}_v(t) = \mathbf{0}$ means equally $\lim_{t\to\infty} \mathbf{e}(t) = \mathbf{0}$, and this also means that $\mathbf{y}(t) \rightarrow \mathbf{y}_d(t)$ as $t \rightarrow \infty$. Therefore, asymptotic tracking can be guaranteed ■

By theorem 2, the control law of RNPC can drive the system output to track the desired output asymptotically despite of parameter/modeling uncertainty by just knowing the bound of uncertainty as represented in Eq.(14). In the implementation of this control, however, the robust control induces a certain problem due to its discontinuous *sgn* function. This problem is usually called the control chattering[8]. In order to circumvent the control chattering, the robust control is modified as

$$r_i(t) = \begin{cases} \text{sgn}(m_i)d_{max,i} & \text{if}(|m_i| > m_i^{lim}) \\ (m_i / m_i^{lim})d_{max,i} & \text{if}(|m_i| \leq m_i^{lim}) \end{cases}, (i=1,\cdots,p) \tag{26}$$

From Eq.(26), the robust control **r**(t) ($r_i(t)$, i=1,⋯,p) is continuously approximated when the vector **m** dependent on the tracking error resides within the boundary layer determined by m_i^{lim}, i=1,⋯,p. The asymptotic convergence of tracking error is guaranteed for only the region outside the boundary layer and hence, the tracking error is driven into bounded value rather than to zero.

4. Simulations

In order to investigate the tracking performance of RNPC, simulations are performed for the position control of two-link robot manipulator. In the study of Park, et al.[9], RNPC was applied to a SISO system which described the translational motion of an underwater wall-ranging robot (UWR) and the simulation results showed the good control performance of RNPC and also robustness to the uncertainty in the dynamics of UWR motion.

The two-link rigid-body manipulator, which Gao and Hung[8] used for their simulation study, is represented by

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{n}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = \boldsymbol{\tau} + \mathbf{f}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\mathbf{p},t), \tag{27}$$

where **θ**, **n**, **τ**, and **f** are all two column vectors. In Eq.(27), **θ** is the joint angle vector, **p** the uncertain parameter vector, M the positive definite inertia matrix, **n** the sum of the centrifugal, Coriolis, and gravitational forces, **τ** the control input torque, **f** the collection of all uncertainties or disturbances. All dimensions such as mass and length of each link are the same as that in the study of Gao and Hung[8] except one assumption that all mass exist as a point mass at the distal end of each link.

The output of the above system is the joint angle vector. Through the same procedure as in Eqs.(4), the future and desired future outputs at time t+h for the system of Eq.(27) having no uncertainty are expressed as

$$\boldsymbol{\theta}(t+h) = \mathbf{z}(t) + \Lambda(h)M^{-1}\left[\boldsymbol{\tau} - \mathbf{n}\right], \tag{28a}$$

$$\boldsymbol{\theta}_d(t+h) = \mathbf{z}_d(t) + \Lambda(h)\ddot{\boldsymbol{\theta}}_d(t), \tag{28b}$$

where $z_i(t) = \theta_i(t) + h\dot{\theta}_i(t)$,

$z_{d,i}(t) = \theta_{d,i}(t) + h\dot{\theta}_{d,i}(t)$ for i=1,2,

$\ddot{\theta}_d^T = [\ddot{\theta}_{d,1} \quad \ddot{\theta}_{d,2}]$, and the 2×2 diagonal matrix Λ(h) has all diagonal elements of $h^2/2$. The future output tracking error is, then, expressed as

$$\mathbf{e}(t+h) = \mathbf{z}_e(t) + \Lambda M^{-1}[\boldsymbol{\tau} - \mathbf{n}] - \Lambda\ddot{\theta}_d(t) \qquad (29)$$

Differentiating the control objective of Eq.(6) by the control input makes the control input τ(t) become

$$\tau(t) = -\Lambda^{-1}M\mathbf{z}_e(t) + \mathbf{n} + M\ddot{\theta}_d(t) + \nu(t). \qquad (30)$$

In Eq.(30), the auxiliary control ν(t) is involved and this is represented as

$$\nu(t) = \Lambda(h)^{-1}M\mathbf{z}_e(t) - M\bar{\mathbf{z}}_e(t)$$

where $\bar{z}_{e,i} = \frac{1}{h^2}e_i + \frac{2}{h}\dot{e}_i$ (for i=1,2).

In order to cope with the uncertainty **f**, the control input is composed of two controls

$$\tau(t) = \tau_c(t) + \hat{\mathbf{f}}(t), \qquad (31)$$

where τ_c is the control input as in Eq.(30) and $\hat{\mathbf{f}}$ is the robust control. For the robust control, the following uncertainty bound is supposed to be known

$$|f_i| \le f_i^{max}, \forall t \ge 0 \text{ (for i=1,2)}$$

The closed loop dynamics of the system of Eq.(27) with the control of Eq.(31) is

$$\ddot{\mathbf{e}}(t) = -\bar{\mathbf{z}}_e(t) + M^{-1}(\mathbf{f} - \hat{\mathbf{f}}) = -\bar{\mathbf{z}}_e(t) + (\mathbf{f}_M - \hat{\mathbf{f}}_M), \qquad (32)$$

where $\mathbf{f}_M = M^{-1}\mathbf{f}$ and $\hat{\mathbf{f}}_M = M^{-1}\hat{\mathbf{f}}$. $\hat{\mathbf{f}}_M$ is called the transformed robust control.

Since we know the bound of **f**, the bound of $\mathbf{f}_M$ can be obtained, and this is represented as

$$|f_{M,i}| \le f_{M,i}^{max}, \forall t \ge 0 \text{ (for i=1,2)}. \qquad (33)$$

With Eq.(32) and Eq.(33), the robust control is derived through procedures from Eq.(19) to Eq.(24) and the transformed robust control is expressed as

$$\hat{f}_{M,i} = \begin{cases} \text{sgn}(m_i)f_{M,i}^{max} & \text{if}(|m_i| > m_i^{lim}) \\ (m_i / m_i^{lim})f_{M,i}^{max} & \text{if}(|m_i| \le m_i^{lim}) \end{cases}, \text{ (for i=1,}\cdots\text{,p)}.$$

Then, finally, the robust control is

$$\hat{\mathbf{f}} = M\hat{\mathbf{f}}_M.$$

For the simulation, the initial position of the joint angle vector is $\theta_1(0)=\theta_2(0)=10°$ and the desired positions are set by

$$\theta_{d,1} = 45° \text{ and } \theta_{d,2} = 0°, \quad \forall t \ge 0.$$

Fig.1 shows the output tracking results of NPC and NPC with the auxiliary control where h is set to 0.15s for all simulations when the system has no uncertainty. Both controls show satisfactory results. The results of NPC have more fast response time but small overshoots in outputs are observed. The results of RNPC without the robust control for the system with uncertainty are shown in Fig.2. The uncertainty (or disturbance) is given by $f_1=50\sin(4\tau_1 t)$ and $f_2=50\sin(4\tau_2 t)$, which is the exactly same form as given by Gao and Hung. As can be seen in Fig.2, the tracking performance of RNPC without the robust control is very poor and unacceptable. Fig.(3a) shows the output tracking performance of RNPC, in this controller, all m_i^{lim} (i=1,2) are given by 0.001, when the system has the uncertainty. Fig.(3b) shows its control behavior. From Fig.(3a), the control performance is improved remarkably, which concludes that RNPC can cope with the uncertainty very well.

For comparison, one method for synthesizing the sliding mode controller (SMC), suggested by Gao and Hung[8], is applied and the tuning parameters of SMC are given by the same values as in Gao and Hung. Figs.(4) show the results of SMC. From Fig.(3a) and Fig(4a), the control performances of both controllers are similar and, from the comparison of the tracking error trends which is not presented in this paper, RNPC shows the slightly better performance. Moreover, comparing Fig.(3b) with Fig.(4b), the oscillations of control torque of SMC are severe, but RNPC shows very smooth behavior of control torque. Based solely on this simulation results, RNPC has more advantages than SMC suggested by Gao and Hung for position control of robot manipulator with uncertainty.

5. Conclusions

In this paper, a robust nonlinear prediction-type controller is suggested. The basic control law of RNPC is developed based on the pure dynamic model of the nonlinear system whereas existing model predictive controllers are derived for explicit models of linear system. The basic control with the auxiliary control provides stable closed loop dynamics of nonlinear systems irrespective of the order of relative degree. The robust control also involved into the basic control drives the outputs of the nonlinear system with parameter/modeling uncertainty to converge to the boundary region around the desired outputs without requiring any restricted conditions against the controller structure and system model.

In the simulation tests, RNPC is applied to the MIMO nonlinear system, i.e., the position control of robot manipulator with uncertain disturbance. The tracking performance of RNPC is very satisfactory and slightly better than that of sliding mode controller suggested by Gao and Hung. It is, therefore, concluded that this controller can be a good candidate among nonlinear controllers that satisfy the control purposes for uncertain nonlinear systems.

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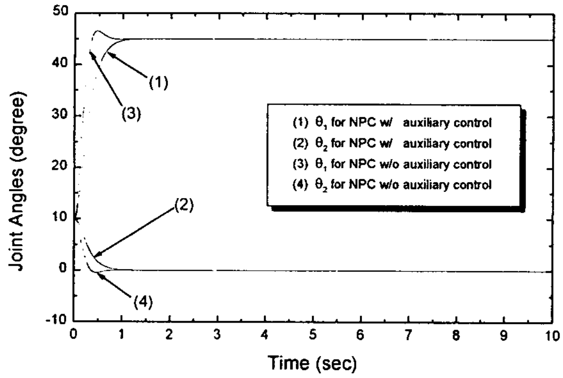

Fig.1 Control results of NPC and NPC with auxiliary control

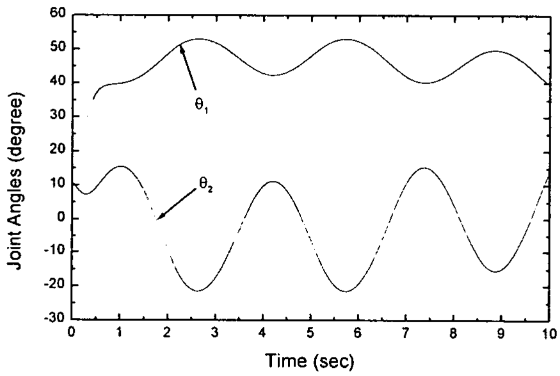

Fig.2 Position control results of RNPC without robust control when disturbances (f_i=50sin(4τ_it), i=1,2) exist

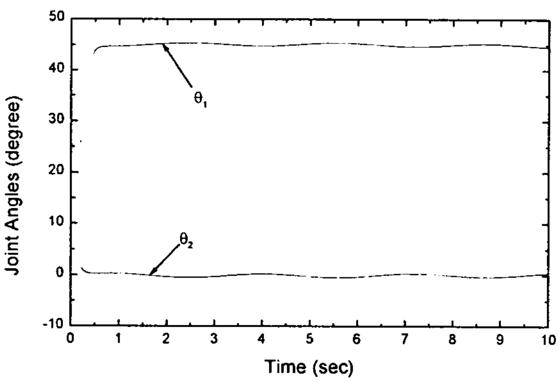

Fig.3(a) Position control results of RNPC when disturbances (f_i=50sin(4τ_it), i=1,2) exist

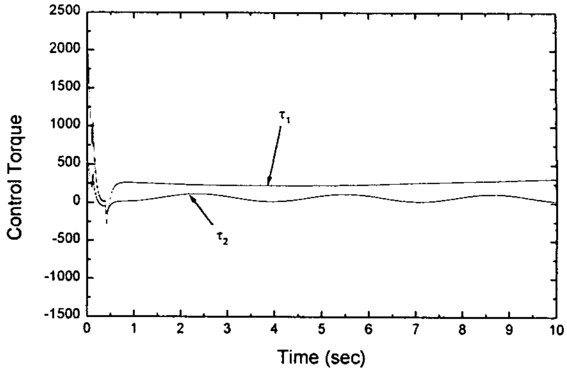

Fig.3(b) Behavior of control torque of RNPC

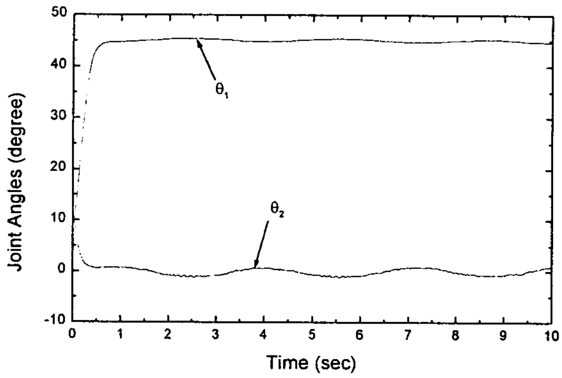

Fig.4(a) Position control results of SMC when disturbances (f_i=50sin(4τ_it), i=1,2) exist

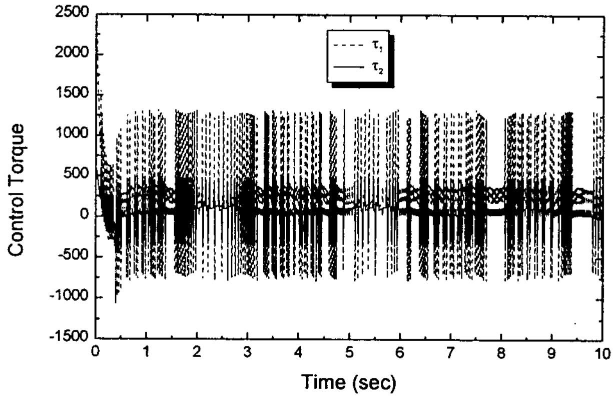

Fig.4(b) Behavior of control torque of SMC