Proceedings of the 13th KACC, October 1998

Robust MILP Model for Scheduling and Design of Multiproduct Batch Processes

Min-ho Suh, Jin-Kwang Bok, Sunwon Park and Tai-yong Lee
Department of Chemical Engineering
Korea Advanced Institute of Science and Technology
Taejon 305-701, Korea
Tel: +82-42-869-3966
Fax: +82-42-869-3910
E-mail: mhsuh@che.kaist.ac.kr

Abstracts

We propose robust MILP model for scheduling and design of multiproduct batch processes in this paper. Recent stochastic modeling approaches considering uncertainty have mainly focused on maximization of expected NPV. Robust model concept is applied to generate solution spectrum in which we can select the best solution based on tradeoff between robustness measure and expected NPV. Robustness measure is represented as penalty term in the objective function, which is Upper Partial Mean of NPV. We can quantify solution robustness by this penalty term and maintain model as MILP form to be computationally efficient. An example illustrates the effectiveness of the proposed model. In many cases sufficient robustness can be guaranteed through a little reduction of expected NPV.

1. Introduction

In recent years we have witnessed an increased interest in the areas of modeling uncertainties in scheduling, design and planning of chemical processes. Uncertain parameters are frequently assumed to be constant scheduling, design and planning of chemical processes for simplicity. But real-world situations are characterized by high degree of uncertainty and we can not cope with future uncertainties such as change in product demands and variations of processing parameter variations using deterministic model. Many researchers have proposed models considering uncertainties[1]-[6],[8]. Ierapetritou and Pistikopoulos[1] proposed a stochastic scheduling and design model to deal with uncertain product demands and processing parameters. Gaussian quadrature approximation was used to calculate expected NPV with the continuous probability function of product demands. Liu and Sahinidis[2] proposed a stochastic planning model with decomposition algorithm. They represented uncertain product demands as discrete random parameters and applied Monte Carlo sampling method. Harding and Floudas[3] proposed a stochastic design model and they applied αBB algorithm to obtain the global optimal solution to NLP problem. Models proposed by above-mentioned three papers could include the uncertain features scheduling, design and planning of chemical processes but objective functions took the form of expected NPV.

In this paper, robustness means that the optimal model solution is near optimal for any realization of the demand scenarios. Markowitz[4] proposed a robust optimization algorithm and Malcolm and Zenios[5] developed a robust optimization model for the expansion of power capacity under uncertain load forecasts. Bok et al.[6] proposed robust investment model for capacity expansion of chemical processing networks.

The objective of this paper is to propose a robust MILP model for scheduling and design of multiproduct batch processes. Model considering scheduling in the stage of design can give higher NPV than the design model with only UIS policy since the overestimation of equipment's size can be avoided by efficient scheduling algorithms. Voudouris and Grossmann[7] proposed an efficient scheduling and design model for multiproduct batch processes but a serious shortcoming of their model was its deterministic nature. We propose a robust model to cope with future uncertainties based on their model[7]. Basic model structure has two-stage stochastic programming framework. Traditional two-stage stochastic optimization model minimizes the sum of the equipment costs in the first stage and the expected operating cost in the second stage. A potential limitation of this approach is that it does not account for the variability of the second stage costs and might lead to solutions where the actual second stage costs are unacceptably high. We use a robustness measure that penalizes second stage costs that are above the expected cost. We can generate solution spectrum by progressively increasing penalty parameter λ in the robust model. We can determine the proper solution considering tradeoff between robustness and expected NPV.

Subrahmanyam et al.[8] proposed a design model of batch plants and also assumed the demand scenario is a particular occurrence or realization of the uncertainty parameter with probability. Scenario based approaches have been prevailed in considering uncertainties[5,6]. Our model assumes that product demands are uncertain and can be predicted as realizable scenarios with their probabilities.

2. Problem Description

The specific features addressed in this paper are as follows:

1. Multiproduct plant
2. Single Product Campaign
3. Zero-wait production policy

Given

1. Production paths and processing times
2. Size factors of each units
3. Product demands and design horizon

Determine

1. Design problem: Equipment sizes
2. Scheduling problem: Production schedule

3. Mathematical Model

The robust MILP model for scheduling and design problem is as follows:

Maximize

(a) Objective function:

= Expected NPV

- Penalty term on variability

$$\Phi = \sum_p w^p NPV^p - \lambda \sum_p w^p \Delta^p \qquad (1)$$

subject to

(b) Deterministic NPV evaluation:

NPV = Revenue from selling the products

- First stage plant cost
- Second stage operating cost

$$NPV^p = R^p(1-tx)(prcoef) \\ -Pc + (Pc/Ny)tx(prcoef) \\ -Oc^p(1-tx)(prcoef) \quad \forall p \qquad (2)$$

In eq.(2) *tx* is the tax rate, *Ny* is the expected life of the plant in years, R^p is the revenue from selling the products and *prcoef* is the present value coefficient with which the future profits are projected to the present. This coeffcient is defined as $prcoef = \{(1+in)^{Ny} - 1)/(in(1+in)^{Ny})\}$ where *in* represents the interest rate.

(c) Positive deviation of p scenario's NPV from the expected NPV:

$$\Delta^p \geq \left(\sum_p w^p NPV^p \right) - NPV^p \quad \forall p \qquad (3)$$

$$\Delta^p \geq 0 \qquad \forall p \qquad (4)$$

Traditional method of representing the variability is using the variance but we use the upper partial mean as our measure of variability of NVPs and it can be justified as follows[9]:

1) Variance is a symmetric risk measure, penalizing costs both above and below the expected NPVs equally. But in chemical processes, Scenarios whose NPV is above the expected NPV should not be penalized so we propose asymmetric variability measure.

2) Using Δ^p, we can generate MILP model to solve computationally easy.

(d) Plant cost and operating cost:

$$Pc = \sum_j \sum_s \hat{c}_{js} y_{js} \qquad (5)$$

$$\hat{c}_{js} = \alpha_j \hat{v}_{js}^{\beta_j} \qquad (6)$$

$$Oc^p = \sum_i \left(\mu_i (\frac{Q_i}{2})(P^p - T_i^p) \right) + \text{mint} \sum_{sv} ord(sv) r_{sv}^p \quad \forall p \qquad (7)$$

The plant cost Pc is the capital investment required for equipment. The cycle operating cost Oc^p are calculated by eq.(7) where the first summation is the inventory cost and the second term is the setup cost paid every time the optimal schedule is repeated.

(e) Scheduling and design related constraints:

$$n_i^p \geq \sum_s \left(\frac{S_{ij} Q_i}{\hat{v}_{js} H} \right) e_{js}^p \quad \forall i,j,p \qquad (8)$$

$$e_{js}^p \leq H y_{js} \qquad \forall j,s,p \qquad (9)$$

$$P^p = \sum_s e_{js}^p \quad \forall j,p \qquad (10)$$

$$H q_i^p = P^p Q_i^p \quad \forall i,p \qquad (11)$$

$$\sum_k NP_{ik}^p = n_i^p \qquad \forall i,p \qquad (12)$$

$$\sum_i NP_{ik}^p = n_k^p \qquad \forall k,p \qquad (13)$$

$$T_i^p = (n_i^p t_{ij'} + (\sum_k NP_{ik}^p Slk_{ikj'})) \quad j' = \text{last machine} \ \forall i,p \qquad (14)$$

$$\sum_i \left(n_i^p t_{ij} + (\sum_k NP_{ik}^p Slk_{ikj}) \right) \leq P^p \quad \forall j,p \qquad (15)$$

$$NP_{ii}^p = n_i^p - 1 \quad \forall i,p \qquad (16)$$

$$H = \sum_{sv} ord(sv) \hat{P}_{sv}^p \qquad \forall p \qquad (17)$$

$$\sum_{sv} \hat{P}_{sv}^p = P^p \qquad \forall p \qquad (18)$$

$$\hat{P}_{sv}^p \leq H r_{sv}^p \qquad \forall sv,p \qquad (19)$$

(f) Integrality constraints:

$$\sum_s y_{js} = 1 \quad \forall j \qquad (20)$$

$$\sum_{sv} r_{sv}^p = 1 \quad \forall p \qquad (21)$$

(g) Variable conditions:

$$y_{js} = \text{binary} \qquad \forall j,s \qquad (22)$$

$$r_{sv}^p = \text{binary} \qquad \forall sv,p \qquad (23)$$

$$n_i^p = \text{integer} \qquad \forall i,p \qquad (24)$$

$$q_i^p,\ NP_{ik}^p,\ P^p \geq 0 \qquad (25)$$

In the model the objective function is defined as expected NPV penalized by Upper Partial Mean (UPM) of each scenario's NPVs. Penalty term plays a key role of limiting the variability of the second operating costs, which represents the solution robustness of the model. Penalty parameter λ enhances solution robustness as it increases. Our objective is to maximize the expected NPV with the variability as small as possible. By settling λ very large, it is possible to make UPM near zero but resulting in substantial loss of expected NPV. Solution spectrum can be obtained by progressively enforcing robustness. We can determine one solution among the alternative solution spectrum based on tradeoff between robustness and expected NPV. We will illustrate the effectiveness of the proposed model through an example.

4. Example

In order to illustrate the effectiveness of the proposed model, consider an example taken from Grossmann and Sargent(1979) for the design of a batch plant that produces two products using three stages with one equipment per stage. In this example only the demands are considered to be uncertain parameters. 3 scenarios are taken to consider the uncertain product demands. The data for this example are shown in Tables 1-3. It is assumed that the equipements are only available in the following set of discrete values {100, 200, 400, 800}. Table 4 shows solutions obtained by deterministic model. Design solutions for each scenario are different from one another and one particular solution may give rise to infeasibility for realization of other scenario. Table 5 shows solution spectrum obtained by the proposed robust model. Expected NPV of the robust model is lower than that of the deterministic model because the robust solution should consider all the realizable demand scenarios. But we can cope with the future demand uncertainty using robust solution that is near optimal for any demand scenario. Solution robustness is manipulated by penalty parameter λ in the model. λ represents conservativeness to the risk of future condition change. A spectrum of solution is generated by progressively enhancing robustness. As λ increases decision makers require larger volume of equipment leading to decrease in the expected NPV. Scheduling and design solution changes as λ increases and UPM decreases to near zero as shown in Table 5. Figure 1 shows the expected NPV and the plant cost vs. UPM. As the UPM increases, the plant cost decreases therefore the expected NPV is decreasing. We can notice the tradeoff between the expected NPV and UPM representing robustness of the solution. Figure 2 shows the percentage reduction in the UPM corresponding to the percentage loss in the expected NPV from the stochastic programming solution. From this plot it can be seen that only 2.18% of expected NPV should be sacrificed to achieve about 98% reduction in the UPM. So we can guarantee a substantial improvement of robustness by a little loss of the expected NPV. A spectrum of solutions obtained by the robust model cannot be obtained by the stochastic model in which the objective function is only the expected NPV.

Note the tradeoff between the expected NPV and robustness. The most important role of the robust model is the generation of spectrum of solution with tradeoff plot as shown in Figure 2. The decision maker can determine a solution according to his preference to the future risk.

5. Conclusions

We proposed a robust MILP model for scheduling and design of mulitiproduct batch processes. The proposed model coped with uncertain future demands and was verified by an application example. We constructed MILP model by defining positive deviation variable that could represent robustness concept appropriately with keeping linearity of the model for the computational efficiency. The example illustrated the generation of solution spectrum in terms of tradeoff between the expected NPV and robustness. The modeling method and the solution analysis with the robustness concept are expected to be applied to other chemical process optimization problems in which tradeoff issues exist. Uncertainty of processing parameters such as size factors and processing times can be considered easily by modifying the proposed robust model.

6. Acknowledgment

The work was supported by the Korea Science and Engineering Foundation (KOSEF) through the Automation Research Center at POSTECH.

References

[1] M. Ierapetritou and E. Pistikopoulos, "Batch Plant Design and Operations under Uncertainty," *Ind. Eng. Chem. Res.*, 35(1996)772-787.

[2] M. Liu and N. Sahinidis, "Optimization in Process Planning under Uncertainty," *Ind. Eng. Chem. Res.*, 35(1996)4154-4165.

[3] S. Harding and C. Floudas, "Global Optimizaiton in Multiproduct and Multipurpose Batch Design under Uncertainty," *Ind. Eng. Chem. Res.*, 36(1997)1644-1664.

[4] H. Markowitz, "Stochastic optimization models in finace," Academic press, New York, 1975.

[5] S. Malcolm and S. Zenios, "Robust optimization for power systems capacity expansion under uncertainty," *J. Oper. Res. Soc.*, 45(1994)1040-1049.

[6] J.-K. Bok, H. Lee and S. Park, "Robust investment model for long-range capacity expansion of chemical processing networks under uncertain demand forecast scenarios," *Comp. Chem. Engng.*, 22(1998)1037-1049.

[7] V. Voudouris and I. Grossmann, "Optimal Synthesis of Multiproduct Batch Plants with Cyclic Scheduling and Inventory Considerations," *Ind. Eng. Chem. Res.*, 32(1993)1962-1980.

[8] S. Subrahrmanyam, J. Pekny, G. Reklaitis, "Design of batch chemical plants under market uncertainty," *Ind. Eng. Chem. Res.*, 33(1994)2688-2701.

[9] S. Ahmed and N. Sahinidis, "Robust Process Planning under uncertainty," *Ind. Eng. Chem. Res.*, 37(1998)1883-1892.

Nomenclature

Indices

i, k=index of product
j=index of processing equipment
p=index of scenario
s=index of discrete sizes for k
sv=index of number of production cycles

Parameters

$\hat{c}_{js}$ = cost of size *s* for potential equipment *j*
H = time horizon in which the demand has to be satisfied
mint = cost in dollars per repetition
Q_i^p = market demand for product *i* in scenario *p*
S_{ij} = size factor of potential equipment *j* for product *i*
Slk_{ikj} = idle time(slack) imposed in equipment *j* when product *k* follows product *i*
t_{ij} = processing time of product *i* at equipment *j*
$\hat{v}_{js}$ = standard volume of size *s* for potential equipment *j*
w^p = probability of scenario *p*
λ = penalty parameter
μ_i = inventory cost per unit mass of inventory of product *I* per unit time
α_j = cost coefficient for equipment *j*
β_j = cost exponent for equipment *j*

Variables

Δ^p = positive deviation from the expected NPV in scenario *p*
n_i^p = number of batches of product *i* during a production cycle
NP_{ik} = number of occurrences of the pair *i-k* in a SPC schedule during a production cycle
Oc^p = operating cost in scenario *p*
Pc = plant cost
P^p = length of production cycle in scenario *p*
q_i^p = amount of product *i* during one production cycle in scenario *p*
r_{sv}^p = binary variable of repetition *sv* in scenario *p*
T_i^p = length of time which is dedicated to the production of product *i*
y_{js} = binary variable of size *s* at equipment *j*

Table 1. Data for Example

Product	Size factor, S_{ij}			Processing time, t_{ij}		
	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
1	2	3	4	8	20	8
2	4	6	3	16	4	4

Table 2. Cost data for Example

Stage	Plant cost coefficient		product	Price
	α_i	β_i		
1	10	0.6	1	5.5
2	10	0.6	2	7.0
3	10	0.6		

Table 3. Demands and Probability data for Example

Scenario	Product		Probability
	1	2	
1	1500	1500	0.3
2	2000	1000	0.5
3	2500	700	0.2

Table 4. Solution of deterministic model for Example

Realizable scenario	Determined Sizes	Expected NPV($)			|deviation|
		Scenario 1	Scenario 2	Scenario 3	
1	{200,400,200}	61930	Infeasible	Infeasible	1480
2	{200,200,400}	Infeasible	59260	Infeasible	1190
3	{200,400,400}	61841	59171	61205	755

Table 5. Solution of Robust model for Example

Penalty parameter, λ	Determined Sizes	Scheduling solution						Expected NPV($)	$\sum_p w^p \Delta^p$
		Cycle time			Number of repeat				
		Sn1	Sn2	Sn3	Sn1	Sn2	Sn3		
0	{200,400,400}	32.258	32.258	32.258	31	31	31	60379	603
1	{200,400,400}	32.258	32.258	32.258	31	31	31	60379	603
2	{200,400,400}	50	32.258	71.429	20	31	14	60013	421
3	{200,400,400}	125	32.258	100	8	31	10	59250	39.6
4	{200,400,400}	125	32.258	100	8	31	10	59250	39.6
5	{200,400,400}	125	32.258	125	8	31	8	59186	24.6
6	{200,400,400}	125	32.258	125	8	31	8	59186	24.6
7	{200,400,400}	125	32.258	125	8	31	8	59186	24.6
8	{400,400,400}	125	32.258	125	8	31	8	59186	24.6
9	{400,400,400}	111.111	32.258	125	9	31	8	59065	10.3
10	{400,400,400}	111.111	32.258	125	9	31	8	59065	10.3
16	{400,400,400}	111.111	32.258	125	9	31	8	59065	10.3
20	{400,400,400}	111.111	32.258	125	9	31	8	59065	10.3

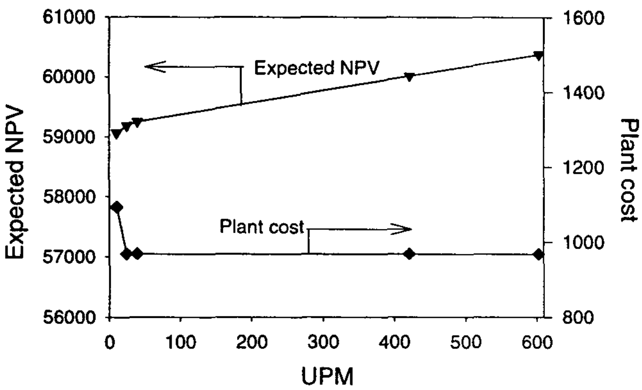

Figure 1. Expected NPV and plant cost for UPM

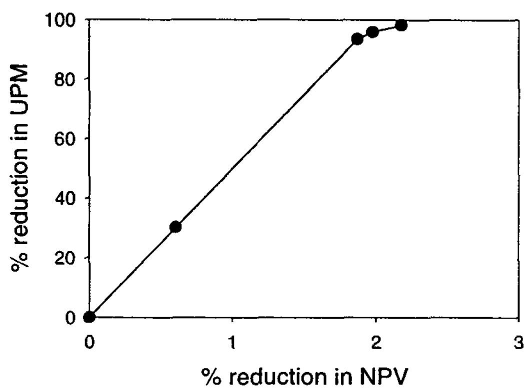

Figure 2. Cost of robustness