ANALYSIS OF THREE DIEMNSIONAL STRUCTURES BY FOLDED PLATE THEORY

-Analysis of Rectangular Plates with In-plane Nx, Ny, Nxy Forces-

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1. INTRODUCTION

Composite materials are, generally, strong in tension. When an element is designed based on tension load, it will have a thin section which is weak against any loading type other than in-plane on-axis tension load. This requires the section modulus to be increased by means of joining thin-walled plates or sandwich panels. Even the one-dimensional element, after the frame is analyzed, requires additional study by the methods explained for thin-walled sections. The thin panels of such sections are weak against the loads normal to these panels. The longitudinal stringers are added between transverse diaphragms to take care of such loads. The diaphragms transmit the loads from stringers to the walls of the beam by means of in-plane shear.

Any curved surface can be considered as continuations of certain types of triangular plates. Therefore, the theory of nonprismatic folded-plates can be applied to any type of shell structures[1].

2. THEORY OF NONPRISMATIC FOLDED PLATE STRUCTURES

Any three-dimensional structural configuration can be approximately represented, with good accuracy, by nonprismatic folded plates, which is composed of sectorial plates. Any sectorial plate problem, with both in and normal to the plane forces, can be solved by the finite difference method, finite element method, and others. The problem then reduces to that of boundaries of two adjoining sectors. Each

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sector may be inclined. Figures 1 and 2 show typical both upper and lower fold lines. Kim, D. H.[1, 2, 3], worked on this problem by finite difference and influence coefficient methods.

The joint forces and displacements at the *n* th fold line can be transformed to a system common to both adjoining plates[4, pp 406-408].

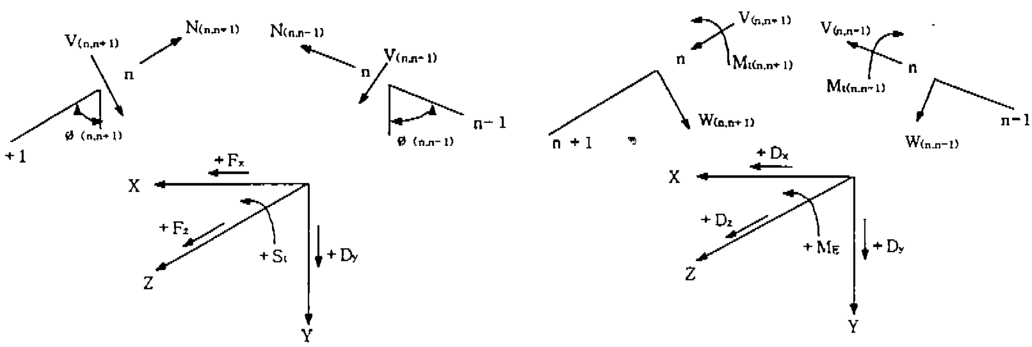
1. When n is at an upper fold line :

$$
\begin{aligned}
F_{x(n,n+1)} &= -N_{t(n,n+1)}\sin\phi_{(n,n+1)} + V_{t(n,n+1)}\cos\phi_{(n,n+1)} \\
F_{y(n,n+1)} &= -N_{t(n,n+1)}\cos\phi_{(n,n+1)} - V_{t(n,n+1)}\sin\phi_{(n,n+1)} \\
F_{x(n,n-1)} &= N_{t(n,n-1)}\sin\phi_{(n,n-1)} + V_{t(n,n-1)}\cos\phi_{(n,n-1)} \\
F_{y(n,n-1)} &= -N_{t(n,n-1)}\cos\phi_{(n,n-1)} + V_{t(n,n-1)}\sin\phi_{(n,n-1)} \\
D_{x(n,n+1)} &= v_{(n,n+1)}\sin\phi_{(n,n+1)} + w_{(n,n+1)}\cos\phi_{(n,n+1)} \\
D_{y(n,n+1)} &= v_{(n,n+1)}\cos\phi_{(n,n+1)} - w_{(n,n+1)}\sin\phi_{(n,n+1)} \\
D_{x(n,n-1)} &= v_{(n,n-1)}\sin\phi_{(n,n-1)} + w_{(n,n-1)}\cos\phi_{(n,n-1)} \\
D_{y(n,n-1)} &= -v_{(n,n-1)}\cos\phi_{(n,n-1)} + w_{(n,n-1)}\sin\phi_{(n,n-1)}
\end{aligned} \tag{1}
$$

where $N_t = \sigma_t h$

2. When n is at a lower fold line :

$$
\begin{aligned}
F_{x(n,n+1)} &= -N_{t(n,n+1)}\sin\phi_{(n,n+1)} - V_{t(n,n+1)}\cos\phi_{(n,n+1)} \\
F_{y(n,n+1)} &= N_{t(n,n+1)}\cos\phi_{(n,n+1)} - V_{t(n,n+1)}\sin\phi_{(n,n+1)} \\
F_{x(n,n-1)} &= N_{t(n,n-1)}\sin\phi_{(n,n-1)} - V_{t(n,n-1)}\cos\phi_{(n,n-1)} \\
F_{y(n,n-1)} &= N_{t(n,n-1)}\cos\phi_{(n,n-1)} + V_{t(n,n-1)}\sin\phi_{(n,n-1)} \\
D_{x(n,n+1)} &= v_{(n,n+1)}\sin\phi_{(n,n+1)} + w_{(n,n+1)}\cos\phi_{(n,n+1)} \\
D_{y(n,n+1)} &= -v_{(n,n+1)}\cos\phi_{(n,n+1)} + w_{(n,n+1)}\sin\phi_{(n,n+1)} \\
D_{x(n,n-1)} &= v_{(n,n-1)}\sin\phi_{(n,n-1)} - w_{(n,n-1)}\cos\phi_{(n,n-1)} \\
D_{y(n,n-1)} &= v_{(n,n-1)}\cos\phi_{(n,n-1)} + w_{(n,n-1)}\sin\phi_{(n,n-1)}
\end{aligned} \tag{2}
$$



Forces Displacements

Figure 1 Notations and sign conventions at the upper fold line.

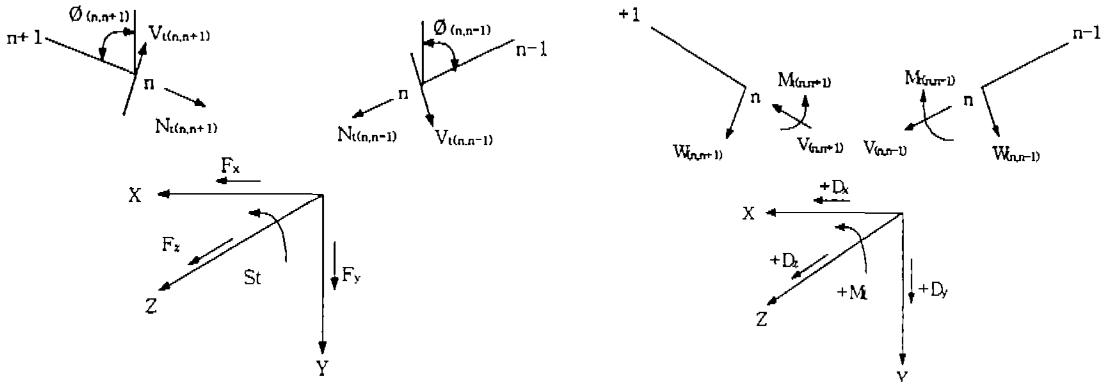


Figure 2. Notations and sign conventios at the lower fold line.

The types of the joint compatibility and joint equilibrium conditions depend on which dependent variables are choosen. If the transverse bending moment, Mt, and the three displacement components, u, v and w, are taken as unknowns. it is necessary to satisfy the slope compatibility condition and the three force equilibrium conditions at each fold as follows.

$$S_{t(n,n+1)} - S_{t(n,n-1)} = 0, \quad F_{xy(n,n+1)} + F_{xy(n,n-1)} = 0,$$
$$F_{x(n,n+1)} + F_{x(n,n-1)} = 0 \quad F_{y(n,n+1)} + F_{y(n,n-1)} = 0 \qquad (3)$$

where $F_{xy} = \tau_{r\theta} h = N_{rt}$.

Since these 'force' expressions are to be written in terms of displacements, the compatibility conditions are qutomatically satisfied. At each fold line, these conditions must be satisfied when the governing differential equations are integrated.

If the finite difference method is used to ingegrte the differential equations, some elaborate work is necessary. A method of solving such a problem was reported by Kim, D. H.[1, 2, 3, 4]. The process of calculation is straightforward. A very high degree of accuracy can be obtained by this method[1, 2].

With the method of analysis as metioned above available, the problem is reduced to solving a plate, either prismatic or non-prismaic, with arbitrary elastic bounday conditions. In this paper, the result of analysis of a rectangular plate, with arbitray elastic boundary conditions is presented. Finite difference method is used for analysis.

3. FINITE DIFFERENCE METHOD (F.D.M.)

The method used in this paper requires the deflection influence surfaces. Since no reliable analytical method is available for the subject problem, F.D.M. is applied to the governing equation of the special orthotropic plates[4, p292].

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}=-q(x,y)+kw+Nx\frac{\partial^2 w}{\partial x^2}+Ny\frac{\partial^2 w}{\partial y^2}+2Nxy\frac{\partial^2 w}{\partial x\partial y} \tag{4}$$

where $D_1=D_{11}$, $D_2=D_{22}$, $D_3=(D_{12}+2D_{66})$.

The number of the pivotal points required, in the case of the order of error Δ^4, where Δ is the mesh size, is five for the central differences. This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, ω, M_x, M_y, are used instead of equation(4)[1, 4].

$$\frac{\partial^2 M_x}{\partial x^2}+4D_{66}\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial M_y}{\partial y^2}=-q(x,y)+kw(x,y)+Nx\frac{\partial^2 w}{\partial x^2}+Ny\frac{\partial^2 w}{\partial y^2}+2Nxy\frac{\partial^2 w}{\partial x\partial y} \tag{5}$$

$$M_x=-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2} \tag{6}$$

$$M_y=-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2} \tag{7}$$

If F.D.M. is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim, D. H.[1, 2, 3] is very efficient to solve such equations.

4. NUMERICAL EXAMINATION

$[A/B/A]_r$ type laminate is considered. The material properties are ;

E_1 = 38.6 GPa,	E_2 = 8.27 GPa,	
ν_{12} = 0.26,	ν_{21} = 0.0557,	a=b=1m.
G_{12} = 4.14 GPa,	t = 0.005×3×3=0.045 m,	

The thickness of a ply is 0.005m. As the r increases, B_{16}, B_{26}, D_{16} and D_{26} stiffnesses decrease and the equations for the special orthotropic plates can be used. For simplicity, it is assumed that A=0°, B=90°, and r=3. Then

D(1,1)=227219 N · m., D(1,2)=1668.02 N · m., D(2,2)=133930.2 N · m. and D(6,6)=31438.12 N · m.

The applied load is the uniform load q=10000 N/m2 plus the wheel loads as shown by Figure 3 and Table 1. The effect of the Nxy on the deflection at the loading points is given in Table 2.

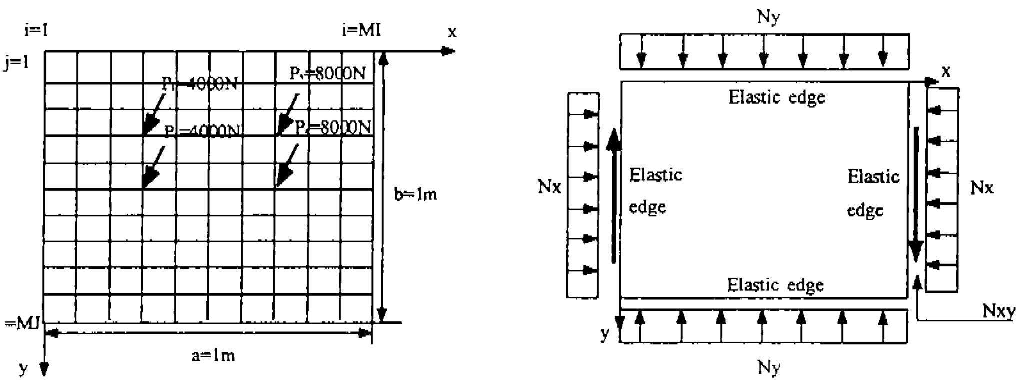


Figure 3. The plate geometry and applied load.

Table 1. Each cases used for numerical example.

Force \ CASE	CASE A	CASE B	CASE C	CASE D
K (N/m²)	9×10^{15}	9×10^{15}	9×10^{15}	9×10^{15}
N_x (N)	1000	1000	1000	1000
N_y (N)	100	100	100	100
N_{xy} (N)	50	100	150	200

Table 2. Deflection of the plate for each case at load points.

(unit : m)

Load Point \ CASE	CASE A	CASE B	CASE C	CASE D
(4,4)	0.281368E-06	0.281368E-06	0.281368E-06	0.281368E-06
(4,6)	0.144080E-06	0.144080E-06	0.144080E-06	0.144080E-06
(8,4)	-0.272102E-07	-0.272102E-07	-0.272102E-07	-0.272102E-07
(8,6)	-0.290340E-07	-0.290340E-07	-0.290340E-07	-0.290340E-07

4. CONCLUSION

One should recall that obtaining the deflection influence coefficients is the first step in design and analysis of a structure. When the plate has concentrated mass or masses, one can simply add these masses to the plate mass and use the same deflection influence surfaces to obtain the natural frequency.

Recently, use of polymeric bridge supports has become quite popular. Unlike the metal hinges and rollers, these polymers behave like elastic support. The actuators for the active control of the bridge, behave, at least partially, as the elastic supports[5].

The finite difference method (F.D.M.) is used to obtain the deflection influence surfaces in this paper. In order to reduce the required number of pivotal points, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, Mx, and My, are used instead of the fourth order partial differential equation for the special orthotropic plate.
If F.D.M. is applied to these equations, the resulting matrix equation is huge in sizes, but the tri-diagonal matrix calculation scheme used by Kim, D. H. is very efficient to solve such problems.
The effect of Nx, Ny, and Nxy on the deflection is thoroughly studied and the result is given in tables to provide a guideline to the practicing engineers.

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